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GATE

Subject : CS 2009 - SOLUTIONS

(Q. NO. 1 – 20) 1 MARKS

- 1. A group G along with an operation '*' i.e. (G,*) has the following properties :
 - i) It is closed
 - ii) It is associative i.e., for all $a, b, c \in G$ we have

 $a^{*}(b^{*}c) = (a^{*}b)^{*}c$

- iii) There exists unique identity element 'e' such that for all $a \in G$ we have $a^*e = e^*a = a$
- iv) There exists a inverse for every element $a \in G$ such that $a^{-1} = b$ when

a * b = b * a = e

Commutativity is not a property of the group. If a group is commutative, it is an Abelian group.

- \therefore (A) is the answer
- 2. A simple connected graph with no odd length cycles is a bipartite graph and a bipartite graph can be coloured using two colours.

The chromatic number of a graph is the minimum number of colors needed to colour the vertices of a graph such that no two adjacent vertices have the same colour.

Thus, the chromatic number of a bipartite graph is 2.

 \therefore (A) is the answer.

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3. Given that the graph is simple, it does not have any self loops or parallel edges. Also it is given that the graph is connected. Hence the degree of all vertices should be from 1 to (n-1). So the degree of atleast two vertices must be same.

 \therefore (B) is the answer.

- 4. A relation R on a set is said to be symmetric if $(x,y) \in R$ then $(y, x) \in R \forall x, y$ in the given set.
 - A relation R on a set is said to be antisymmetric if $(x,y) \in R$ then $(y, x) \notin R \forall x, y$ in the given set unless x = y.

The given relation is not symmetric because $(x,y) \in R$ and $(z, y) \in R$ but $(y, x) \notin$ and $(y, z) \notin R$. R is not antisymmetric because both (x, y) and (z, x) are present in R

 \therefore (D) is the answer.

5. $(1217)_8 = (001010001111)_8$

To convert it into a base 16 numbers, group 4 digits from LSB to MSB

$$\begin{array}{c} 0010 \\ 2 \\ 8 \\ F \end{array}$$

:. $(1217)_8 = (28F)_{16}$ (B) is the answer.

6.

F = AB + C F = (A +C) (B+C) - on simplification F = $\overline{(A+C)(B+C)}$ ∴ F = $\overline{(A+C)(B+C)}$

using only 2 input NOR Gates, we get the following circuit



∴ We require minimum of 3 2-input NOR gates.(B) is the answer

- A CPU generally handles the interrupt only after execution of the current instruction.
 (C) is the answer.
- 8. Belady's anomaly occurs in FIFO page replacement algorithm. It is an abnormal behaviour for some exceptional reference strings. Belady's anomaly states that in FIFO some times when we increase the number of page frames, the number of page faults also increases.

(A) is the answer.

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- 9. A page table must contain page frame number. Virtual page number is used as an index in the page table to get the corresponding page frame number.(B) is the answer.
- 10. Even in the worst case, the selection sort performs O(n) swaps. Thus when swapping is an expensive operation, selection sort is preferred for sorting.(A) is the answer.
- 11. $S \rightarrow aSa|bSb|a|b$

The language generated by the above grammar over $\Sigma = \{a, b\}$ is $\{a, b, aaa, aba, bab, bbb, ababa,\}$ i.e. all odd length palindromes are generated (B) is the answer.

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12. Bellman-Ford single source shortest path algorithm only finds whether any negative weighted cycle is reachable from the source. It cannot find any other negative weighted cycle which is not reachable from the source.

If a graph contains a negative cycle (i.e. a cycle whose edges sum to a negative value) that is reachable from the source, then there is no cheapest path : any path that has a point on the negative cycle can be made cheaper by one more walk around the negative cycle. In such a case, the Bellman Ford algorithm can detect negative cycles and report their existence.

 \therefore (B) is the answer.

13. If a problem π_A belongs to both NP and NP-Hard, then π_A is a NP-Complete problem.



- (C) is the answer.
- The given regular expression describes the set of all strings containing at least two 0's.
 (C) is the answer.
- 15. (A) is true. A minimal DFA is always unique.
 - (B) is true We can convert a NFA to an equivalent PDA.
 - (C) is true CFLs are not closed under complementation. Hence, complement of a CFL is a recursive language. Ching. Excelling. Leading.
 - (D) is false. A non deterministic PDA can never be converted to a deterministic PDA and vice versa.
 - (D) is the answer.

16. # Ram chips needed =
$$\frac{\text{Memory Capacity}}{\text{Chip Size}}$$

= $\frac{256 \text{ KB}}{32 \text{ K} \times 1\text{b}}$
= 64

(C) is the answer

17. Regular expressions are used in lexical analysis phase of a compiler. PDA is used in syntax analysis making use of context free grammar. Data flow analysis is done in code optimization. Registers are allocated to the data in code generation.

(B) is the answer.

18. x = 15. Let x be stored at memory location 1234. f(f, & X) = (5, 1234)

t	f	*f_p
f(4, 1234)	f(4,1234) + *f _ p	f (4, 1234)
5	5+3= 8	5
f (3,1234)	f(3,1234) + * f _ p	f (3, 1234)
3	3 + 2 = 5	3
f (2, 1234)	f (2, 1234) + * f _ p	f (2, 1234)
2	2 + 1 = 3	2
f (1, 1234)	f (1,1234) + * f _ p	f (1, 1234)
	1+ 1=2	

for n = 1, $f_p = 1$ and return 1 Return value is f = 8(B) is the answer.

- The order of coupling between modules (stongest to weakest) is: I-II-III-IV-V (A) is the answer.
- 20. The table formed from the given code will be as follows:

ah	cd	
aU	ef	ah
ik		gn

Number of rows in each column g. Excelling. Leading.

column 1 : 2	(ab, ik)
column 2: 3	(cd, ef, ik)

column 3:2 (cd, gh)

Number of column in each row :

Row	1:2	(ab, cd)
Row	2:3	(ab, ef, gh)
Row	3:2	(ik, gh)
(C) is t	the answer	

21.

value is even. $P(O) = \frac{90}{100} P(E)$... (1) given we know that for a die, P(O) + P(E) = 1... (2) $\therefore \frac{90}{100} P(E) + P(E) = 1$ On solving equations (1) & (2) $P(E) = \frac{10}{19}$ $P(O) = \frac{9}{19}$ and Also it given that the probability of getting any even numbered face is the same. $\therefore P(2) = P(4) = P(6) = \frac{1}{3} \left(\frac{10}{19} \right) = \frac{10}{57}$ P(E / face value > 3) = 0.75 given $\frac{P(E \cap facevalue > 3)}{P(facevalue > 3)} = 0.75$ The face value that is even and greater than 3 is 4 and 6 $\therefore \frac{P(4,6)}{P(facevalue > 3)} = 0.75$ $\therefore P(facevalue > 3) = \frac{0.75}{P(4) + P(6)} \text{ Excelling. Leading.}$ $=\frac{0.75}{\frac{10}{57}+\frac{10}{57}}$ $\therefore P(facevalue>3) = 0.468$

(B) is the answer

22. A generator is an element that generates all the elements in a group when raised to its positive powers.

Consider element a $a^1 - a$

 $a^{1} = a$ $a^{2} = a * a = a$ $a^{3} = a * a * a = a$

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Let P(O) be the probability that the face value is odd and P(E) be the probability that the face

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\therefore a is not a generator
Consider element b
\mathbf{b}^1 = \mathbf{b}
b^2 = b * b = a
b^3 = b^2 * b = a * b = b
b^4 = b^2 * b^2 = a * a = a
c and d are not generated by b
∴ b is not a generator
Consider element c
c^1 = c
c^2 = c * c = b
c^3 = c^2 * c = b * c = d
c^4 = c^3 * c = c^2 * c^2 = a
\therefore c is a generator
Note: If an element is a generator, them its inverse is also a generator. In this group, the identity
element is a as x * a = x \forall x in the element set of the given cyclic group.
To find the inverse of c,
\therefore x * c = e
\therefore x * c = a
Also, d * c = a
                                                                   (H)
 \therefore x = d i.e.
                   c^{-1} = d
                                         SHF
 ∴ d is also a generator.
 c, d are generators.
 (C) is the answer.
 The given statement states that if an ornament is of gold or silver, then it is precious. However
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don't get confused with the conjunction 'and' here as an ornament cannot be both of gold as well

- as silver. Hence the option (B) does not make any sense. (D) is the correct answer.
- 24. The given operation ' \Box ' actually is Q P Q \rightarrow P $\equiv \neg Q \lor P$

 $Q \rightarrow I \equiv \neg Q \lor I$ $\therefore P \Box Q \equiv P \lor \neg Q$ (B) is the answer

25.
$$\int_{0}^{\pi/4} \frac{(1 - \tan x)}{(1 + \tan x)} dx$$
$$= \int_{0}^{\pi/4} \left(\frac{\pi}{4} - x\right) dx$$

23.

$$= \left[\left| -\ell \operatorname{n} \operatorname{sec} \left(\frac{\pi}{4} - x \right) \right| \right]_{0}^{\pi/4}$$
$$= \left[0 - \left(-\ell \operatorname{n} \left| \operatorname{sec} \left(\frac{\pi}{4} \right) \right| \right) \right]$$
$$= \ell \operatorname{n} \sqrt{2}$$
$$= \frac{1}{2} \ell \operatorname{n} 2$$
$$\therefore (D) \text{ is the answer.}$$

26.
$$\neg \forall x (P(x)) \equiv \exists x (\neg P(x))$$

 \therefore (B) is the answer

27. The minimum length of an input string is 3 and the string is 101. Initially, A=0, B=0 On input 1 : A=0, B=1 and output = 0KHAR Now A = 0, B=1On input 0 : A = 1, B=0 and output = 0 Now A=1, B=0On input 1 : A = 0, B = 1 and output = 1which is the desired state (A) is the answer. Coaching. Excelling. Leading. The pipeline will be as follows : 28.



Here if we count the total number of clock cycles needed, we get 23 cycles. (B) is the answer.



: Each set has 4 blocks. Using LRU policy,



30.

		R1 (3)	R2 (2)	R3 (3)	R4 (2)
	t = 0	3	0	1	1
	t = 1	3	0	0	1
	t = 2	1	0	0	0
block P1	t = 3	1	0	0	0
	t = 4	0	0	0	0
unblock P1	t = 5	1	1	0	0
	t = 6	1	1	1	0
	t = 7	1	0	2	0
block P1	t = 8	2	0	2	1
unblock P1	t = 9	2	1	3	0
	t = 10				

At t = 3, the process P1 has to wait because its need is greater than the availability for resource R1. Available R1 = 1 but P1 needs 2 units of R1. So Pl is blocked. Whenever the resource is available P1 is unblocked again.

Similar is the case at t = 8

As we can see that all the processes have completed their execution, there is no possibility of deadlock.

(A) is the answer.

31. Using SSTF Policy,



32. Statement (I) is not necessarily true. Another process may or may not make the transition A immediately.

Statement (II) is true as any process entering the ready state will not affect any other process in the running state.

Statement (III) is true as we have a back transition C from running to ready state i.e. when a higher priority process preempts another low priority running process.

Statement (IV) is false.

 \therefore (C) is the answer.

33. The given solution satisfies mutual exclusion as test-and-set is an atomic operation. Hence, only one process can enter the critical section at any given time.

 \therefore IV is false.

But there is no such ordering possible for the processes to enter a CS.

n > 3

n ≤

 \therefore III is also false

Test - and - set makes sure that deadlock doesn't occur but it does not avoid starvation.

 \therefore II is also false

Only I is true

(A) is the answer.

34. Multilevel page tables are used when the current page table is very large and occupies more space in memory. Multilevel page tables help to reduce the size of traditional page tables in memory.

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(B) is the answer.

35.
$$T(n) = T\left(\frac{n}{3}\right) + cn$$

Using Master's theorem,

$$a = 1, b = 3, k=1, p=0$$

 $b^{k} = 3^{1}$

$$a < b^k$$
 as $1 < 3$

If
$$p \ge 0$$
, $T(n) = \theta (n^k (logn)^p)$

Here p = 0

 $\therefore \quad \mathbf{T}(\mathbf{n}) = \theta \left(\mathbf{n}^1 (\log \mathbf{n})^0 \right)$

$$\therefore$$
 T(n) = θ (n)

(A) is the answer. Coaching. Excelling. Leading.

36. Hash function is : h (k) = k mod 10 Linear probing is used to resolve collision 12 % 10 = 2 8 % 10 = 8 13 % 10 = 3 2 % 10 = 2 — collision \therefore Probe at 2 + 1 = 3 — Again collision \therefore Probe at 3 + 1 = 4 3 % 10 = 3 — Collision Probe at 3 + 1 = 4— Again collision

Harsh Table

0	
1	
2	12
3	13
4	2
5	3
6	23
7	5
8	18
9	15

... Probe at 4 + 1 = 5 $23\% \ 10 = 3$ \rightarrow collision Successive collisions at locations 4 and 5 as well ... Probe at 5 + 1 = 6 $5\% \ 10 = 5$ \rightarrow collision ... Probe at 5 + 1 = 6 \rightarrow Collision again ... Probe at 6 + 1 = 7 $15\% \ 10 = 5$ \rightarrow Collision Successive collisions at locations 6, 7, 8 as well ... Probe at 8 + 1 = 9Option (C) is the resultant hash table (C) is the answer.

37. An AVL tree is a height balanced binary search tree with a balance factor BF for each node. Balance factor of an AVL tree is calculated as

> BF = |L-R| and BF should be less than or equal to 1. where L - height of the left sub tree R - height of the right sub tree

For 7 nodes, AVL Tree will be as follows:



Also the property of balance factor is satisfied for all nodes. For leaf nodes, BF is 0 whereas for rest of the nodes, it is 1

Maximum height of the AVL tree is 3 (longest path from root node to leaf node).

(B) is the answer.

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38. In Kruskal's algorithm, we sort the edges in increasing order of their edge weights. we add these edges one by one into the minimum spanning tree and reject if it forms a cycle.

(D) is the incorrect sequence of edges added to the MST as edge (b, c) whose weight is 4 is added before the edge (a,c) whose weight is 3.

 \therefore (D) is the answer

39. As per the given conditions, the recurrence relation becomes :

$$T(n) = T\left(\frac{n}{4}\right) + T\left(\frac{3n}{4}\right) + O(n)$$

On solving, the solution converges to T (n) = θ (nlogn) It is similar to the recurrence relation T (n) = 2T (n/2) + O(n)

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and T (n) = \theta (nlogn)
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(B) is the answer.

Note :

In quick sort :

(i) When $\left(\frac{n}{k}\right)$ th element is selected as pivot, the recurrence relation becomes

$$T(n) = T\left(\frac{n}{k}\right) + T\left(\frac{(k-1)n}{k}\right) + O(n)$$

and solution converges to

 $T(n) = \theta (n \log n)$

(ii) When (n-k)th element is selected as pivot, the recurrence relation becomes T(n) = T(n-k) + T(k) + O(n)and the solution converges to $T(n) = O(n^2)$ which becomes the worst case of quick sort.

40.
$$L_1 = \{a^m b^m c a^n b^n \mid m, n \ge 0\}$$

L₁ is a CFL For each a, pop one b for c, no operation Then agian for each a, pop one b L₂ = { $a^i b^j c^k | i, j, k \ge 0$ } L₂ is a regular language The regular expression for L₂ is $a^* b^* c^*$ Now L = L₁ \cap L₂ = CFL \cap Regular language L = CFL

 \therefore L = CFL

CFLs are intersected with regular languages

 \therefore L is a CFL

(C) is the answer.

41. The given DFA accepts the language

 $L=\{00,100, 1100, 10100, 100100, 000, 1000, \dots...\}$

i.e. it accepts all strings over $\{0,1\}$ that end with 00

(C) is the answer

42. (I) is true

(II) is false. To implement recursion, we need dynanic memory allocation.

(III) is false. L attributed definitions are such where all inherited attributes' values depend only on :

• inherited attributes of parent and

• attributes of left siblings.

And these can be evaluated in bottom - up manner

(IV) is true

(B) is the answer.



 \therefore Only schedules S2 and S3 are conflict serializable

(B) is the answer

44. Here the question asks to count the number of leaf node splits in a B+ tree.If we follow left biasing, we get a total of 5 splits out of which 4 are leaf node splits.If we follow right biasing, we get a total of 4 splits out of which 3 are leaf node splits.

Since the question clearly states the MAXIMUM number of leaf node splits, we will follow left biasing to get 4 as the answer.

Following are the 4 leaf node splits

- 1. After inserting 6
- 2. After inserting 4
- 3. After inserting 2 (there will be one internal node split and one leaf node split)
- 4. After inserting 1



Insert 2 (2 splits) 1 internal node split + 1 leaf node split.



- \therefore We have 4 leaf node splits using left biasing
- (C) is the answer.
- 45. (I) describes the division operation in relational algebra whereas (II) describes the equivalent division operation in tuple relational calculus.
 - (A) is the answer
 - RSA follows public key cryptography (I) is true For encryption, Cypher Text = (Plain text)^e mod n S Leading. For decryption, Plain text = (Cypher text)^d mod n
 - (III) is true

46.

as $ed = 1 \mod \phi(n)$ or $d^{-1} = e \mod \phi(n)$

- (B) is the answer
- 47. The maximum packet lifetime is 64sec. So after every 64 sec, a new sequence number is generated.

:. Minimum permissible rate
$$= \frac{1}{64}$$

= 0.015 per second

(A) is the answer.

- 48. For G(x) to detect odd bit erross, (1 + x) must be a factor of G (x)(C) is the answer
- 49. A data flow diagram (DFD) is a graphical representation of the "flow" of data through an information system, modelling its process aspects. Data flow diagrams are also known as bubble charts. Statements (I) and (III) are true
 (C) is the answer
- 50. Statement (I) is false.Statement (II) is true.Statement (III) is true.(B) is the answer.
- 51. The smallest division in a magnetic disk that carries information is a sector. Sectors are combined together to form a track. Tracks which lie on the same dimension of the platters form a cylinder. Each platter of the disk has two surfaces (upper surface and lower surface). A read write head is used to access the disk. To access a particular sector of a track, the head has to reach the track whose sector is to be accessed and wait for rotation of the platter so that the required sector comes under it.

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The given address is <400, 16, 29>
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400 is the cylinder number, 16 is the surface number and 29 is the sector number.

The disk has

- 63 sectors per track
- 20 surfaces (10 platters * 2 (each platter has 2 surfaces))
- 1000 cylinclers

So the address will correspond to the sector number

 $400 \times 20 \times 63 + 16 \times 63 + 29 = 505037$

(C) is the answer. Coaching. Excelling. Leading.

52. (A) corresponds to the sector number

0×20×63 +15×63+31=976

(B) corresponds to the sector number

 $0 \times 20 \times 63 + 16 \times 63 + 30 = 1038$

(C) corresponds to the sector number

 $0 \times 20 \times 63 + 16 \times 63 + 31 = 1039$

(D) corresponds to the sector number

 $0 \times 20 \times 63 + 17 \times 63 + 31 = 1102$

We want the address of 1039th sector which is given by option (C)

 \therefore (C) is the answer

53. Given two sequences X [m] an Y [n] of lenghts m and n respectively, we compute their longest common subsequence (LCS) *l* (m,n) using dynamic programming as follows :

 $l (i, j) = 0, \text{ if either } i = 0 \text{ or } j = 0; // \text{ if either of the sequences are empty} = 1 + l (i - 1, j - 1); \qquad \text{if } i, j > 0 \text{ and } X [i - 1] = Y [j - 1] = \max (l (i - 1, j), l (i, j - 1)); \qquad \text{if } i, j > 0 \text{ and } X [i - 1] ! = Y [j - 1] (C) \text{ is the answer.}$

54. (B) is the answer

(D) is false because it states that L[2,3] to be computed before L[1,4] which is not necessarily true if we use Row Major Order.

55. SELECT P.pid FROM Parts P WHERE P. color<> 'blue' It selects the pids of all non-blue parts.

SELECT C.sid FROM Catalog C WHERE C.pid NOT IN It selects all the suppliers who have supplied a blue part.

SELECT S.sname FROM Suppliers S WHERE S. Sid NOT IN It selects the names of suppliers who have not supplied any blue parts.

So none of the options are true.

Even option (D) is false as it selects all the suppliers who have not supplied 'ONLY' blue parts As a side note, the given query also selects suppliers who have not supplied any parts.

56. Since it is given that (sname, city) forms a key for the suppliers relation and no other functional dependencies are implied other than those implied other than those implied by primary and candidate keys.

:. We have only the following functional dependency:

 $\{\text{sname, city} \rightarrow \text{sid, street} \}$

Candidate key is (sname, city)

Suppliers is in 2NF as no proper subset of candidate key functionally determines a non-key attribute

Suppliers is in 3NF as no non-key attribute functionally determines another non-key attribute.

Suppliers is in BCNF as we have only one candidate key and he following candition does not hold:

Proper subset of one candidate key Proper subset of another candidate key

... The suppliers relation is in BCNF

(A) is the answer.

57. Transmission delay (T_t) = $\frac{L}{BW}$ = $\frac{1000b}{10^6 bps}$ = $10^{-3} \sec$ Propagation delay (P_t) = 25 msec

i.e. 25 frames can be transmitted by the sender before the first frame reaches the receiver

bits to represent 25 frames = $\log_2 25$

= 5 bits

 \therefore (D) is the answer

58. WS = 2^i = 2^5

=

32

 \therefore 32 frames are sent \therefore Tt for 32 frames = 32 msec RTT is the time between which a frame is sent and its acknowledgement is received.

RTT =
$$T_t$$
 (for frame) t + P_t (for frame) + T_t (for ACK) + P_t (for ACK)
= 1+25+1+25 {Assuming frame size for Biggy backing is also 1000 bits}

 $\therefore \text{ Waiting time} = 52 - 32$ = 20 msec

as for the 32 msec, the sender was transmitting frames.



(C) is the answer

59. For a binary max - heap, we should have Every parent \geq its children



(C) is the answer.

60. The obtained max heap is



(D) is the answer

 \setminus