MATHEMATICS
FOR
Junior Secondary School

2

AKADALEARN

JSS2

MATHEMATICS

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FIRST TERM
MATHEMATICS
JSS2

<u>Week 1 & 2</u>

Topic: WHOLE NUMBERS

Factors and Prime factors (revision)

40 ÷ 8 = 5 and 40 ÷ 5 = 8

8 and 5 divide into 40 without remainder.

8 and 5 are **factors** of 40.

A **prime number** has only two factors, itself and 1, 2, 3, 5, 7, 11, 13, ... are prime numbers.

1 is not a prime number.

Standard form

Standard form is a way of writing down very large or very small numbers easily. $10^3 = 1000$, so $4 \times 10^3 = 4000$. So 4000 can be written as 4×10^3 . This idea can be used to write even larger numbers down easily in standard form.

Small numbers can also be written in standard form. However, instead of the index being positive (in the above **<u>Example</u>**, the index was 3), it will be negative.

The rules when writing a number in standard form is that first you write down a number between 1 and 10, then you write × 10(to the power of a number).

<u>Example</u>

Write 81 900 000 000 000 in standard form: 81 900 000 000 000 = 8.19×10^{13}

It's 10^{13} because the decimal point has been moved 13 places to the left to get the number to be 8.19

<u>Example</u>

Write 0.000 001 2 in standard form:

0.000 001 2 = 1.2×10^{-6}

It's 10^{-6} because the decimal point has been moved 6 places to the right to get the number to be 1.2

On a calculator, you usually enter a number in standard form as follows: Type in the first number (the one between 1 and 10). Press EXP . Type in the power to which the 10 is risen.

Manipulation in Standard Form

This is best explained with an **Example**:

Example

The number p written in standard form is 8×10^5

The number q written in standard form is 5×10^{-2}

Calculate p × q. Give your answer in standard form.

Multiply the two first bits of the numbers together and the two second bits together:

$8 \times 5 \times 10^5 \times 10^{-2}$

$= 40 \times 10^3$ (Remember $10^5 \times 10^{-2} = 10^3$)

The question asks for the answer in standard form, but this is not standard form because the first part (the 40) should be a number between 1 and 10.

$= 4 \times 10^4$

Calculate p ÷ q.

Give your answer in standard form.

This time, divide the two first bits of the standard forms. Divide the two second bits. $(8 \div 5) \times (10^5 \div 10^{-2}) = 1.6 \times 10^7$

Express the following fractions in standard form

a. 0.000 07

b. 0.000 000 022

c. 0.075

Whole Numbers in Standard Forms

A number is said to be in standard form if it is re-written as a figure between 1 and 10 and then multiplied by a power of ten without changing its original value. i.e. $P \times 10^x$.

Note

When expressing numbers in standard form, point are either carried from the left hand side (LHS) or right hand side (RHS) of it. While the point carried from the left hand side turns negative the point from right hand side turns positive.

Another very important thing to note is that when expressing either decimal number or whole number in standard form, points are carried until they are between the 1st and 2nd value. **Example 1**; Express 263,000,000 in standard form.

Solution

The value above is a whole number so you carry point (imaginary) from (RHS) towards (LHS). Let's do it!
= 2.63×10^8. Answer

Example 2; Express 0.0006927 in standard form.

Solution

The value above is a decimal number so points are carried from the left hand side (LHS) – (RHS).

= 6.927×10^{-4}.

Example 3; Express 34.694 in standard form.

Solution

Even though the value above is also a decimal number, points here will be carried from (RHS) – (LHS).

The result will be; 3.4694×10^1.

Ordinary Form

Ordinary form is the opposite of standard form. When you are expressing numbers in ordinary form it means going the other way round to get your answer.

For **Example**; Express 3.4694×10^1 in ordinary form.

Solution

You are going to carry the point once from (LHS) – (RHS). Why? Because 10 is raised to power of 1.

$3.4694 \times 10^1 = 34.694$. (Answer)

Assessment

1. Express the following in standard form;

(a) 54000

(b) 0.0003164

(c) 263.478

(d) 0.00000364

(e) 600.84

2. Find the value of A if $0.000046 = A \times 10^{-5}$

3. What is the value of n if $0.0000094 = 9.4 \times 10^n$?

4. Express the following in ordinary form;

(a) 2.83×10^{8}

(b) 4.765×10^{-3}

(c) 1.278×10^{2}

(d) 9.87×10^{-9}

Rounding off numbers

You have learnt how to round off numbers to the nearest thousand, hundred, tens, etc.

Remember that the digits 1, 2, 3, 4 are rounded down and the digits 5, 6, 7, 8, 9 are rounded up.

Round off the following to the nearest

i. thousand ii. hundred iii. ten

a. 12 835

b. 46 926

c. 28 006

Significant figures

Significant figures begin from the first non-zero digit at the left of a number. As before, the digits 5, 6, 7, 8, 9 are rounded up and 1, 2, 3, 4 are rounded down. Digits should be written with their correct place value.

Read the following **Example**s carefully.

a. 546.53 = 500 to 1 significant figure (s.f.)

543.52 = 550 to 2 s.f.

543.52 = 547 to 3 s.f.

546.52 = 546.5 to 4 s.f.

b. 8.0296 = 8 to 1 s.f.

8.0296 = 8.0 to 2 s.f.

In this case the zero must be given after the decimal point. It is important.

8.0296 = 8.03 to 3 s.f.

8.0296 = 8.030 to 4 s.f.

Notice that the fourth digit is zero. It is significant and must be written

Decimal Places

Decimal places are counted from the decimal point. Zeros after the point are significant and are also counted. Digits are rounded up or down as before. Place value must be kept.

Read the following **Example**s carefully.

a. 14.902 8= 14.9 to 1 decimal place (d.p.)

b. 14.902 8 = 14.90 to 2 d.p.

3. 14.902 8 = 14.903 to 3 d.p.

Prime Factors

Prime Factors

Let's look at the number 32. We can multiply 4 times 8 to get 32, so 4 and 8 are factors of 32. But 4 and 8 are like the frosting and the cream in the donut – they are parts, but they are not the smallest possible parts. The numbers 4 and 8 can each be divided evenly by another number – the number 2. The 2 is a **prime number** – a number divisible only by 1 and itself. That means 2 is a prime factor of 32. A **prime factor** is a factor that is also a prime number. In other words, it is one of the smallest components of the number, and it can only be divided by 1 and by itself.

A factor that is a prime number: one of the prime numbers that, when multiplied, give the original number.

Example: The prime factors of 15 are 3 and 5 (3×5=15, and 3 and 5 are prime numbers). In number theory, the **prime factors** of a positive integer are the prime numbers that divide that integer exactly. The prime factorization of a positive integer is a list of the integer's prime factors, together with their multiplicities; the process of determining these factors is called integer factorization. A Prime Number can be divided evenly **only** by 1 or itself. And it must be a whole number greater than 1.

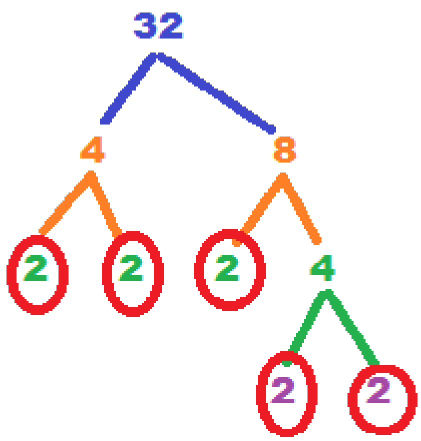
The first few prime numbers are: 2, 3, 5, 7, 11, 13, and 17 ..., and we have a prime number chart if you need more.

Factors

"Factors" are the numbers you multiply together to get another number:

Factor Trees

When trying to determine the basic ingredients of a donut, we look at the recipe. When trying to determine the basic ingredients of a number, the prime factors, we can make a factor tree. Look at this picture of the factor tree for 32.



- The first branch shows that 32 is equal to 4 times 8.
- The next branch shows that 4 is equal to 2 times 2. Both these numbers are prime numbers, so this branch is finished.
- The 8 is broken into 2 times 4. Since 2 is a prime number, its branch is done.
- The 4 is broken into 2 times 2. Now all the numbers on the ends are prime.
- When we look at the circled numbers, we see all the prime factors of 32.
- The prime factorization of 32 is 2 x 2 x 2 x 2 x2.

Prime Factorization

"Prime Factorization" is finding **which prime numbers** multiply together to make the original number.

Here are some **Example**s:

Example 1: What are the prime factors of 12 ?

It is best to start working from the smallest prime number, which is 2, so let's check: 12 ÷ 2 = 6

Yes, it divided evenly by 2. We have taken the first step!

But 6 is not a prime number, so we need to go further. Let's try 2 again: 6 ÷ 2 = 3

Yes, that worked also. And 3 **is** a prime number, so we have the answer: **12 = 2 × 2 × 3**

As you can see, **every factor** is a **prime number**, so the answer must be right.

Note: **12 = 2 × 2 × 3** can also be written using exponents as **12 =** 2^2 **× 3**

Example 2: What is the prime factorization of 147 ?

Can we divide 147 evenly by 2?

147 ÷ 2 = 73½

No it can't. The answer should be a whole number, and 73½ is not.

Let's try the next prime number, 3:

147 ÷ 3 = 49

That worked, now we try factoring 49, and find that 7 is the smallest prime number that works:

49 ÷ 7 = 7

And that is as far as we need to go, because all the factors are prime numbers.

147 = 3 × 7 × 7

(or **147 = 3 × 7^2** using exponents)

Example 3: What is the prime factorization of 17 ?

Hang on ... **17 is a Prime Number**.

So that is as far as we can go. **17 = 17**

Another Method

We showed you how to do the factorization by starting at the smallest prime and working upwards.

But sometimes it is easier to break a number down into **any factors** you can ... then work those factor down to primes.

Example: What are the prime factors of 90 ?

Break 90 into 9 × 10

- The prime factors of 9 are **3 and 3**
- The prime factors of 10 are **2 and 5**

So the prime factors of 90 are **3, 3, 2 and 5**

Perfect Squares

In **mathematics**, a **square** number or **perfect square** is an integer that is the **square** of an integer; in other words, it is the product of some integer with itself. For **Example**, 9 is a **square**number, since it can be written as 3 × 3. **Square** numbers are non-negative.

Examples of Perfect Square

16 can be expressed as a perfect square as 4 x 4 (the product of 2 equal integers)

36 can be expressed as a perfect square as 6 x 6

81 can be expressed as a perfect square as 9 x 9

169 can be expressed as a perfect square as 13 x 13

Squares and square roots

Square roots

$7^2 = 7 \times 7 = 49$.

In words ‘ the **square** of 7 is 49’. We can turn this statement round and say. ‘the **square root** of 49 is 7’.

In symbols, $\sqrt{49} = 7$. The symbol √ means *the square root of*.

To find the square root of a number, first find its factors.

Example

Find $\sqrt{11\,025}$.

Method: Try the prime numbers 2, 3, 5. 7, …

Working:

3	11 025
3	3 675
5	1 225
5	245
7	49
7	7
	1

$11\,025 = 3^2 \times 5^2 \times 7^2$

$= (3 \times 5 \times 7) \times (3 \times 5 \times 7)$

$= 105 \times 105$

Thus $\sqrt{11\,025} = 105$

It is not always necessary to write a number in its prime factors.

Example

$\sqrt{6\,400}$

$6400 = 64 \times 100$

$= 8^2 \times 10^2$

Thus $\sqrt{6\,400} = 8 \times 10 = 80$

The rules for divisibility can be useful when finding square root.

Assessment

Find by factors the square roots of the following:

1. 225
2. 194
3. 342
4. 484

Week 3

Topic: LCM and HCF of Whole Numbers

Common factors

The number 12, 21 and 33 are all divisible by 3. We say that 3 is a **common factor** of 12, 21 33.

There may be more than one common factor of a set of numbers. For **Example**, both 2 and 7 are common factors of 28, 42 and 70. Since 2 and 7 are common factors and are both prime numbers, then 14 (= 2 x 7) must also be a common factors of the set of numbers.

1 is a common factor of all numbers.

Lowest Common Multiples (LCM)

L.C.M is least common multiple, the smallest number which is exactly divisible by all the given numbers

There are two methods to find L.C.M of given numbers, they are:

- Prime factorization method.
- Division Method.

How to find L.C.M of given numbers by prime factorization method?

Factorization

Follow the steps below to find L.C.M of given numbers by prime factorization method.

1. Express the given numbers as product of their prime factors.
2. Find highest index in all the prime factors of given numbers.
3. The product of all the prime factors with respective highest indices is the L.C.M of given numbers.

Example:

L.C.M of 14, 42, 36

1. Express the numbers as product of prime factors.14 = 2*736 = 3^2 *2^2 42 = 2*3*7
2. The highest index of 2, 3, 7 are 2, 2, 1 respectively
3. The product of all the prime factors with the respective highest indices

Example.

Ex: L.C.M of 12,98,188

```
2|12,98,188
2|6,49,94
  |3,49,47
```

The product of divisors and remaining numbers = 2*2*3*49*47*= 27636

Hence, the L.C.M of 12,98,188 = 27636

The least common multiple, or LCM, is another number that's useful in solving many math problems. Let's find the LCM of 30 and 45. One way to find the least common multiple of two numbers is to first list the prime factors of each number.

30 = 2 × 3 × 5
45 = 3 × 3 × 5

Then multiply each factor the greatest number of times it occurs in either number. If the same factor occurs more than once in both numbers, you multiply the factor the greatest number of times it occurs.

Then multiply each factor the greatest number of times it occurs in either number. If the same factor occurs more than once in both numbers, you multiply the factor the greatest number of times it occurs.

2: one occurrence
3: two occurrences
5: one occurrence
*2 × 3 × 3 × 5 = 90 <— **LCM***

After you've calculated a least common multiple, always check to be sure your answer can be divided evenly by both numbers.

EXAMPLES

Find the LCM of these sets of numbers.

3, 9, 21
Solution: List the prime factors of each.
3: 3
9: 3 × 3
21: 3 × 7
Multiply each factor the greatest number of times it occurs in any of the numbers. 9 has two 3s, and 21 has one 7, so we multiply 3 two times, and 7 once. This gives us 63, the smallest number that can be divided evenly by 3, 9, and 21. We check our work by verifying that 63 can be divided evenly by 3, 9, and 21.

12, 80
Solution: List the prime factors of each.

12: 2 × 2 × 3
80: 2 × 2 × 2 × 2 × 5 = 80
Multiply each factor the greatest number of times it occurs in either number. 12 has one 3, and 80 has four 2's and one 5, so we multiply 2 four times, 3 once, and five once. This gives us 240, the smallest number that can be divided by both 12 and 80. We check our work by verifying that 240 can be divided by both 12 and 80.

30, 60, 90 are all common multiples of 6, 10 and 15. 30 is the lowest number that 6, 10 and 15 will divide into. We say that 30 is the lowest common multiple of 6, 10 and 15.

The LCM of 4, 5 and 6 is 60 (not 120).

a. Express the number as a product of prime factors;

b. Find the lowest product of factors which contains all the prime factors of the numbers.

Example

Find the LCM of 8, 9 and 12.

8 = 2 x 2 x 2

Any multiple of 8 must contain 2 x 2 x 2.

9 = 3 x 3

Any multiple of 9 must contain 3 x 3.

12 = 2 x 2 x 3

Any multiple of 12 must contain 2 x 2 x 3.

The lowest product containing all the three is

2 x 2 x 2 x 3 x 3

The LCM of 8, 9 and 12 is 2 x 2 x 2 x 3 x 3 = 72

Assessment

Find the LCM of the following:

a. 4 and 6

b. 6 and 8

c. 9 and 12

d. 8, 10 and 15

e. 10, 12 and 15

HCF of Whole Numbers

Rules of divisibility

Table 1.2 gives some rules for divisors of whole numbers.

Any whole number is exactly divisible by …

2 if its last digit is even or 0

3 if the sum of its digit is divisible by 3

4 if its last two digits form a number divisible by 4

5 if its last digit is five or zero

6if its last digit is even and the sum of its digits is divisible by 3

8 if its lat three digits forma number divisible by 8

9if the sum of its digit is divisible by 9

10 if its last digit is 0

Table

There is no easy rule for division by 7.

Notice the following:

a. If a number m is divisible by another number n, m is also divisible by the factors of n. For **Example**, a number divisible by 8 is also divisible by 2 and 4.

b. If a number is divisible by two or more numbers, it is also divisible by the LCM of these numbers. For **Example**, a number divisible by both 6 and 9 is also divisible by 18, 18 is the LCM of 6 and 9.

Example

Test the following numbers to see which are exactly divisible by 9. a. 51 066 b. 9 039

Solution

a. 5 + 1 + 0 + 6 + 6 = 18

18 is divisible by 9.

Thus 51 066 is divisible by 9.

b. 9 + 0 + 3 + 9 = 21

21 is not divisible by 9.

Thus 9 039 is not divisible by 9.

Highest Common Factor (HCF)

2, 7 and 14 are common factors of 28, 42 and 70; 14 is the greatest of three common factors. We say that 14 is the **highest common factor** of 28, 42 and 70.

To find the HCF of a set of numbers:

Express the number as a product of prime factors;

b. Find the common prime factors

c. Multiply the current prime factor together to give the HCF.

<u>Example</u>

Find the HCF of 18, 24 and 42.

18 = 2 x 3 x 3

24 = 2 x 2 x 2 3

42 = 2 x 3 x 7

The common prime factors are 2 and 3.

The HCF = 2 x 3 = 6.

Find the HCF of 216

2 | 216

2 | 108

2 | 54

3 | 27

3 | 9

3 | 3

....

1   1

2 | 288

2 | 144

2 | 72

2 | 36

2 | 18

3 | 9

3 | 3

....

1   1

In index notation

$216 = 2^3 x 3^3$

$288 = 2^5 x 3^2$

2^3 is the lowest power of two contained in the two numbers. Thus the HCF contains 2^3.

3^2 is the lowest power of 3 contained in the tow numbers. The HCF contains 3^2.

$216 = (2^3 x 3^3) x 3$

$288 = (2^2 x 3^3) x 2^2$

The HCF = $2^2 x 3^3$ = 8 x 6 = 72.

Examples on highest common factor (H.C.F) are solved here step by step.

Let's follow 10 **Example**s on highest common factor (H.C.F).

1. Find the highest common factor (H.C.F) of 15 (Fifteen) and 35 (Thirty five).

   Solution:

   Factors of 15 (Fifteen) = 1, 3, 5 and 15.

   Factors of 35 (Thirty five) = 1, 5, 7 and 35.

   Therefore, common factor of 15 (Fifteen) and 35 (Thirty five) = 1 and 5.

   Highest common factor (H.C.F) of 15 (Fifteen) and 35 (Thirty five) = 5.

2. Find the highest common factor (H.C.F) of 21 (Twenty one) and 35 (Thirty five).

   Solution:

   Factors of 21 (Twenty one) = 1, 3, 7and 21.

   Factors of 35 (Thirty five) = 1, 5, 7 and 35.

   Therefore, common factor of 21 (Twenty one) and 35 (Thirty five) = 1 and 7.

Highest common factor (H.C.F) of 21 (Twenty one) and 35 (Thirty five) = 7.

3. Find the highest common factor (H.C.F) of 30 (Thirty) and 24 (Twenty four).

<u>Solution</u>:

Factors of 30 (Thirty) = 1, 2, 3, 5, 6, 10, 15 and 30.

Factors of 24 (Twenty four) = 1, 2, 3, 4, 6, 8, 12 and 24.

Therefore, common factor of 30 (Thirty) and 24 (Twenty four = 1, 2, 3, and 6.

Highest common factor (H.C.F) of 30 (Thirty) and 24 (Twenty four = 6.

<u>Assessment</u>

Using the steps above, solve the questions below.

4. Find the highest common factor (H.C.F) of 33 (Thirty three) and 55 (Fifty five).

5. Find the highest common factor (H.C.F) of 50 (Fifty) and 70 (Seventy).

6. Find the highest common factor (H.C.F) of 12 (Twelve), 18 (Eighteen) and 24 (Twenty four).

<u>Week 4</u>

Topic: Fractions

<u>Expressing Fractions as Decimals, Ratio and Percentages</u>

<u>Converting Fractions to Ratios</u>

To understand the relationship between fractions and ratios, consider a pizza cut into six slices. If you only one slice has pepperoni, then you could say the pizza is 1/6 pepperoni. The ratio between pepperoni and non-pepperoni slices is 1:6.

To convert a fraction to a ratio, first write down the numerator, or top number. Second, write a colon. Thirdly, write down the denominator, or bottom number. For **<u>Example</u>**, the fraction 1/6 can be written as the ratio 1:6.

Changing a common fraction to a decimal fraction

Divide the numerator of the fraction by its denominator.

For **<u>Example</u>**,

5/8 = 5.00 ÷ 8

=0.625 (a terminating decimal)

To change a decimal fraction to a common fraction

Write the decimal fraction as fraction with a numerator and denominator which is power of 10. Simplify if possible.

For **<u>Example</u>**

0.85 = = =

To change a fraction to a percentage

Multiply the fraction by 100.

For **<u>Example</u>**, = (×100) = % =57%

0.145 = (0.145×100)% = 14.5%= 14

To change percentage to a fraction

Divide the percentage by 100.

For **<u>Example</u>**,

92% =

= 0.92 (as a decimal fraction)

= (as common fraction)

Change the following to common fractions in their lowest terms

- 2
- 55
- 312
- 264
- 004
- 876

Arithmetic in the Home and Office

Bankers want people to save money. They give extra savings to encourage savings. The extra money is called **interest**

For <u>Example</u> a person saves N10 000 in a year. If the interest rate is 8% per annum (i.e. 8% per year) the saver will have N10, 800 at the end of the year: the original 10,000 plus N 800 interest from the bank. Interest that is paid that way is called **simple interest.**

<u>Example 1</u>

Find the simple interest on N60, 000 for 5 years at 9% per annum.

Yearly interest = 9% of N60,000

= x 60, 000

Interest for five years = N5400 x 5 = N27,000

Income tax

Most people have to pay a part of their income to the government. The part they pay is called **income tax.** This process is called the **PAYE (pay as you earn)** the government uses taxes to pay for public service such as defence, education, health and transport.

For any wage or salary earner,

Taxable income = total income – allowances

<u>Example</u>

A man has a total income of N52, 800 per month. He has three young children. He claims N3, 700 for a dependent relative. Calculate the amount of the tax he pays.

First calculate the allowance s.

Personal allowance = N6, 000

Child allowance = 3 x N2, 500 = N7, 500

Dependent relative = N3, 700

Total allowance = N17, 000

Second: calculate the taxable income

Taxable income = total income – allowances

= N52, 800 – 17,200

= N35, 600

Third: calculate the tax on the taxable income

Taxable income = N35, 600 = N20, 000 + N10, 600

Tax = 10% of N20, 000 + of N15, 000

= N2000 + N2, 340

= N4, 340

Discount Buying

A discount is a reduction in price. Discounts are often given for paying in cash.

<u>Example</u> 4

A radio costs N5, 400. A 12 % discount is given for cash. What is the cash price ?

Either,

Discount = 12% of N5, 400 = x N5, 400

= x N5, 400

= N675

Cash price = N5, 400 – N675 = N4, 725

These discounts are often given for buying in bulk

Commission

Commission is the payment for selling an item. For **Example**, insurance agents get commission for selling insurance. The more insurance they sell, the more commission they get. Factories often employ sales representatives to sell their goods to shop and traders. The sales representative often receives a proportion of the value of the goods they sell. This proportion is their commission.

Example

A sales representative works for an electric fan company. He gets a commission of 14k in the naira. In one week he sells four table fans at N10, 500 and nine small fans at N5, 400 each. Calculate his commission.

Total sales = 4x N10, 500 + 9 x N5, 400

= N42, 000 + N48, 000

= N90, 000

He gets 14k for every naira.

Commission = N90, 000 x 14k

= N1268400k

Assessment

1. A sales representative sells Peak milk products for Wamco Ltd. He gets a commission of 5% on Peak Tin and 10% on Peak Filled per carton sold . In one week he sells 200 cartons of Peak Tin and 150 cartons of Peak filled. A carton of Peak tin costs £2500 and Peak filled costs £2200 Calculate his commission.
2. A Bag of Rice costs N8500. A 15 % discount is given for cash on every 5 bags purchased at a time. If Tolu bought 30 bags (of 5 per time), how much commission does she get.
3. Find the simple interest on N85,000 for 5 years at 12% per annum.
4. Change 85% to fraction.
5. Kemi bought a 8 packets of biscuit and she gets an additional one. How many packs does she need to buy to get extra 10 packets of biscuit.

<u>Week 5</u>

Topic: Approximation

Rounding Off to Decimal Places

When numbers are rounded off, the digit s 1, 2, 3, 4 are rounded down the and the digits 5, 6, 7, 8, 9 are rounded up.

Round off 124.25 a. to two significant figures b. to one decimal place .

124. 25 = 120 to 2 s.f.

125. 25 = 124.3 to 1 d.p

Rounding off to the nearest whole number

Rules for rounding decimals to the nearest whole number:

- To round a decimal to the nearest whole number analyze the digit at the first decimal place i.e., tenths place.
- If the tenths place value is 5 or greater than 5, then the digit at the ones place increases by 1 and the digits at the tenths place and thereafter becomes 0.

<u>For Example:</u>

(i) 9.63 →

In 9.**<u>6</u>**3 analyze the digit at the tenths place. Here 6 is more than 5. Therefore we have to round the number up to the nearest whole number 10.

(ii) 78.537 →

In 78.**<u>5</u>**37 analyze the digit at the tenths place. Here 5 is equal to 5. Therefore we have to round the number up to the nearest whole number 79.

If the tenths place value is less than 5, then the digit at the ones place remains the same but the digits at the tenths place and thereafter becomes 0.

<u>For Example;</u>

(i) 7.21 →

In 7.**<u>2</u>**1 analyze the digit at the tenths place. Here 2 is less than 5. Therefore we have to round the number down to the nearest whole number 7.

(ii) 13.48 →

In 13.**4**8 analyze the digit at the tenths place. Here 4 is less than 5. Therefore we have to round the number down to the nearest whole number 13.

Rounding off to decimal places

We can **round** decimals to a certain accuracy or number of decimal places. This is used to make calculation easier to do and results easier to understand, when exact values are not too important.

First, you'll need to remember your place values:

To round a number to the **nearest tenth,** look at the next place value to the right (the hundredths). If it's 44 or less, just remove all the digits to the right. If it's 55 or greater, add 11 to the digit in the tenths place, and then remove all the digits to the right.

(In the **Example** above, the hundredths digit is a 44, so you would get 51.051.0.)

To round a number to the **nearest hundredth,** look at the next place value to the right (the thousandths this time). Same deal: If it's 44or less, just remove all the digits to the right. If it's 55 or greater, add 11 to the digit in the hundredths place, and then remove all the digits to the right.

(In the **Example** above, the thousandths digit is an 88 , so you would get 51.0551.05 .)

In general, to round to a certain place value, look at the digit to the right of that place value and make a decision.

Example:

5.18375.1837 to the nearest hundredth would be 5.185.18 (round down since 3<53<5),

but to the nearest thousandth, it is 5.1845.184 (round up because 7≥57≥5).

Order matters when calculating and rounding (vs. rounding then calculating):

3.7+2.6→4+3→73.7+2.6→4+3→7 (rounding first to nearest whole number, then adding)

3.7+2.6→6.3→63.7+2.6→6.3→6 (adding first and then rounding at the end.)

Which is correct? They both are. Which one you choose to use, depends upon your purpose. The second one is more accurate, because it's closer to the real answer (6.36.3). In general, it's more accurate to calculate first and then round. However, sometimes if you're doing mental math and you need a quick and very approximate answer, it's easier to round and then calculate.

The number on the left has been rounded off, either to two or to three significant places. Decide which number should go in the boxes.

ASSESSMENT

- 241269 f 241000
- 279 f 280
- 925 s.f 3.9
- 354 s.f 73.4
- 25 s.f 2.30

Week 6

Topic: Algebraic Expressions – Factors and Factorization

Definition of Algebraic Expression

In mathematics, an **algebraic expression** is an expression built up from integer constants, variables, and the algebraic operations (addition, subtraction, multiplication, division and exponentiation by an exponent that is a rational number) For **Example**, {\displaystyle 3x^{2}-2xy+c} is an algebraic expression. Since taking the square root is the same as raising to the power.

Remember that in algebra, letters can be whole or fractional, positive or negative.

1. Just as 5a is short for 5 x a, so – 5a is short for (-5) x a.{\displaystyle {\tfrac {1}{2}}}{\displaystyle {\sqrt {\frac {1-x^{2}}{1+x^{2}}}}}
2. Just as m is short for 1 x m, so – m is short for (-1) x m.
3. Algebraic terms and number can be multiplied together. For **Example**,

4 x (-3x) = (+4) x (-3) x X

= – (4 x3)x X = -12 x X

=-12X

(-2y) x (-8y) = (-2) x y x (-8) x y

= (-2) x (-8) x y x y

=+ (2×8) x y^2

= + $16y^2$ or just $16y^2$

Division with algebraic expression is also possible.

18a ÷ (-6) = =-( ) a

= (-3) x a= -3a

=() = = =() = 11

Factorization of simple algebraic expressions

A factor is a number or quantity that when multiplied with another number produces a given number or expression. For **Example**, the factors of 12 are 1, 2, 3, 4, 6 and 12.

12 = 1×12

12 = 2×6

12 = 3×4

Any number can be expressed in the form of its factors as explained shown above.

In terms of its prime factors 12 can be expressed as:

12 = 2 × 3 × 2 × 1

Similarly an algebraic expression can also be expressed in the form of its factors. An algebraic expression consists of variables, constants and operators. An algebraic expression consists of terms separated by addition operation. Consider the following algebraic expression:

3xyz−16×2−yz

This expression consists of 3 terms 3xyz, −16×2 and −yz. Each term of this algebraic expression can be expressed in the form of its factors as:

3xyz=3.x.y.z, −16×2=−1.2.2.2.2.x.x and −yz=−1.y.z.

Algebraic Expressions can be factorized using many methods. The most common methods used for factorization of algebraic expressions are:

- **Factorization using common factors**
- **Factorization by regrouping terms**
- **Factorization using identities**

Let us discuss these methods one by one in detail:

Factorization using common factors

In order to factorize an algebraic expression, the highest common factors of the terms of the given algebraic expression are determined and then we group the terms accordingly. In simple terms the reverse process of expansion of an algebraic expression is its factorization.

To understand this more clearly let us take an **Example**.

Example- −3y2+18y

Solution- The algebraic expression can be re-written as

−3y2+18y=−3.y.y+3.6.y

⇒−3y2+18y=−3.y(y+6)

Consider the algebraic expression -3y(y – 6), if we expand this we will obtain -3y2 + 18y.

· Factorization by regrouping terms

In some algebraic expressions it is not possible that every term has a common factor. For instance consider the algebraic expression 12a + n -na – 12. The terms of this expression do not have a particular factor in common but the first and last term have a common factor of '12' similarly second and third term has n as common factor. So the terms can be regrouped as:

$\Rightarrow$12a + n − na − 12= 12a − 12 + n − an

$\Rightarrow$12a − 12 − an + n = 12(a −1) −n(a −1)

After regrouping it can be seen that (a−1) is a common factor in each term,

$\Rightarrow$12a + n −na − 12=(a−1) (12 − n)

Thus by regrouping terms we can factorize algebraic expressions.

· Factorizing Expressions using standard identities

An equality relation which holds true for all the values of variables in mathematics is known as an identity. Consider the following identities:

$(a+b)^2=a^2+b^2+2ab$

$(a-b)^2=a^2+b^2-2ab$

$a^2-b^2=(a+b)(a-b)$

On substituting any value of a and b, both sides of the given equations remain the same. Therefore, these equations are identities

<u>Example</u>: **Factorize** 9 x 2 + $4m^2$ + 12mx**.**

<u>Solution</u>: Observe the given expression carefully. This expression has three terms and all the terms are positive. Moreover, the first and the second term are perfect squares. The expression fits the form (a+b)2=a2+b2+2ab where a = 3x, b = 2m.

9×2 + $4m^2$ + 12mx = $(3x)^2$ + $(2m)^2$ + 2.3 x. 2m

Therefore, $9x^2 + 4m^2 + 12mx = (3x + 2m)^2$

To change a Fraction to a decimal

Divide the numerator of the Fraction by its denominator

<u>Example</u> – 5/8 = 5.0 ÷ 8 = 0.625

1/9 = 0.111

To change a decimal fraction to a common fraction

Write the decimal fraction as a fraction with a denominator which is a power of 10. Simplify if possible

e.g 0.85 = 85/100 = 17/20

To change a fraction to a percentage

Multiply the fraction by 100

For **<u>Example</u>** – 4/7 = (4/7 x 100%) = 571/7%

To change a percentage to fraction

50% = 50/100 = 0.5

Ratio

Suppose that two shirts N600 and N800. The ratio of their prices is 600:800.

600/800 = 6/8 = 3/4 = 3:4

Decimal Fractions

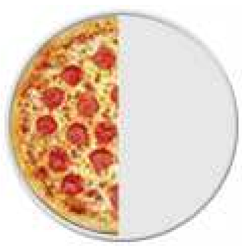
We have seen in previous lessons how to extend the place value system to include decimal fractions. The number 3.549 is a way of writing 3 units + 5/10 + 4/100 + 9/1000, or simply 3 units + 549/1000:

units decimal tenth hundredths thousandths

↓ ↓ ↓ ↓ ↓

3 . 5 4 9

The decimal point acts as a place-holder between the whole-number part and the fractional part of the number.

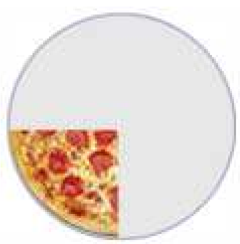


A Half can be written…

As a fraction:½

As a decimal:0.5

As a percentage:50%



A Quarter can be written…

As a fraction:1/4

As a decimal:0.25

As a percentage:25%

Addition and Subtraction

Be very careful to set out your work correctly. Units must be under units, decimal points under decimal points, … and so on. For **Example**, 24.8 + 6.5 is set out as

24.8 24.8

+ 6.5 not 6.5

——— ———

31.3

After you have set out your work correctly, add and subtract in the same way as you do for whole numbers, but remember to write down the decimal point when you come it.

Assessment

a. 0.2 + 0.6

b. 0.6 – 0.5

c. 1.3 + 0.8

d. 15.86

+ 5.15

————

e. 0.56

– 0.18

————

Positive and Negative Numbers

ADDITION:

If the signs are the same then you add the two numbers and keep the

Ex. 6 + 2 = 8 or −6 + −2 = −8

If the signs are different, subtract the two numbers and take the sign

of the larger number.

Ex. −6 + 2 = −4 or 6 + −2 = 4

SUBTRACTION:

Change the sign of the second number, then add the two numbers using the rules for addition, above.

Ex. 6 – 2 = 6 + (−2) = 4

Ex. −6 − −2 = −6 + (+2) = −4

Ex. $-6 - 2 = -6 + (-2) = -8$

Ex. $6 - -2 = 6 + (+2) = 8$

Exercise

FACTORIZATION OF BINOMIALS

Binomials are expressions with only two terms being added.

$2x^2 - 4x$ is an **Example** of a binomial. (You can say that a negative 4x is being added to $2x^2$.)

First, factor out the GCF, 2x. You're left with 2x (x – 2). This is as far as this binomial can go. Any binomial in the form 1x +/- n cannot be factored further.

When you have a binomial that is a variable with an even exponent, added to a negative number that has a square root that is a natural number, it's called a perfect square.

$x^2 - 4$ is an **Example** of this. It can be expressed as the product of the square root of the variable plus the square root of the positive constant, and the square root of the variable minus the square root of the positive constant.

Basically, take the square root of the variable. You'll end up with x. Then square root the 4. You'll end up with 2. If you add them together, you'll get x+2. Subtract them, and you'll get x-2. Multiply the two, and you'll get (x+4)(x-4). You've just factored a perfect square.

If you multiply (x+2)(x-2) together using FOIL, you'll end back up with x^2-4.

(FOIL: First Outer Inner Last, a way of multiplying two binomials together. Multiply the first terms of the binomials (x and x in this case), then the outer two (x and -2), then the inner two (2 and x), then the last terms (2 and -2), then add them all up. $x^2 - 2x + 2x - 4 = x^{\wedge 2} - 4$.)

This can be done again if one of the binomials is a perfect square, as in this instance:

$x^4 - 16 = (x^2 + 4)$ $\mathbf{(x^2 - 4)}$ $= (x^2 + 4)(x + 2)(x - 2)$.

This can be factored further if you bring in irrational numbers, see step [9].

How to factor binomials in the form of (x^3 **+** b^3):

Just plug into $(a - b)(a^2 + ab + b^2)$. For **Example**, $(x^3 + 8) = (x - 2)(x^2 + 2x + 4)$.

How to factor binomials in the form of (x^3 **–** b^3):

Plug into $(a + b)(a^2 - ab + b2)$. Note that the first two signs in the expression are switched.

$(x^3 - 8) = (x + 2)(x^2 - 2x + 4)$.

ALGEBRAIC FRACTIONS

Adding and subtracting

Algebraic fractions are simply fractions with algebraic expressions on the top and/or bottom.

When adding or subtracting algebraic fractions, the first thing to do is to put them onto a common denominator (by cross multiplying).

e.g. $\frac{1}{(x + 1)} + \frac{4}{(x + 6)}$

$= \frac{1(x + 6) + 4(x + 1)}{(x + 1)(x + 6)}$

$= \frac{x + 6 + 4x + 4}{(x + 1)(x + 6)}$

$= \frac{5x + 10}{(x + 1)(x + 6)}$

Solving equations

When solving equations containing algebraic fractions, first multiply both sides by a number/expression which removes the fractions.

Example

Solve $\frac{10}{(x + 3)} - \frac{2}{x} = 1$

multiply both sides by x(x + 3):

$\therefore \frac{10x(x + 3)}{(x + 3)} - \frac{2x(x + 3)}{x} = x(x + 3)$

$\therefore 10x - 2(x + 3) = x^2 + 3x$ [after cancelling]

$\therefore 10x - 2x - 6 = x^2 + 3x$

$\therefore x^2 - 5x + 6 = 0$

$\therefore (x - 3)(x - 2) = 0$

$\therefore$ either x = 3 or x = 2

Multiplication

To multiply algebraic fractions, factorise the numerators and denominators. Then cancel the factors common to the numerator and denominator before applying multiplication to obtain the answer.

Example

Simplify

a. 7 X b/8

b. p/14 X 6/p

c. x – 3/8 X 12/x – 3

Solution

a. 7 X b/8 = ab/56

b. p/14 X 6/p = 3/7

c. x – 3/8 X 12/x – 3 = 3/2

FRACTIONS WITH BRACKETS

x + 6/3 is a short way of writing (x + 6)/3 or 1/3(x + 6).

Notice that all the terms of the numeratorare divided by 3.

X + 6/3 = (x + 6)/3 = 1/3(x + 6) = 1/3x + 2

Example

Simplify

a. x + 3/5 + 4x – 2/5

Solution

x + 3/5 + 4x – 2/5

= (x + 3) + (4x – 2)/5

= x + 3 + 4x – 2/5

= 5x + 1/5

Assessment

Simplify the following

a. 2a – 3/2 + a + 4/2

b. 3b + 4/3 + 2b – 5/3

c. 4c – 3/5 – 2c + 1/5

Week 7

Topic: ARITHMETIC IN THE HOME AND OFFICE

Personal Arithmetic

Interest

Bankers want people to save money. They give extra payments to encourage saving. The extra money is called **interest**.

For **Example**, a person saves N10 000 in a bank for a year. If the interest rate is 8% per annum (i.e. 8% per year), the saver will have N10 800 at the end of the year: the original N10 000 plus N800 interest from the bank. Interest that is paid like this is called **simple interest**.

Example

Find the simple interest on N60 000 for 5 years at 9% per annum.

Yearly interest = 9% of N60 000

= 9/100 X N60 000 = N5 400

Interest for 5 years = N5 400 X 5 = N27 000

Exercise

Find the interest on the following:

1. N40 000 for 1 year at 5% per annum
2. N70 000 for 1 year at 4% per annum
3. N10 000 for 3 years at 6% per annum
4. N10 000 for 2 years at 4% per annum
5. N10 000 for 4 years at 4½ per annum

Sometimes people borrow money. When someone borrows money that person has to pay interest to the lender.

Example

A man borrows N1 600 000 to buy a house. He is charged interest at a rate of 11% per annum. In the first year he paid the interest on the loan. He also paid back N100 000 of the money he borrowed. How much did he pay back altogether? If he paid this amount by monthly installments, how much did he pay per month?

Interest on N1 600 000 for 1 year

= 11% of N1 600 000

= 11/100 X N1 600 000

= 11 X N16 000

=N176 000

Total money paid in 1st year

=N176 000 + N100 000

= N276 000

Monthly payments = N278 000 ÷ 12

= N23 000

(Notice that the man now owes N1 500 000. Interest will be paid on this new amount in the second year.)

Exercise

Find the total amount to be paid back (i.e. loan + interest) on the following loans:

a. N500 for 2 weeks at N100 interest per week.

b. N2 000 for 3 weeks at N1 on each N10 interest per week

c. N1 000 for one year at 9% simple interest per annum.

INCOME TAX

Most people have to pay part of their income to the government. The part they pay is called **income tax**. This process is often called PAY E (Pay As You Earn). The government uses taxes to pay for public services such as defence, education, health and transport.

The method of calculating taxes varies in different countries and tax rates change from time to time. However, the method is usually similar to the following **Example**.

Typical PAY E tax system

1. Each month, all earners pay tax on their taxable income. Table below contains typical rates of tax, for various income bands.

Tax bands on monthly income	Rate of tax
First N20 000	10%
Over N20 000 and up to N40 000	15%

Over N40 000 and up to N60 000	20%
Over N60 000	25%

2. For any wage or salary earner, taxable income = total income – allowances.

3. Allowances are as follows:

a. personal allowance: N6 000

b. child allowance: N2 500 for each child under 16 t-years of age (for am maximum of 4 children)

c. dependent relative allowance: maximum of N4 000.

If both husband and wife are working, the child allowance can only be claimed by one parent

Example

A man has a total income of N52 800 per month. He has 3 young children. He claims N3 700 for a dependent relative. Calculate the amount of tax he pays.

First: calculate the allowances.

Personal allowance = N6 000

Child allowance = 3 X N2 500 = N7 500

Dependent relative = N3 700

Total allowances = N17 200

Second: calculate the taxable income.

Taxable income = total income – allowances

= N52 800 – N17 200

= N35 600

Third: Calculate the tax on the taxable income, using the tax band table sighted above, using it.

Taxable income = N35 600 =N20 000 + N15 600

Tax = 10% of N20 000 + 15% of N15 600

= N2 000 + N2 340

= N4 340

Notice the following:

1. Everyone gets the personal allowance of N6 000. Thus anyone earning N6 000 a month or less will pay no tax.

2. Always work in the following order:

First – find the allowances,

Second – calculate the taxable income,

Third – calculate the tax.

Exercise

A person with 5 children earns N61 320 per month.

a. Calculate the tax allowances.

b. Calculate the taxable income

c. Calculate the amount of tax paid

d. Calculate the income after tax is paid.

Household Arithmetic

DISCOUNT BUYING

A **discount** is a reduction in price. Discounts are often given for paying in cash.

Example

A radio costs N5 400. A 12½% discount is given for cash. What is the cash price?

Either,

Discount = 12½% of N5 400

= 12½/100 X N5 400

= N675

Cash price = N5 400 – N675 = N4 725

or,

cash price = (100% – 12½ %) of N5 400

= 87½ % of N5 400

= 87½ /100 X N5 400

= 7/8 X N5 400

= N4 725

Discounts are often given buying in bulk.

INSTALLMENT BUYING

An installment is a part of payment. Many people find it easier to buy expensive items by paying instalments.

Example

The cost of a DVD player is either N34 000 in cash or deposit of N4 000 and 12 monthly payments of N2 750. Find the difference between the installment price and the cash price.

Installment price = deposit + installment

= N4 000 + 12 X N2 750

= N4 000 + N33 000

= 37 000

Price difference = N37 000 – N34 000

= N 3 000

Buying by installment is called **hire purchase**. The buyer hires the bus of an item before paying for it completely. This is why hire purchase is more costly than paying in cash.

CIVIC ARITHMETIC

Value Added Tax (VAT)

A proportion of the money paid for certain goods and services is given to the government. The part which is given to the government is called **Value Added tax (VAT)**, and the goods and services are called **VATable** items.

In Nigeria 5% of the cost of VATable items is given to the government as VAT. The government uses money collected as VAT to improve services such as education, health and transport.

Example

An advertisement for a table says that its price is 'N15 300 plus VAT'. How much does the customer pay?

Amount paid by customer = 105% of N15 300

= N15 300 X 105/100

= N16 065

Note: The difference between N16 065 and N15 300 is N765. The Government receives N765 as VAT.

BILLS AND CHARGES

Electricity bills

Electricity bills are based on the number of units of electricity used. Also there is usually a standing charge, or Demand charge, and VAT is charged. The table below gives typical charges.

Cost per unit	N8
Standing charge	N150 per quarter
VAT	5% of total bill

Example

A company's electricity meter reading changed from 45 243 to 50 548 in one quarter. Use the table above to calculate the company's electricity bill.

Number of units used = 50 584 – 45 243

= 5 341

Cost at N8 per unit = N8 X 5 341

= N42 728

Standing charge = N150

Total charges = N42 728 + N150

= N42 878

Total bill, including VAT = N42 878 X 105%

= N42 878 X 105/100

= N45 021.90 (calculator)

= N45 022 to nearest naira

Postal charges

Postal charges vary according to the weight of the letters and parcels being posted by and their destination. The table below gives some typical charges.

Letters up to 20 g

Letters within a state	N20
Letter between states	N50

Letters outside Nigeria	N100

Parcel within Nigeria

≤ 1kg	N200
> 1kg and ≤ 2kg	N400

COMMERCIAL TRANSACTION

Profit and loss

A trader buys a kettle for N800 and sells it at a profit of 15%. Find his actual profit and the selling price.

Profit = 15% of N800 = 15/100 X N800

= N 12 000/100

= N120

Selling price = N800 + N120

= N920

Example

A hat is bought for N250 and sold for N220.

What is the loss percent?

Actual loss = N250 – N220

= N30

The ratio, loss : cost = N30:N250

= 30 : 250

= 30/250

Thus the loss is 30/250 of the cost price

Percentage loss = 30/250 X 100%

= 12%

Notice that the loss or gain is calculated as a percentage of the cost price.

COMMISSION

Commission is payment for selling an item. For **<u>Example</u>**, insurance agents get commission for selling insurance. The more insurance they sell the more commission they get. Factories often employ sales representatives to sell their goods to shops and traders. The sales representatives often receive a proportion of the value of the goods they sell. This proportion is their commission.

<u>ASSESSMENT</u>

A sales representative works for an electric fan company. He gets a commission of 14k in the naira. In one week he sells four table fans at N5 400 each. Calculate his commission.

Total sales = 4 X N10 500 + 9 X N5 400

= N42 000 + N48 600

= N90 600

He gets 14k for every naira.

Commission = 90 600 X 14k

= 1 268 400k

= N12 684

Week 8

Topic: APPROXIMATION AND ESTIMATION

Rounding off numbers

To 'round off' or 'approximate' a number to a desired degree of accuracy, we

a. round the number *up* if the next digit is 5 or more

b. round the number *down* if the next digit is less than 5.

We represent approximately equal to as and approximately as '~'

<u>Examples</u>:

1. 73 is close to 70 if approximating to or rounding in "tens". So

73 ~ 70 (Read as 73 is approximately equal to 70)

2. 86 is close to 90 when rounded off to the nearest "tens", or approximate to the nearest "tens"

86 ~ 90

3. 650 ~ 700 when rounded off to the nearest hundreds

4. 26432 rounded off to

a. nearest 100 is 264 / 32 ~ 26400

b. nearest 10,000 is 2 / 6432 ~ 30000

Strategy to round off numbers

1. Put a line where you want to round off. In the above **<u>Example</u>** 3 to round off 650, put a big line after 6, because 6 is in the hundreds place and you want to round to the nearest hundred.

6/50

2. The digit before the big line (6 in this case) will go up by 1, and the rest of the digits after the line will become 0, since the number after the line is 5. So the answer is 700.

Estimation

Estimating numbers

<u>Example</u>

A school has 21 classes. One of the second-year classes has 34 students. Estimate the number of students in the school.

First method

Round off the number to 1 s.f.

Number of classes ≅ 20

Number of students per class ≅ 30

Number of students in school ≅ 20 X 30

≅ 600

Second method

Assume the second-year class represents every class in the school.

Number of students in school

≅ 21 X 34

≅ 714

≅ 700 to 1 significant figure

Notice that there can be varying answers when estimating. In **Example** 3, different methods give 600 to 700 students. However, both numbers are of the same order of size. Knowing that the school contains 600 to 700 students gives a good idea aof the size of the school. Thus, either number is a useful estimate.

Example

A student writes 47 words in five lines of writing. His **Exercise** book has 28 lines on each page.

a. Approximately how many words does he write on one page?

b. How many pages will a 750-word essay take up?

Solution

a. Five lines of writing contain 47 words.

One line of writing contains 47/5 ≅ 9 words.

28 lines of writing contain approximately 9 X 28 = 252 words.

Approximately 250 words are written on each page.

Assessment

1. A school has 27 classes. One of the classes has 33 students. Estimate the numbers of students in the school.

2. A college has 689 students. The average mass of a student is approximately 51 kg. Estimate the total mass of all the students in the school.

Estimate quantities using traditional measures

Market traders often use traditional measures to estimate quantities. For **Example**, dry measures such a s rice, beans are sometimes measured in cups, basins and sacks.

Liquids such as palm oil, groundnut oil and kerosene are measured in jars, bottles and tins. Medicine is often measured in teaspoons.

Assessment

1. Assume that for dry rice and beans,

20 cups ≅ 1 basin

20 basin ≅ 1 sack

a. A sack contains 100kg of rice. Estimate the number of kilograms of rice in 1 basin. Estimate the number of grams of rice in 1 cup.

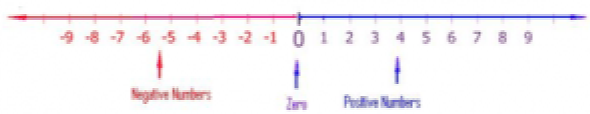
b. A trader buys a sack of rice for N600, she sells it as at N200 per basin. Estimate her profit on a sack of rice.

Week 9

Topic: DIRECTED NUMBERS – MULTIPLICATION AND DIVISION

Adding and Subtracting Direct numbers

Numbers can be shown on a number line which extends above and below zero. This gives positive and negative numbers.



The signs + and – show the direction from 0. Positive and negative numbers are called **directed numbers**.

To add a positive number, move to the right on the number line.

Example

(+1) + (+3) = +4

(–3) + (+5) = +2

To subtract a positive number, move to the left on the number line.

Example

(+5) – (+3) = +2

(+3) – (+7) = –4

(–1) – (+2) = –3

To add a negative number, move to the left on the number line. This is equivalent to subtracting a positive number of the same value.

Multiplication of directed numbers

Positive multipliers

Multiplication is a short way of writing repeated additions. For **Example**

3 x 4 = 3 lots of 4

= 4 + 4 + 4

= 12

With directed numbers.

(+4) + (+4) + (+4) = 3 lots of (+4)

= 3 x (+4)

The multiplier is. It is positive. Thus,

(+3) x (+4) = (+4) + (+4) + (+4) = +12

(+3) x (+4) = +12

Negative Multipliers

Find the next four terms in each of the following patterns.

a. +15, + 12, + 9, +6, _, _, _, _

Example: Tank Levels Rising/Falling

The tank has 30,000 liters, and 1,000 liters are taken out every day. What was the amount of water in the tank **3 days ago**?

We know the amount of water in the tank changes by −1,000 every day, and we need to subtract that 3 times (to go **back 3 days**), so the change will be:

$-3 \times -1{,}000 = +3{,}000$

The full calculation is:

$30{,}000 + (-3 \times -1{,}000) = 30{,}000 + 3{,}000 = 33{,}000$

So 3 days ago there were 33,000 liters of water in the tank.

Multiplication Table

Here is **another way** of looking at it.

Start with the multiplication table (just up to 4×4 will do):

x	1	2	3	4
1	1	2	3	4
2	2	4	6	8
3	3	6	9	12
4	4	8	12	16

Now see what happens when we head into **negative territory**!

Let’s go **backwards** through zero:

x	1	2	3	4
-4	-4	-8	-12	16
-3	-3	-6	-9	-12
-2	-2	-4	-6	-8
-1	-1	-2	-3	-4
0	0	0	0	0
1	1	2	3	4
2	2	4	6	8
3	3	6	9	12
4	4	8	12	16

Look at the “4” column: it goes **-16, -12, -8, -4, 0, 4, 8, 12, 16**. Getting 4 larger each time.

Look over that table again, make sure you are comfortable with how it works, because …

What About Multiplying 3 or More Numbers Together?

Multiply two at a time and follow the rules.

Example: What is (−2) × (−3) × (−4) ?

First multiply (−2) × (−3). Two like signs make a positive sign, so:

(−2) × (−3) = +6

Next multiply +6 × (−4). Two unlike signs make a negative sign, so:

+6 × (−4) = -24

Result: (−2) × (−3) × (−4) = −24

Division with directed numbers

When directed numbers are multiplied together two **like** signs give a b positive **result;**

Two **unlike** signs give a **negative** result.

Example

(+3) x (+8) = +24

(−3) x (−8) = +24

(+3) x (−8) = −24

(−3) x (+8) = −24

The same rule is true for division. For **Example**

(+24) ÷ (−3) = (+8)

(−24) ÷ (−3) = (+8)

(+24) ÷ (−3) = (−8)

(−24) ÷ (+3) = (−8)

ASSESSMENT

Divide -36 by 9

-36 ÷ 9 = − 36/+9 = − (36/9) = -4

JSS2
MATHEMATICS
SECOND TERM

Week 1

Topic: EXPANSION OF ALGEBRAIC EXPRESSIONS

Directed algebraic terms

Remember that in algebra, letters stand for numbers. The number can be whole or fractional, positive or negative.

1. Just as 5a is short for 5 x a, so -5a is short for (-5) x a.

2. Just as m is short for 1 x m, so −m is short for (-1) x m.

3. Algebraic terms and numbers can be multiplied together. For **Example**,

4 X (−3x) = (+4) x (−3) X x

= −(4 x 3) X x = −12 X x = −12x

(−2y) x (−8y) = (−2) X y X (−8) X y

= (−2) X (−8) X x X y

= +(2 X 8) X y^2

= +16y^2 or just 16^2

4. Division with directed numbers is also possible. For **Example**,

18a ÷ (−6) = (+18) X a/(−6)

= − (18/6) X a

= (−3) X a = −3a

−33x^2 /−3x = (−33) X x X x/(−3) X x

= +(33/3) X x = 11x

Substitution

Since letters in algebra stand for numbers, it is always possible to substitute values for the letters. Just as in football, the manager might substitute player NO 4 for players No 23, so in algebra we might substitute a value such as 8 or -5 for an unknown x.

Example

Find the value of

a. 4x b. xy – 5y when x = 2 and y = 3.

Solution

a. Substitute the value 2 for x, i.e. use the value 2 instead of x.

When x = 2, 4x = 4 X x

= 4 X 2 = 8

b. xy – 5y = x X y – 5 X y

When x = 2 and y = 3,

xy – 5y = 2 X 3 – 5 X 3

= 6 – 15 = -9

Example

What is the value of p – q/p when p = -5 and q = +10?

Notice that p – q/p is the same as(p – q)/q.

Simplify the top line before dividing.

When p = -5 and q = +10,

P – q/p = (-5) – (+10)/(-5) = -15/-5 = +(15/5) = +3

Removing Brackets

3 X (7 + 5) means first add 7 and 5, then multiply the result by 3. Suppose a pencil costs 70 naira and a rubber costs 50 naira.

Cost of a pencil and a rubber

= 70 naira + 50 naira

= (70 + 50) naira = 120 naira

If three students each buy a pencil and a rubber, then,

Total cost = 3 X (70 + 50) naira

= 3 X 120 naira

= 360 naira

There is another way to find the total cost. Three pencils cost 3 X 70 naira. Three rubbers cost 3 X 50 naira. Altogether, total cost = 3 X 70 naira + 3 X 50 naira

= 210 naira + 150 naira

= 360 naira

Thus, 3 X (70 + 50) = 3 X 70 + 3 X 50.

This shows that brackets can be removed by multiplying the three into both the 70 and the 50.

Usually, we do not write the multiplication sign. We just write 3(70 + 50).

Say 3(70 + 50) as ‘3 into (70 + 50)’

3(70 + 50) = 3 X 70 + 3 X 50

In general, using letters for numbers,

a(x + y) = ax + ay

Notice also that:

3(70 + 50) = 3 X 20 = 60

and 3 X 70 – 3 X 50 = 210 – 150 = 60

Thus, 3(70 – 50) = 3 X 70 – 3 X 50

Again, using letters for numbers,

a(x + y) = ax + ay

Example

Remove brackets from the following:

a. 8(2c + 3d)

b. 4y(3x – 5)

Solution

a. 8(2c + 3d) = 8 X 2c + 8 X 3d

= 16c + 24d

b. 4y(3x – 5) = 4y X 3x – 4y X 5

= 12xy – 20y

Expanding algebraic expression

The expression (a + 2)(b – 5) means (a + 2) X (b – 5) means (a + 2) X (b – 5). The terms in the first bracket, (a + 2), multiply each term in the second bracket, (b – 5). Just as:

X(b − 5) = bx − 5x

So, writing (a + 2) instead of x,

(a + 2)(b − 5) = b(a + 2) − 5(a + 2)

The brackets on the right-hand side can now be removed.

(a + 2)(b − 5) = b(a + 2) − 5(a + 2)

= ab + 2b − 5a − 10

ab + 2b − 5a − 10 is the product of (a + 2) X (b − 5). We often say that the **expansion** of (a + 2)(b − 5) is:

ab + 2b − 5a − 10

Example

Expand the following:

a. (a + b)(c + d)

b. (6 − x)(3 + y)

Solution

a. (a + b)(c + d) = c(a + b) + d(a + b)

= ac _ bc + ad + db

b. (6 − x)(3 + y) = 3(6 − x) + y(6 − x)

= 18 − 3x + 6y − xy

We sometimes call this **binomial expansion,** since each bracket contain two terms (*bi-nomial means two-names*).

Exercise

Expand the following:

1. (p + q)(r + s)

2. (x + 8)(y + 3)

3. (4 + 5a)(3b + a)

4. (a − b)(c + d)

Week 2

Topic: Simple Equations

Simple Equations

However, when most people talk about equations, they mean algebraic equations. These are equations that involve letters as well as numbers. Letters are used to replace some of the numbers where a numerical expression would be too complicated, or where you want to generalize rather than use specific numbers.

Algebraic equations are solved by working out what numbers the letters represent. We can turn the two simple equations above into algebraic equations by substituting x for one of the numbers:

$2 + 2 = x$

We know that $2 + 2 = 4$, which means that x must equal 4. The equation answer is therefore $x = 4$.

$5 + 3 > 3 + x$

We know that $5 + 3 = 8$. The equation tells us that 8 is greater than (>) $3 + x$.

Take 3 away from 8, to get 5.

We can see that x must be less than 5 or x is 4 or less. $x < 5$ or $x <= 4$

We cannot say more precisely what x is with the information that we are given.

There is no magic about using the letter x. You can use any letter you like, although x and y are commonly used to represent the unknown elements of equations.

Terms of an Equation

A term is a part of the equation that is separated from other parts by an addition or subtraction sign.

Terms may be just numbers, or they may be just letters, or they may be a combination of letters and numbers, such as 2x, 3xy or 4×2.

In a term involving letters and numbers, the number is known as the coefficient, and the letter as the variable.

Terms that have exactly the same variable are said to be like terms, and you can add, subtract, multiply or divide them as if they were simple numbers.

ASSESSMENT

The equation $2x + 3x$ is equal to $5x$, simply 2 lots of x plus 3 lots of x to make 5 lots of x ($5x$).

$5xy - xy = 4xy$

$5y \times 3y = 15y$.

You cannot add or subtract unlike terms. However, you can multiply them by combining variables and multiplying the coefficients together.

So, for **Example**, $3y \times 2x = 6xy$ (because $6xy$ simply means 6 times x times y).

You can divide unlike terms by turning them into fractions and cancelling them down. Start with the numbers, then the letters.

So, for **Example**, $6xy \div 3x =$

$$\frac{6xy}{3x} = \frac{2xy}{x} = \frac{2y}{1} = 2y$$

<u>Week 3</u>

Topic: Linear Inequalities

INEQUALITIES

In mathematics we use the equals sign, =, to show that quantities are the same. However, very often, quantities are different, or **unequal**. For **<u>Example</u>**, a mother is always older than her child their ages are always different. We say that there is inequality in their ages. This chapter explains the use of inequalities in arithmetic, algebra and in everyday life. It also introduces the inequality symbols.

Greater than, less than

The sum 5 + 3 = 8 is a simple **equality**. However, as we know, quantities are often not equal. For **<u>Example</u>**:

$5 + 5 \neq 8$

where ≠ means 'is not equal to'. We can also write:

$5 + 5 > 8$

where > means 'is greater than. Similarly we can write the following:

$3 + 3 \neq 8$

$3 + 3 < 8$

where < means 'is less than'.

≠, >, < are **inequality symbols**. They tell us that quantities are not equal. The > and < symbols are more helpful than ≠. They tell us more. For **<u>Example</u>**, $x \neq 0$ tell us that x does not have the value 0; x can be any positive or negative number. However, $x<0$ tell us us that x is less than 0; x must be a negative number.

<u>Assessment</u>

Answer the following questions

1. Write either > or < instead of the words.
2. 6 is less than 11 —–
3. -1 is greater than -5 —–-
4. 0 is greater than -2.4 —–-

5. −3 is less than +3
6. x is greater than 12
7. State whether each of the following is true, T, or false, F.
8. 13 > 5 → T
9. 19 < 21 →
10. −2 < −4 →
11. −15 > 7 →
12. 3 +9 < 10 →

The symbols can be used to change word statements into algebraic statements. See

<u>Example</u>s below.

1. The distance between two villages is over 18km. Write this as an algebraic statement.

Let the distance between the villages be d km. Then, $d > 18$

A statement like $d > 18$ is called an **inequality.**

2. I have x naira. I spent N200. The amount I have left is less than N50. Write an inequality in x.

I spend N200 out of x naira.

Thus I have $x - 200$ naira left.

Thus $x - 200 < 50$.

Not greater than, not less than

In most towns there is a speed limit of 50 km/h. If a car, travelling at s km/h, is within the limit, then s is not greater than 50. If $s < 50$ or if $s = 50$, the speed limit will not be broken. This can be written as one inequality:

$s \leq 50$

where ≤ means 'less than or equal to'. Thus, not greater than means the same as less than or equal to.

In most countries, voters in elections must not be less than 18years of age. If a person of age a years is able to vote, then a is not less than 18. The person can vote if $a > 18$ or if $a = 18$. This can be written as one inequality:

$a \geq 18$, where ≥ means 'is greater than or equal to'. Thus, not less than means the same as greater than or equal to.

Graphs of inequalities

Linear inequalities

Inequalities like 3x > – 12 and 2x – y ≤ 7 have unknowns, or **variables,** with an index of 1 9i.e. x = x^1 and y = y^1). Inequalities with variables of index 1 are called **linear inequalities.**

3x > – 12 is a **linear inequality in one variable** (x); 2x – y ≤ is a linear inequality in two variables (x and y). This chapter is restricted to linear inequalities in one variable.

Linear Inequalities in one variable

When working with linear equations involving one variable whose highest degree (or order) is one, you are looking for the one value of the variable that will make the equation true. But if you consider an inequality such as x + 2 < 7, then values of x can be 0, 1, 2, 3, any negative number, or any fraction in between. In other words, there are many solutions for this inequality. Fortunately, solving an inequality involves the same strategies as solving a one variable equation. So even though there are an infinite number of answers to an inequality, you do not have to work any harder to find the answer. To review how to solve one variable equations,

However, there is one major difference that you must keep in mind when working with any inequality. If you multiply or divide by a negative number, you must change the direction of the inequality sign. You'll see why this is the case soon.

Let's go back and look at x + 2 < 7. If this were an equation, you would only need to subtract 2 from both sides to have x by itself.

x + 2 < 7

– 2 – 2

....

x < 5

Keep in mind that the new rule for inequalities only applies to multiplying or dividing by a negative number. You can still add or subtract without having to worry about the sign of the inequality.

But what would happen if you had -2x ≥ 10? Before solving, If you let x = -5 or -6 or any other value that is less than -5, then the inequality will be true. So you would write your solution as x ≤ – 5. In the process of solving this inequality using algebraic methods, you would have something that looks like the following:

-2/-2 10/-2

x -5

Let's Practice

2x + 3 > -11

Begin by getting the variable on one side by itself by subtracting 3 from both sides. Then divide both sides by 2. Since you are dividing by a positive 2, there is no need to worry about changing the sign of the inequality.

2x + 3 > -11

2x > -14

x > -7

4 – 3x 20

The solution to this problem begins with subtracting 4 from both sides and then dividing by -3. As soon as you divide by -3, you must change the sign of the inequality.

4 – 3x 20

-3x 16

x   -16/3

5x – 7 > 3x + 9

Thissolution will require a little more manipulation than the previous **Example**s. You have to gather the terms with the variables on one side and the terms without the variables on the other side.

5x – 7 > 3x + 9

-3x   -3x

2x – 7 > 9

+7   +7

2x > 16

x > 8

There is another type of inequality called a double inequality. This is when the variable appears in the middle of two inequality signs. This is simply a shortcut way of writing two separate inequalities into one and using a shorter process for finding the solution.

Basic Rules of Inequalities

Rule 1

If a > b then b < a, i.e. if a is greater than b then b is less than a, If a < b then b > a and if a is less than b then b is greater than a

Rule 2

If a > b and b > e then a > e, e. g. if 6 > 4 and 4 > 2 then 6 > 2, If a < band b < e then a < e, e. g. if 3 < 7 and 7 < 10 then 3 < 10

Rule 3

If $a > b$ then $a + e > b + c$ or $a - c > b - c$, If $a < b$ then $a + e < b + c$ or $a - c < b - c$

i.e. we can add to or subtract from both sides of an inequality the same quantity without changing the sense (or sign) of the inequality.

Rule 4

If $a > b$ and c is a positive number, i.e. $c > 0$ then $ac > bc$ and $a/c > b/c$, If $a < b$ and $c > 0$ then $ac < bc$ and $a/c < b/c$

i.e. both sides of an inequality can be multiplied or divided by the same positive number without changing the sense of the inequality.

Rule 5

If $a> b$ and c is negative i.e. $c < 0$ then $ac < bc$ and $a/c <b/c$, If $a < b$ and $c < 0$ then $ac > bc$ and $a/c > b/c$

Note: Both sides of an inequality can be multiplied or divided by a negative number, but the sense of the inequality is reversed.

The sense of an inequality is changed if both sides are multiplied or divided by the same negative number.

Rule 6

If $a > b$ and $c > d$ then adding the inequalities $a + c > b + d$, If $a < b$ and $c < d$ then $a + c < b + d$

i.e. Inequalities having the same sense can be added side by side to each other without changing the sense of the inequalities.

Rule 7

If $a > b$ and $c > d$ then either $a - c > b - d$ or $c - a > d - b$ is true but not the two of them are true at the time.

Similarly if $a < b$ and $c < d$ then either $a - c < b - d$ or $c -a < d - b$ is true but not the two of them.

Rule 8

If $a > b > 0$ and $c > d > 0$ or $a < b < 0$ and $c < d < 0$ then $ac > bd$

Rule 9

If a > b and n > 0 then $a^n > b^n$

e.g. 5 > 3 and 2 > 0

$5^2 > 3^2$ i.e.25 > 9

If a > b and n < 0 then $a^n < b^n$

If a< b and n > 0 then $a^n < b^n$

If a < b and n < 0 then $a^n > b^n$

e.g. 4 < 6 and -2 < 0

$4^{-2} > 6^{-2}$

i.e. 1/16 > 1/36

Example

-2 6x – 1 10

The strategy for solving this inequality is not that much different than the other **Example**s. Except in this case, you are trying to isolate the variable in the middle rather than on one side or the other. But the process for getting the x by itself in the middle, you should add 1 to all three parts of the inequality and then divide by 6.

-2 6x – 1 10

-1 6x 11

-1/6 x 11/6

Graphing One-Variable Inequalities

Before graphing linear inequalities, we summarized below the different forms of inequalities, with its corresponding interval form and graph:

Let's take a look at the inequality symbols and their meanings again.

> **Greater Than**

≥ **Greater Than or Equal To** (The line underneath the greater than sign indicates also equal to)

< **Less Than** (Tip: To remember this sign, if you open the sign up a little more, it would look like a capital L for less than)

≤ **Less Than or Equal To** (The line underneath the less than sign indicates also equal to)

Graphing Symbols

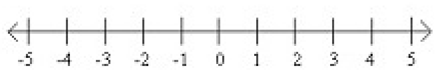

Greater Than (The open circle indicates that this is **NOT Equal to** the numeral graphed.


Greater Than or Equal To (The closed circle indicates that this is **Equal to** the numeral graphed.


Less Than (The open circle indicates that this is **NOT Equal to** the numeral graphed.


Less Than or Equal To (The closed circle indicates that this is **Equal to** the numeral graphed.

There are just a few important concepts that you must know in order to graph an inequality. Let's review a number line.

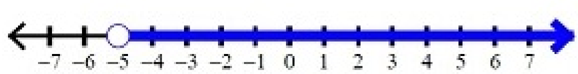


The negative numbers are on the left of the zero and the positive numbers are on the right.

Example

$r > -5$

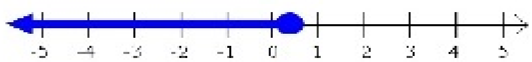
This is read as "r is greater than -5." This means it includes all numbers greater than, or to the right, of -5 but does not include -5 itself. We will have to show this by using an open circle and having the arrow shoot out to the right.



Example 2

x≤ 0.4

This is read as “x is less than or equal to 0.4.” This time we include the 0.4 by using a closed circle and the arrow will shoot out to the left. The number 0.4 is in between the 0 and the 1 on a number line.



Here is a summary of the important details in graphing inequalities.

Make sure you read the inequality starting with the variable!

“greater than” or “greater than or equal to” – arrow shoots out to the right

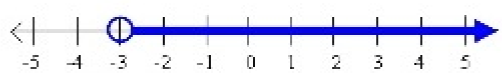
“less than” or “less than or equal to” – arrow shoots out to the left will have open circles ≤ and ≥ will have closed circles.

Questions

1. How is this read ?

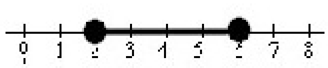
A. Less than or equal to B. greater than or equal to C. Less than D. Greater than

2. What inequality value can be read from the graph below?



A. −3 < *p* B. −3 > p C. −3 p D. −3 p

3. What inequality value can be read from the graph below?



Let’s try a couple <u>**Example**</u>s.

<u>Solution</u> of inequalities

Balance method

Consider a compound in which 23 people live. T any one time there may be x people in the compound. If all 23 people are in the compound, then x = 23. This is an **equation**.

If some people have left the compound, then x < 23. This is the **inequality.**

The equation has only one solution: $x = 23$. The inequality has many solutions: if $x < 23$, then x could be 0, 1, 2, 3, . . ., 20, 21, 22.

Notice that negative and fractional values of x are impossible in this **Example**.

Inequalities are solved in much the same way as equations. We use the **balance method.**

However, there is one important difference to be shown later on.

Example

Solve the inequality $x + 4 < 6$.

$x + 4 < 6$

subtract 4 from both sides.

$x + 4 - 4 < 6 - 4$

$x < 2$

$x < 2$ is the solution.

When solving inequalities, we do not normally try to list the values of the unknown.

Exercise

1. Solve the following inequalities. Sketch a graph of each solution.
2. $x - 2 < 3$
3. $x + 3 \geq 6$
4. $a + 5 > 7$
5. $y - 3 \leq 5$

Example

Find the values of x that satisfy the inequality $3x - 3 > 7$, such that x is an integer.

Note: an **integer** is any whole number

-3, 0, 22 are **Example**s of integers.

$3x - 3 > 7$

Add 3 to both sides.

$3x > 10$

Divide both sides by 3.

$x > 3\frac{1}{3}$

But x must be an integer.

Thus x can have values 4, 5, 6, ….

x = 4, 5, 6, … is the solution.

Multiplication and division by negative numbers

Consider the following true statement: 5 > 3. Multiply both sides of the inequality by -2.

This gives -10 > -6.

But this is a false statement. In fact, -10 is less than -6.

Similarly, dividing both sides of 15 > -12 (true) by -3 gives – 5 > 4 (false, since -5 < 4).

In general, **when multiplying or dividing both sides of an inequality by a negative number, reverse the inequality sign** to keep the statement true.

For **<u>Example</u>** if -2x ≥ 14 is true, then dividing both sides by -2 gives the equivalent true statement x ≤ -7.

<u>Example</u>

Solve 5 – x > 3

Either

5 – x > 3

Subtract 5 from both sides.

-x > -2

Multiply both sides by -1 and reverse the inequality.

(-1) X (-x) < (-1) X (-2)

x < 2

or:

5 – x > 3

Add x to both sides.

5 > 3 + x

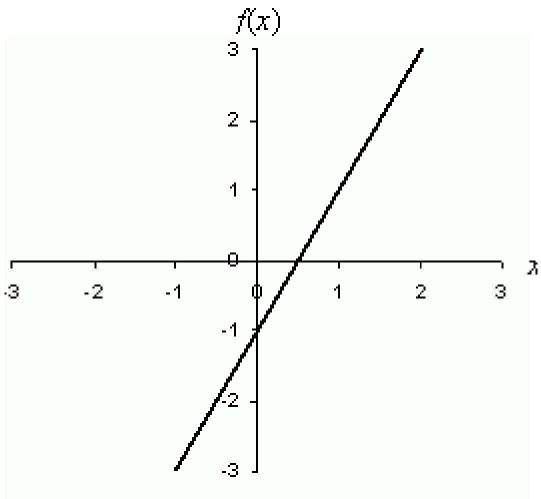
Subtract 3 from both sides.

2 > x

thus, x < 2

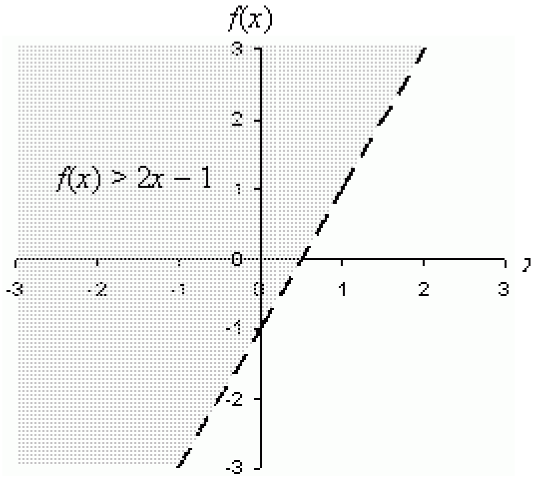
The second method in **Example** above shows that the rule of reversing the inequality sign when multiplying by a negative number is correct.

A **linear inequality** involves the relationship between linear functions, just as do linear equations. The difference, however, is that linear inequalities relate two linear functions using the symbols <, ≤, >, or ≥, which correspond respectively to less than, less than or equal to, greater than, and greater than or equal to. Because these relationships do not involve a strict equality, solutions for expressions that contain them are more complex than similar expressions that do involve strict equality. For the most part, the same rules we have used for linear equations also apply to linear inequalities–a few nuances must be considered, however.



Let's consider a simple linear inequality: $f(x) > 2x - 1$. This expression simply means that the output of the function f are those values that are greater than $2x - 1$. Let's look at this inequality graphically–first, we must plot the line $2x - 1$, which is the boundary between values that satisfy the inequality and those that do not.

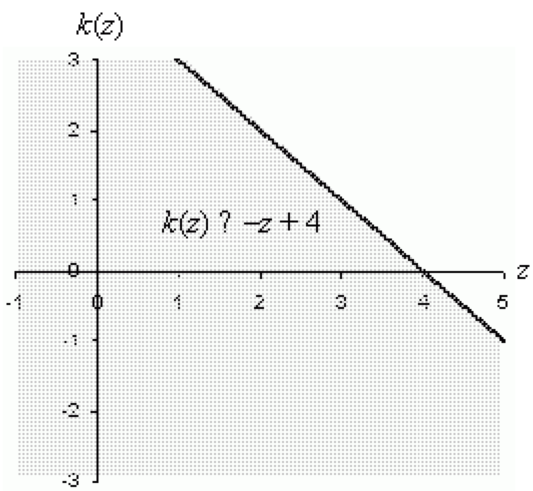
The line in the above graph defines the function (or equation) $f(x) = 2x - 1$; what we want to find, however, is $f(x) > 2x - 1$. This is to say, the values of $f(x)$ that satisfy this inequality are those that are *larger* than those corresponding to the expression $2x - 1$. Since $f(x)$ and $2x - 1$ are both sets of values along the vertical axis, the plot of $f(x) > 2x - 1$ must be all values above the line, but not including the line. (If the expression was $f(x) \geq 2x - 1$, then the graph *would* include the line.) To display this set of values for $f(x)$ on the graph, we shade the region above the line, but we make the line broken to indicate that the line is not part of the region that satisfies the expression $f(x) > 2x - 1$.



If the inequality was $f(x) < 2x - 1$ or $f(x) \leq 2x - 1$, then the region below the line $2x - 1$ would be shaded (and the line would or would not be solid depending on which expression was used).

Practice Problem: Graph the inequality $k(z) \leq -z + 4$.

Solution: First, let's draw an appropriate set of axes and graph the line $-z + 4$. We'll use a solid line because the line itself is part of the solution set (since ≤ is used). Next, we'll shade the area below the line. The result should look like the following graph.



We can perform a "necessary but not sufficient" check of the result to give us more confidence in this answer: pick any point in the shaded region and use those values to see if the inequality holds for that point. Let's try the point (4, 0):

$-z + 4 = -4 + 4 = 0$

$k(4) = 0 \leq 0$ Inequality holds

If the linear inequality is expressed entirely in terms of the independent variable, then the solution is all values on one side or the other of some vertical line $x = c$, where c is some constant. For instance, let's look at the inequality $-7x - 5 \leq -2 + x$. We can manipulate inequalities using the same rules of algebra we apply to algebraic equations. There is one difference, however: if both sides of an inequality are multiplied by a negative number, you must reverse the direction of the inequality. We can see this with simple numbers: 4 < 6, yet –4 > –6. Let's express this rule generally for functions $f(x)$ and $g(x)$ and a constant value $-c$ (where $c > 0$):

$f(x) < g(x) \to -cf(x) > -cg(x)$

$f(x) > g(x) \to -cf(x) < -cg(x)$

$f(x) \leq g(x) \to -cf(x) \geq -cg(x)$

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$f(x) \geq g(x) \to -cf(x) \leq -cg(x)$

Now, let's return to our **Example**.

$-7x - 5 \leq -2 + x$

$-7x - 5 - x \leq -2 + x - x$

$-8x - 5 \leq -2$

$-8x - 5 + 5 \leq -2 + 5$

$-8x \leq 3$

Now, recall the new rule:

$-8x \leq 3$

$(-8x) \geq (3)$

$x \geq$

This is the solution to the inequality. Note, however, that this is a range of values, not just a single value. We call the set of solutions to an equation or inequality the **Solution set**. Let's take a graphical look at the solution set for this inequality. Note that because our solution involves only the independent variable, we can plot the results using a number line instead of a planar graph. When doing so, use an open circle () if the endpoint of the solution set is *not* included, and use a closed circle () if it *is* included. We use an arrow (a ray) to indicate the solution set.

Note that we use a solid line because the inequality is of the form "greater than or equal to." The shaded region in the graph, including the solid line, is the solution set of the inequality $-7x - 5 \leq -2 + x$.

Practice Problem: Find and graph the solution set of $3x - 4 < 1 - x$.

Solution: First, we can manipulate the inequality to find a corresponding solution set in terms of the independent variable x.

$3x - 4 < 1 - x$

$3x - 4 + 4 < 1 - x + 4$

$3x < 5 - x$

$3x + x < 5 - x + x$

$4x < 5$

$x < 5/4$

We can check this result by using a value that satisfies $x < 5/4$; let's try $x = 0$.

$3(0) - 4 < 1 - (0)$

$-4 < 1$ Inequality holds

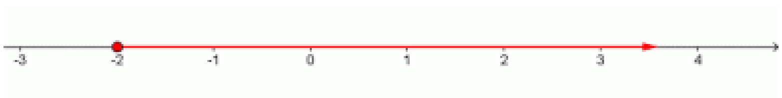
Now, let's graph the result. Note that we use an open circle at $x = 5/4$ because the solution set is a strict inequality (the < symbol is used).

Week 4&5

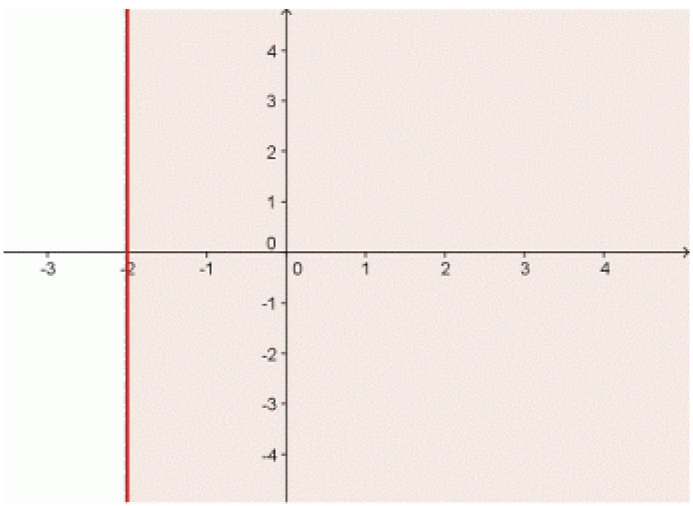
Topic: Linear Inequality (Graphical Representation)

Linear Inequality (Graphical Representation)

Inequalities with one variable can be plotted on a number line, as in the case of the inequality $x \geq -2$:

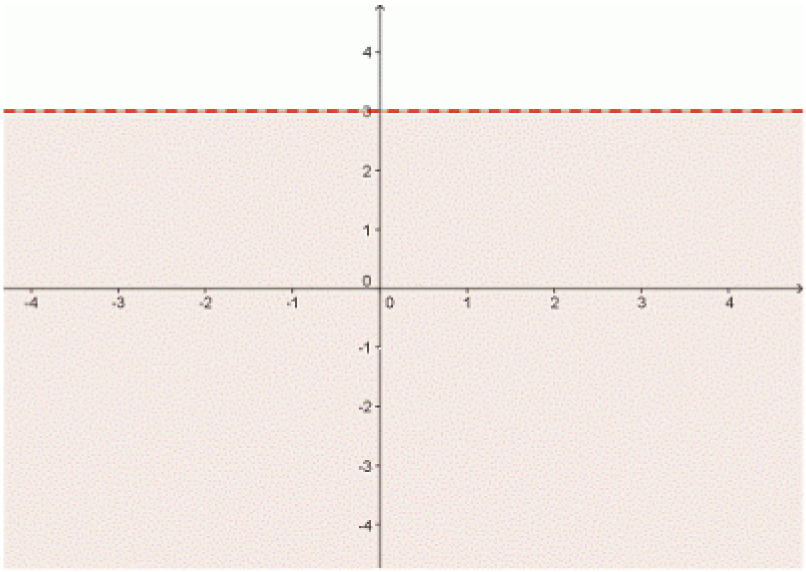


Here is another representation of the same inequality $x \geq -2$, this time plotted on a coordinate plane:



On this graph, we first plotted the line $x = -2$, and then shaded in the entire region to the right of the line. The shaded area is called the **bounded region**, and any point within this region satisfies the inequality $x \geq -2$. Notice also that the line representing the region's boundary is a solid line; this means that values along the line $x = -2$ are included in the solution set for this inequality.

By way of contrast, look at the graph below, which shows $y < 3$?



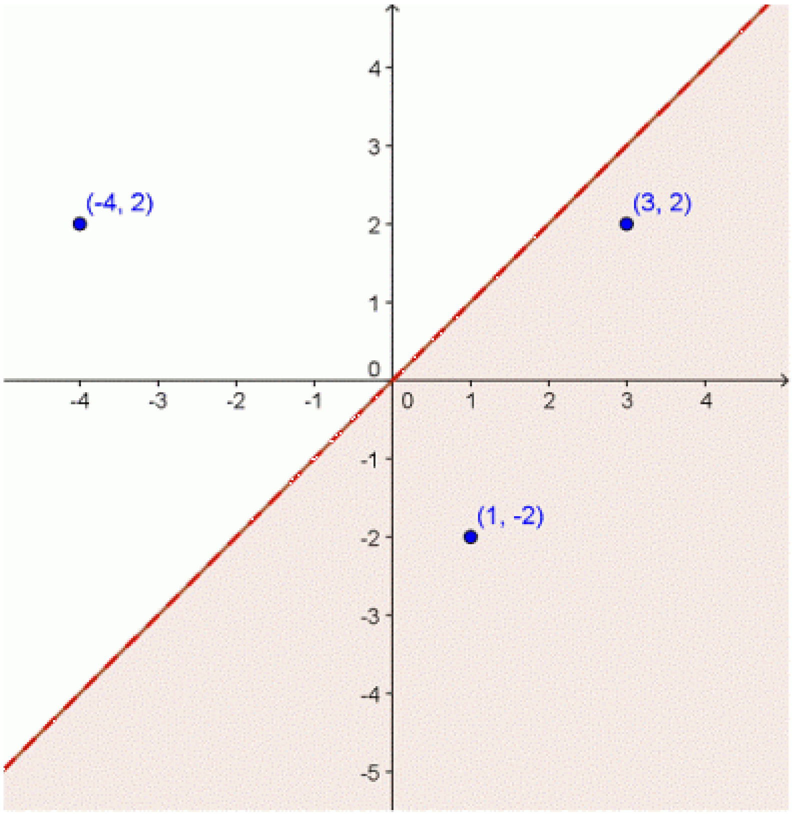
In this inequality, the **boundary line** is plotted as a dashed line. This means that the values on the line *y* = 3 are not included in the solution set of the inequality.

Notice that the two **Example**s above used the variables *x* and *y*. It is standard practice to use these variables when you are graphing an inequality on a (*x*, *y*) coordinate grid.

Two Variable Inequalities

There's nothing too compelling about the plots of $x \geq -2$ and $y < 3$, shown above. We could have represented both of these relationships on a number line, and depending on the problem we were trying to solve, it may have been easier to do so.

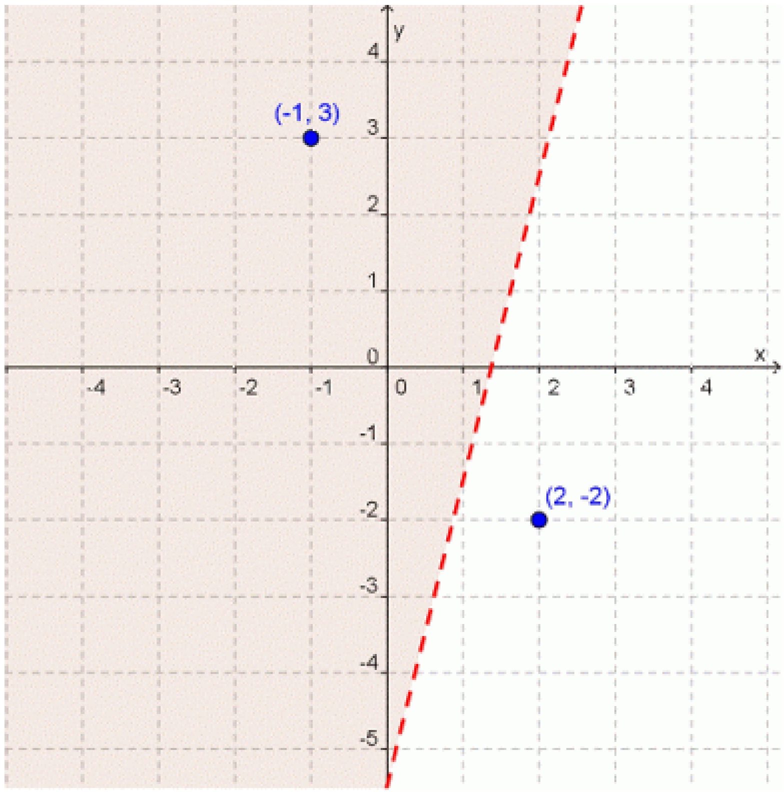
Things get a little more interesting, though, when we plot linear inequalities with two variables. Let's start with a basic two-variable inequality: $x > y$.



The boundary line is represented by a dotted line along $x = y$. All of the points under the line are shaded; this is the range of points where the inequality $x > y$ is true. Take a look at the three points that have been identified on the graph. Do you see that the points in the boundary region have x values greater than the y values, while the point outside this region do not?

Plotting other inequalities in standard $y = mx + b$ form is fairly straightforward as well. Once we graph the boundary line, we can find out which region to shade by testing some ordered pairs within each region or, in many cases, just by looking at the inequality.

The graph of the inequality $y > 4x - 5.5$ is shown below. The boundary line is the line $y = 4x - 5.5$, and it is dashed because our y term is “greater than,” not “greater than or equal to.”



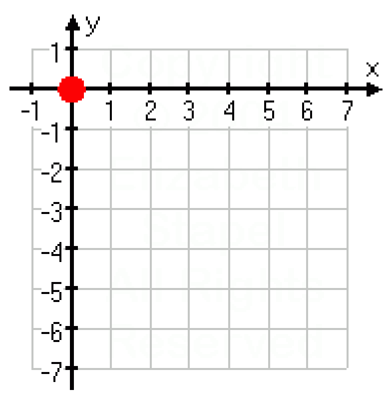
Graph of Cartesian plane (the axis)

Finding the location of (5, 2) and then drawing its dot is called “plotting the point (5, 2)”.

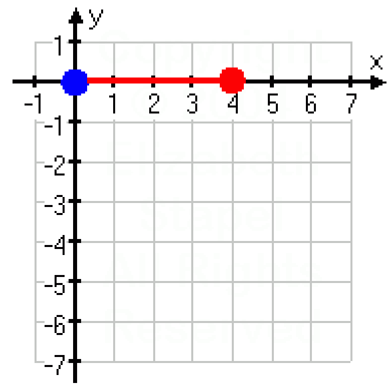
When plotting, remember that the first number is for the horizontal axis and the second number is for the vertical axis. You always go “so far over or back” and then “so far up or down”.

- **Plot the point (4, −5).**

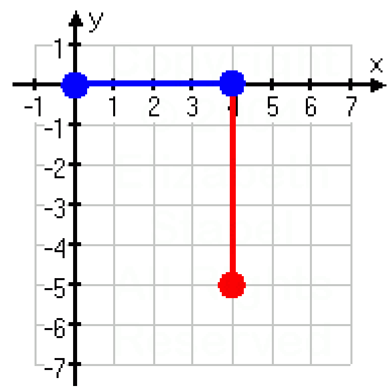
I will start at the origin:



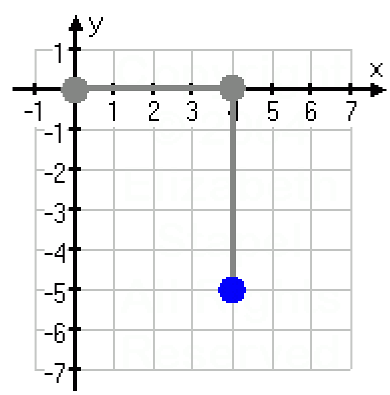
...then I'll count over four units on the horizontal *x*-axis:



...then I'll count *down* five units paralell to the *y*-axis:



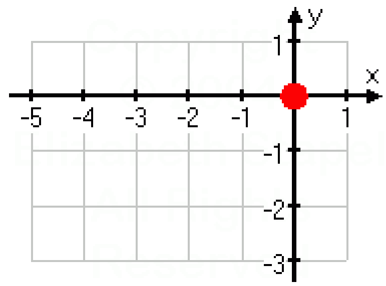
...and then I'll draw my dot:



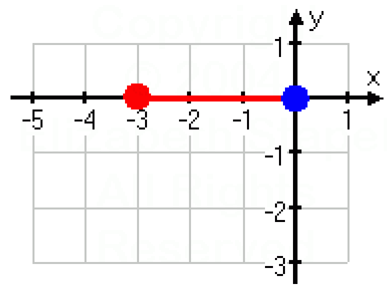
As you can see above, a negative *y*-coordinate means that you'll be counting *down* the *y*-axis, not up.

- **Plot the point (–3, –1).**

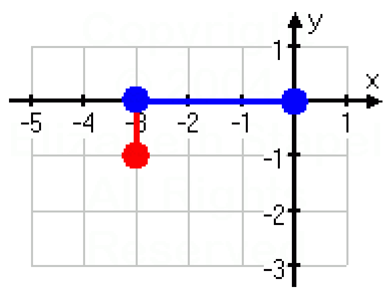
I'll start at the origin:



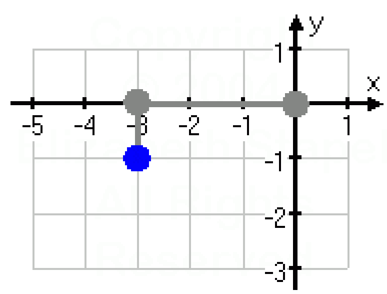
...then I'll count *backwards* three units along the *x*-axis:



...then I'll count *down* one unit parallel to the *y*-axis:



...and then I'll draw my dot:

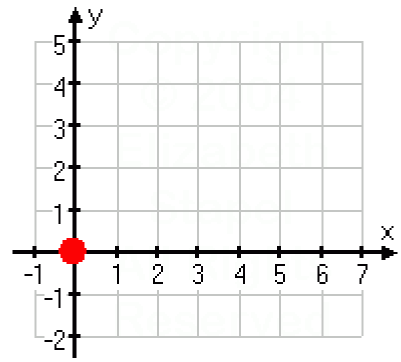


<u>ASSESSMENT</u>

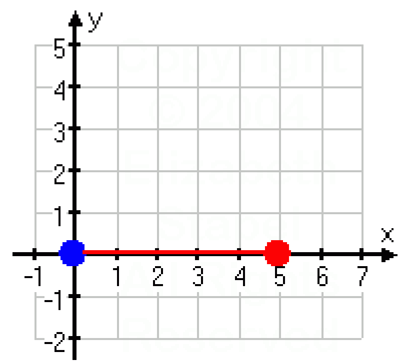
Suppose you were told to locate "(5, 2)" (pronounced as "the point five two" or just "five two") on the plane. Where would you look? To understand the meaning of "(5, 2)", you have to know the following rule: The *x*-coordinate (the number for the *x*-axis) *always* comes first. The first number (the first coordinate) is *always* on the horizontal axis.

ANSWER

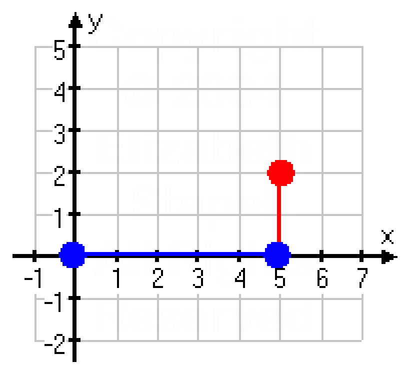
So, for the point (5, 2), you would start at the "origin", the spot where the axes cross:



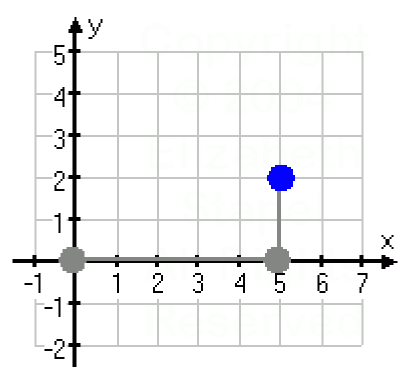
...then count over to "five" on the x-axis:



...then count up to "two", moving parallel to the y-axis:



...and then draw in the dot:



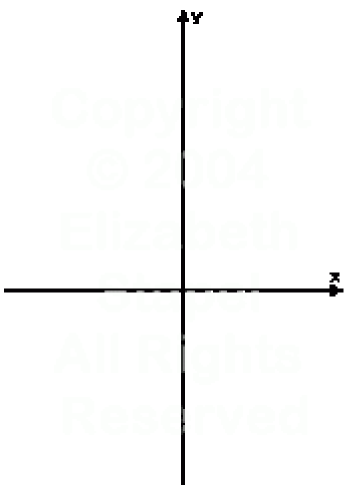
Week 6 & 7

Topic: GRAPHS OF LINEAR EQUATIONS

Linear equations and graphs

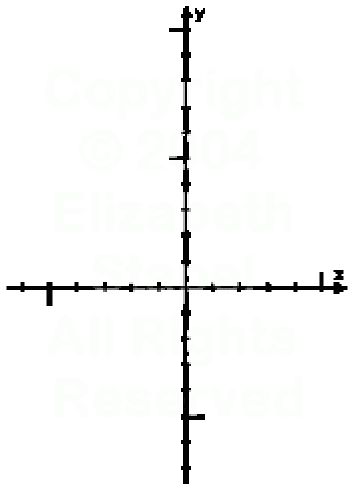
Now that you have your points, you need to draw your axes. REMEMBER TO USE YOUR RULER! If you don't use a ruler, you will have messy axes and inconsistent scales on the axes, and your points will NOT line up properly. Don't "fake it" with your graphs. Get in the habit now of drawing neatly. It will save you so much trouble down the line! (And, no, using graph paper is not the same as, nor does it replace, using a ruler!) -2011 All Rights Reserved

Also, make sure you draw your axes large enough that your graph will be easily visible. On a standard-sized sheet of paper (8.5 by 11 inches, or A4), you will be able to fit two or three graphs on a page. If you are fitting more than three graphs on one side of a sheet, then you're probably drawing them too small. Here are my axes:



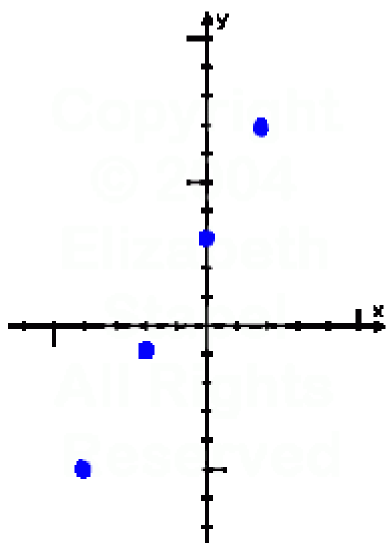
Remember that the arrows indicate the direction in which the values are increasing. Your book (and even your teacher) may draw things incorrectly, but that's no excuse for you. Arrows go on the upper numerical ends of each axis, and NOWHERE ELSE (unless you have an educator who *wants* it drawn wrong; then just remember the right way for later courses).

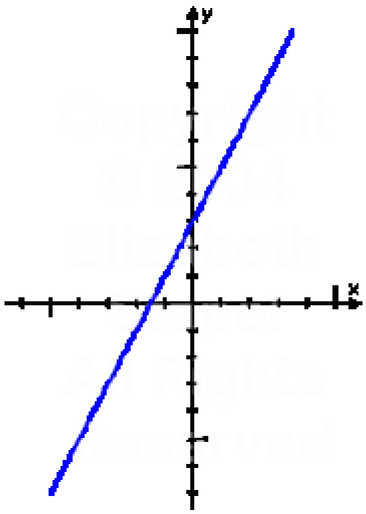
Once I've drawn my axes, I have to label them with an appropriate scale. "Appropriate" means "one that is neat and that fits the numbers I'm working with". For instance, considering the values I'm working with, I'll count off by ones. But if I were doing a graph for a word problem about government waste, I would probably count off by hundred thousand or maybe even by millions. Adjust the scales and axes to suit the case at hand. And ALWAYS use a ruler to make sure that your tick-marks are even! Here's my scale:



Note that I've made every fifth tick-mark a bit longer. This isn't a rule, but I've often found it helpful for counting off the larger points; it's more of a time-saver than anything else.

Now I'll plot (draw) the points I'd computed in my T-chart:



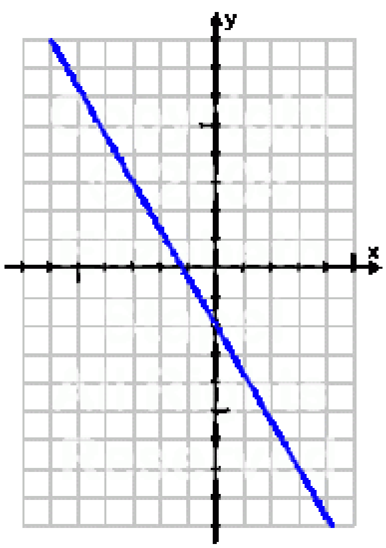


Graph $y = (-5/3)x - 2$

First I'll do the chart.

X	-6	-3	0	3
(-5/3)x	10	5	0	-5
-2	-2	-2	-2	2-
Y = (-5/3)x - 2	8	3	-2	-7

Since I am multiplying x by a fraction, I will x-values that are multiples of 3, so the denominator will cancel out and I will not have fractions. Then I will plot my point and draw my graph.



First I will do the chart Graph $y = 7 - 5x$

x	-1	0	1	2	3

7	7	7	7	7	7
–5x	5	0	–5	–10	–15
Y=7 – 5x	2	7	2	–3	–8

This equation is an **Example** of a situation in which you will probably want to be particular about the *x*-values you pick. Because the *x* is multiplied by a relatively large value, the *y*-values grow quickly. For instance, you probably wouldn't want to use *x* = 5 or *x* = –3. You could pick larger *x*-values if you wished, but your graph would get awfully tall.

And as you can see, the graph is pretty tall and already.

Form of linear equation

y = mx + c

Another way of arranging the equation y = 4x – 7 is to put the variables in alphabetical order, equating to zero: 4x – y – 7 = 0. This equation is in the form ax + by + c = 0, where the graph y = 4x – 7 is also the graph Of 4x – y – 7 = 0.

ASSESSMENT

The equations above are all in the form y = mx + c, where x and y are variables and m and c are constants. For **Example**, the equation y = 4x – 7, so m = 4 and c = 7.

ANSWER

ax + by = c = 0

Week 8

Topic: STRAIGHT-LINE GRAPHS

Linear Graphs

A **graph** is a picture that represents numerical data. Most of the graphs that you have been taught are **straight-line** or **linear graphs**. This topic shows how to use linear graphs to represent various real-life situations.

If the rule for a relation between two variables is given, then the graph of the relation can be drawn by constructing a table of values.

To plot a **straight line graph** we need to find the coordinates of *at least two points* that fit the rule.

Example

Plot the graph of $y = 3x + 2$.

Solution

Construct a table and choose simple x values.

X	-2	-1	0	1	2
Y					

In order to find the y values for the table, substitute each x value into the rule y = 3x + 2

When x = -2, y = 3(-2) + 2

= -6 + 2 = = -4

When x = -1, y = 3(-1) + 2

= -3 + 2 = 1

When x = 0, y = 3 X 0 + 2

= 0 + 2 = 2

When x = 1, y = 3 X 1 + 2

= 3 + 2 = 5

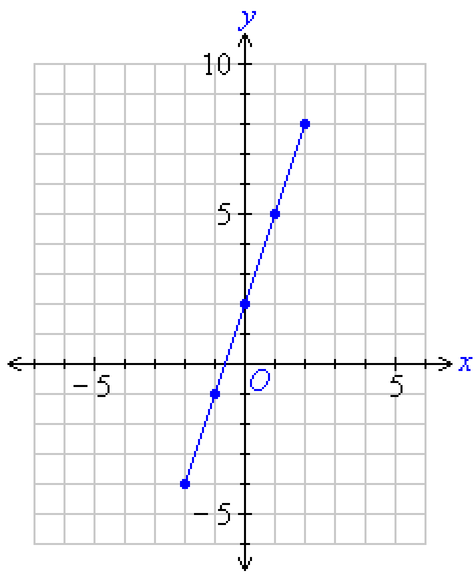
When x = 2, y = 3 X 2 + 2

= 6 + 2 = 8

The table of values obtained after entering the values of y is as follows:

X	-2	-1	0	1	2
Y	-4	-1	2	5	8

Draw a Cartesian plane and plot the points. Then join the points with a ruler to obtain a straight line graph.



Setting out:

Often, we set out the solution as follows.

Y = 3x + 2

When x = -2, y = 3(-2) + 2

= -6 + 2 = -4

When x = -1, y = 3(-1) + 2

= -3 + 2 = = -1

When x = 0, y = 3 X 0 + 2

= 0 + 2 = 2

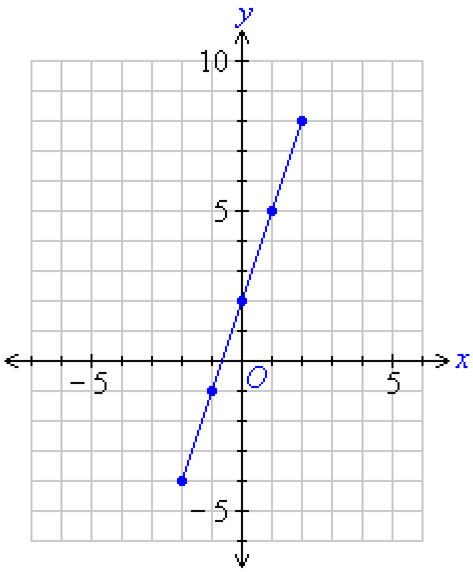
When x = 1, y = 3 X 1 + 2

= 3 + 2 = 5

When x = 2, y = 3 X 2 + 2

= 6 + 2 = 8

X	-2	-1	0	1	2
Y	-4	-1	2	5	8



Example

Plot the graph of *y*= −2*x*+ 4.

Solution

Y = -2x + 4

When x = -2, y = -2(-2) + 4

= 4 + 4 = 8

When x = -1, y = -2(-1) + 4

= 2 + 4 = 6

When x = 0, y = -2 X 0 + 4

= 0 + 4 = 4

When x = 1, y = -2(1) + 4

= -2 + 4 = 2

When x = 2, y = -2(2) + 4

= -4 + 4 = 0

When x = 3, y = -2(3) + 4

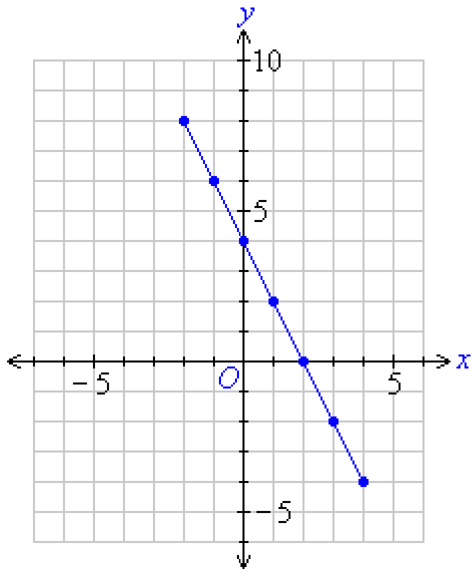
= -6 + 4 = -2

When x = 4, y = -2(4) + 4

= -8 + 4 = -4

x	-2	-1	0	1	2	3	4

y 8 6 4 2 0 -2 -4

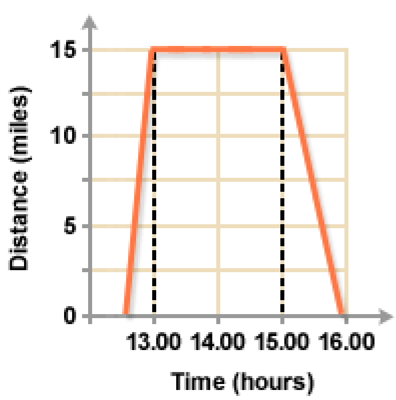


Distance-time graphs

We use distance-and-time graphs to show journeys. It is always very important that you read **all** the information shown on these type of graphs.

A graph showing one vehicle's journey

If we look at the graph shown below, you can see that the time in hours is along the horizontal, and the distance in miles is on the vertical axis. This graph represents a journey that Jan took, in travelling to Glasgow and back, from Aberdeen.



Important points to note are:

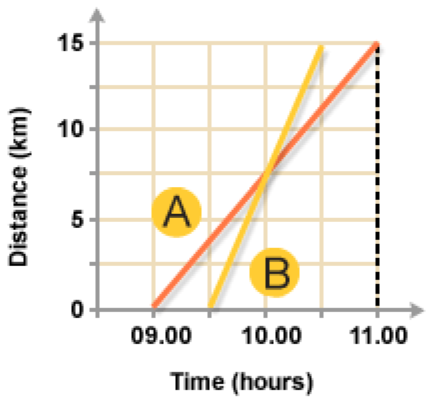
It took half an hour to travel a distance of miles

Between 1pm and 3pm there was no distance travelled. This means that the car had stopped.

The journey back, after 3pm, took one hour.

A graph showing two different journeys in the same direction

The next graph shows two different journeys. You can see that there is a difference with the steepness of the lines drawn. Remember that, the steeper the line, the faster the average speed. We can calculate the average speeds, by reading distances from the graph, and dividing by the time taken.



Line A: How long does journey A last, and what distance is travelled?

The journey takes 2 hours, and the distance travelled is 15km.

Line B : How long does journey B last, and what distance is travelled?

The journey takes 1 hour, and the distance travelled is also 15 km.

This means that the average speeds are:

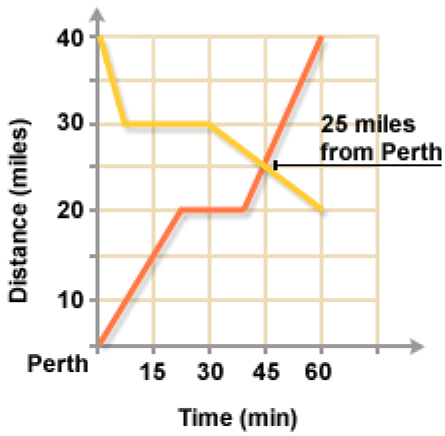
A: 15/2 = 7.5 km per hour

B: 15/1 km per hour

You will also notice from the graph that the two lines cross. This means that, if the two vehicles were travelling along the same route, they would have met at that point, which was just after 10am. The vehicle on journey B overtook vehicle

A graph showing two journeys in the opposite direction

A different pair of journeys is shown below. It is important to note that one journey begins at a distance of , and the other at a distance of miles, from Perth. In fact, what is happening is that one journey is travelling away from, and the other is travelling towards Perth. Again, the two journeys meet. This time it is miles from Perth.



You will also see that the two journeys contain stops.

If we were to calculate the average speeds for each total journey we would have to include this time as well.

A graph showing a journey (or journeys) should have time on the horizontal axis, and distance from somewhere on the vertical axis.

A line moving up, as it goes from left to right, shows a journey moving away from a place, and a line moving down, as it goes from left to right, represents a journey towards a place.

A horizontal line is a break or rest.

Two lines, sloping the same way, cut: then an overtaking has taken place.

Two lines, sloping opposite ways, cut: a meeting has taken place.

Speed-time graphs

A **speed-time graph,** velocity-time graph, shows how the speed of an object varies with time during a journey.

There are two very important things to remember about velocity – time graphs.

The distance traveled is the area under the graph.

The gradient or slope of the graph is equal to the acceleration. If the gradient is negative, then there is a deceleration. We may use the equations(1) or some rearrangement of this equation.

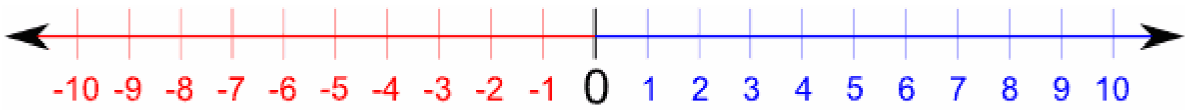
GRAPHS – CARTESIAN PLANE AND COORDINATES

The position of points

A **graph** is a picture of numerical data. We used graphs in statistics in class 1, where they represented number patterns. Here we extend graphs to identifying and drawing the position of points.

Points on a line

Writing numbers down on a Number Line makes it easy to tell which numbers are bigger or smaller.



Numbers on the left are stronger than numbers on the right.

Example

5 is smaller than 8

-1 is smaller than 1

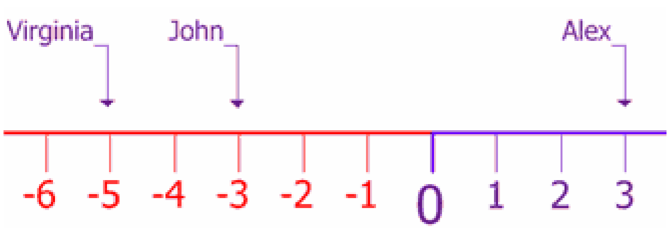
-8 is smaller than -5

Example

Example: John owes $3, Virginia owes $5 but Alex doesn't owe anything, in fact he has $3 in his pocket. Place these people on the number line to find who is poorest and who is richest.

Having money in your pocket is positive, owing money is negative.

So John has "−3", Virginia "−5" and Alex "+3"



Now it is easy to see that Virginia is poorer than John (−5 is less than −3) and John is poorer than Alex (−3 is smaller than 3), and Alex is, of course, the richest!

Plotting Points on a Cartesian Plane

A Cartesian plane (named after French mathematician Rene Descartes, who formalized its use in mathematics) is defined by two perpendicular number lines: the ***x*-axis**, which is horizontal, and the ***y*-axis**, which is vertical. Using these axes, we can describe any point in the plane using an ordered pair of numbers.

The Cartesian plane extends infinitely in all directions. To show this, math textbooks usually put arrows at the ends of the axes in their drawings.

The location of a point in the plane is given by its coordinates, a pair of numbers enclosed in parentheses: (x, y). The first number x gives the point's horizontal position and the second number y gives its vertical position. All positions are measured relative to a "central" point called the origin, whose coordinates are (0, 0). For **Example**, the point (5, 2) is 5 units to the right of the origin and 2 units up, as shown in the figure. Negative coordinate numbers tell us to go left or down. See the other points in the figure for **Example**s.

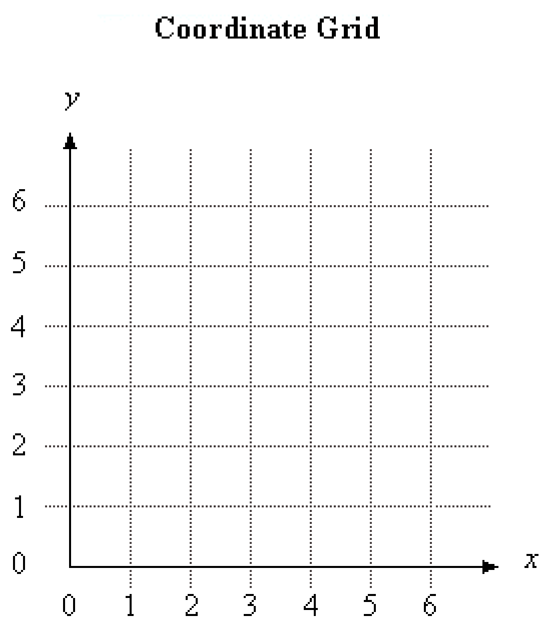
The Cartesian plane is divided into four quadrants. These are numbered from I – IV, starting with the upper right and going around counterclockwise. (For some reason everybody uses roman numerals for this).

In Quadrant I, both the *x*– and *y*–coordinates are positive; in Quadrant II, the *x*–coordinate is negative, but the *y*–coordinate is positive; in Quadrant III both are negative; and in Quadrant IV *x* is positive but *y* is negative.

Points which lie on an axis (i.e., which have at least one coordinate equal to 0) are said not to be in any quadrant. Coordinates of the form (*x*, 0) lie on the horizontal *x*–axis, and coordinates of the form (0, *y*) lie on the vertical *y*–axis.

Coordinate Graphing

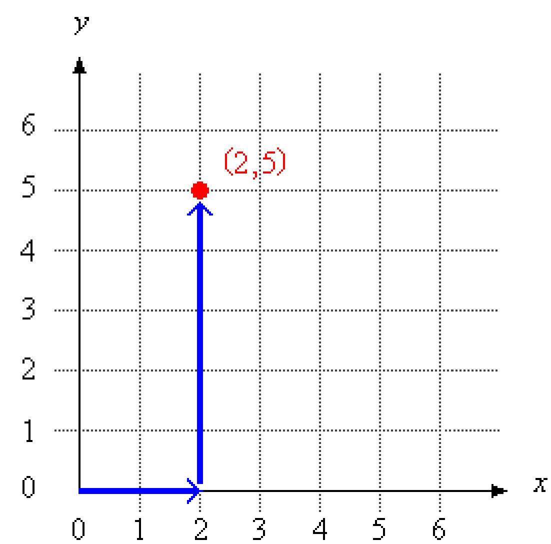
Coordinate graphing sounds very dramatic but it is actually just a visual method for showing relationships between numbers. The relationships are shown on a **coordinate grid**. A coordinate grid has two perpendicular lines, or **axes**, labeled like number lines. The **horizontal axis** is called the ***x*–axis**. The **vertical axis** is called the ***y*–axis**. The point where the *x*–axis and *y*–axis intersect is called the **origin**.



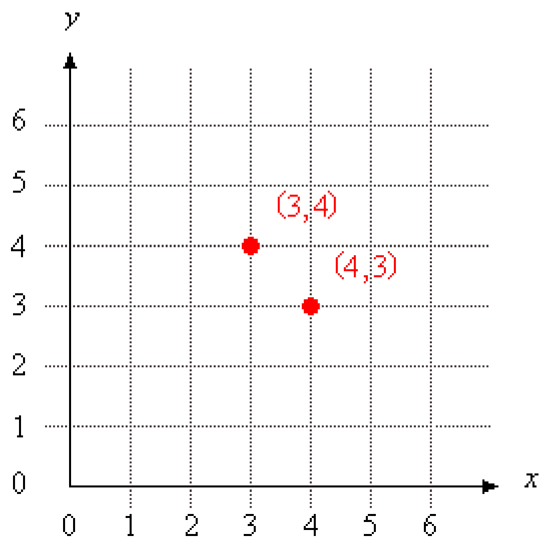
The numbers on a coordinate grid are used to locate points. Each point can be identified by an **ordered pair** of numbers; that is, a number on the *x*–axis called an ***x*–coordinate**, and a number on the *y*–axis called a **y–coordinate**. Ordered pairs are written in parentheses (*x*–

coordinate, *y*-coordinate). The origin is located at (0,0). Note that there is no space after the comma.

The location of (2,5) is shown on the coordinate grid below. The *x*-coordinate is 2. The *y*-coordinate is 5. To locate (2,5), move 2 units to the right on the *x*-axis and 5 units up on the *y*-axis.



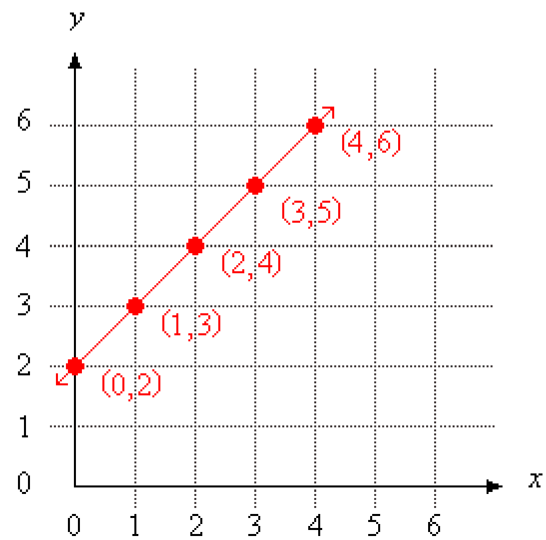
The order in which you write *x*- and *y*-coordinates in an ordered pair is very important. The *x*-coordinate always comes first, followed by the *y*-coordinate. As you can see in the coordinate grid below, the ordered pairs (3,4) and (4,3) refer to two different points!



The function table below shows the *x*– and *y*–coordinates for five ordered pairs. You can describe the relationship between the *x*– and *y*–coordinates for each of these ordered pairs with this rule: the x-coordinate plus two equals the y-coordinate. You can also describe this relationship with the algebraic equation $x + 2 = y$.

***x*–coordinate**	***x* + 2 = *y***	***y*–coordinate**	**ordered pair**
0	**0** + 2 =**2**	2	(0,2)
1	**1** + 2 =**3**	3	(1,3)
2	**2** + 2 =**4**	4	(2,4)
3	**3** + 2 =**5**	5	(3,5)
4	**4** + 2 =**6**	6	(4,6)

To graph the equation $x + 2 = y$, each ordered pair is located on a coordinate grid, then the points are connected. Notice that the graph forms a straight line. The arrows indicate that the line goes on in both directions. The graph for any simple addition, subtraction, multiplication, or division equation forms a straight line.



Plotting Points

To plot a point means to raw its position on a Cartesian plane. The easiest way to plot a point is as follows:

1. Start at the origin.

2. Move along the x-axis by an amount and in a direction given by the x-coordinate of the point.

3. Move up or down parallel to the y-axis by an amount and in a direction given by the y-coordinate.

ASSESSMENT

Example. A car starts on a journey. It accelerates for 10 seconds at It then travels at a constant speed for 50 seconds before coming to rest in a further 4 seconds.

a. Sketch a velocity – time graph.

b. Find the total distance traveled.

c. Find the deceleration when the car is coming to a stop at the end.

d. Find the average speed.

ANSWER

a. We may rearrange (1) to obtain v = u + at = 0 + 3 X 10 = 30m/s. Hence we may draw a straight line from (0,0) to (3,30). During the second part the car is travelling at a constant speed of 30m/3. Hence we can draw a straight line to (3,30)+(50,0)=(53,30). During the last part, which takes a further 4 seconds the car comes to a rest, and it's final velocity will be zero. Hence we can draw a straight line to (57,0). We can now draw the velocity time graph.

b. Distance travelled = Area under the graph. The graph is a trapezium so use the formula for the area of a trapezium: ½(a + b) X h = 1/2 (57 + 50) X 30 = 1535m

c. During the final part of the journey the velocity decreases from 30 to 0 in 4 seconds so a = v – u/t = (0 – 30)/4 = -7.5m/s^2 → deceleration = 7.5m/s^2

d. Average speed = Total Distance/Total Time = 1535/57 = 26.93m/s^2.

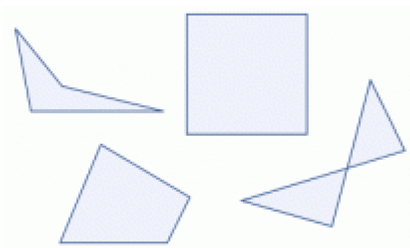
Week 9

Topic: PLANE FIGURES OR SHAPES

Quadrilateral just means "four sides", (*quad* means four, *lateral* means side).

Any four-sided shape is a Quadrilateral.

But the sides have to be **straight**, and it has to be **2-dimensional**.

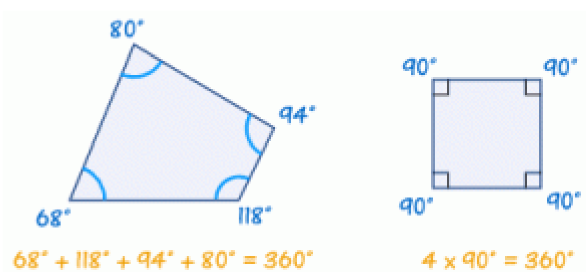


Properties

Four sides (edges)

Four vertices (corners)

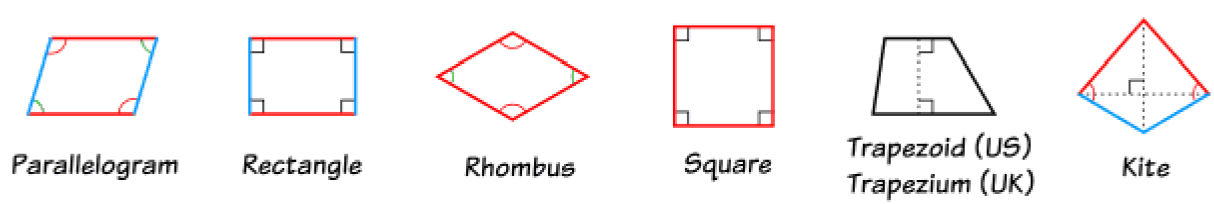
The interior angles add up to **360 degrees**:



Try drawing a quadrilateral, and measure the angles. They should add to **360°**

Types of Quadrilaterals

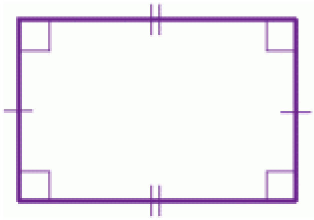
There are special types of quadrilateral:



Some types are also included in the definition of other types! For **<u>Example</u>** a **square**, **rhombus** and **rectangle** are also *parallelograms*.

Let us look at each type in turn:

The Rectangle

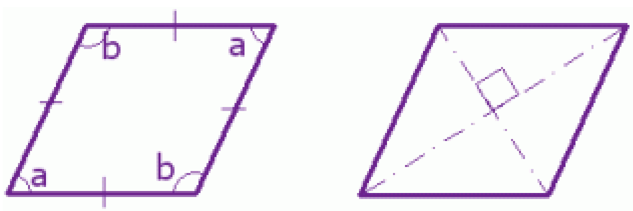


 means “right angle” | and || show equal side.

A rectangle is a four-sided shape where every angle is a right angle (90°).

Also **opposite sides** are parallel and of equal length.

The Rhombus



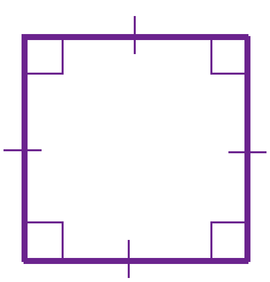
A rhombus is a four-sided shape where all sides have equal length.

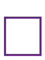
Also opposite sides are parallel *and* opposite angles are equal.

Another interesting thing is that the diagonals (dashed lines in second figure) meet in the middle at a right angle. In other words they “bisect” (cut in half) each other at right angles.

A rhombus is sometimes called a **rhombus** or a **diamond**.

The Square



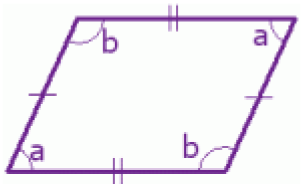
 means “right angle” | shows equal side.

A square has equal sides and every angle is a right angle (90°)

Also opposite sides are parallel.

A square also fits the definition of a **rectangle** (all angles are 90°), and a **rhombus** (all sides are equal length).

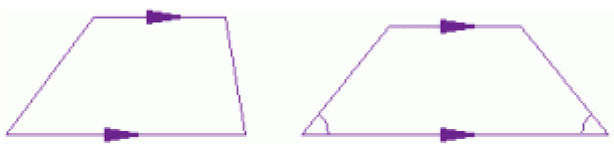
The Parallelogram



A parallelogram has opposite sides parallel and equal in length. Also opposite angles are equal (angles “a” are the same, and angles “b” are the same).

NOTE: Squares, Rectangles and Rhombuses are all Parallelograms!

The Trapezoid

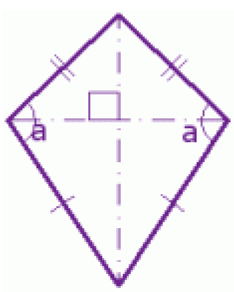


A trapezoid (called a trapezium in the UK) has a pair of opposite sides parallel.

It is called an **Isosceles** trapezoid if the sides that aren’t parallel are equal in length and both angles coming from a parallel side are equal, as shown.

And a **trapezium** (UK: trapezoid) is a quadrilateral with NO parallel sides:

The Kite



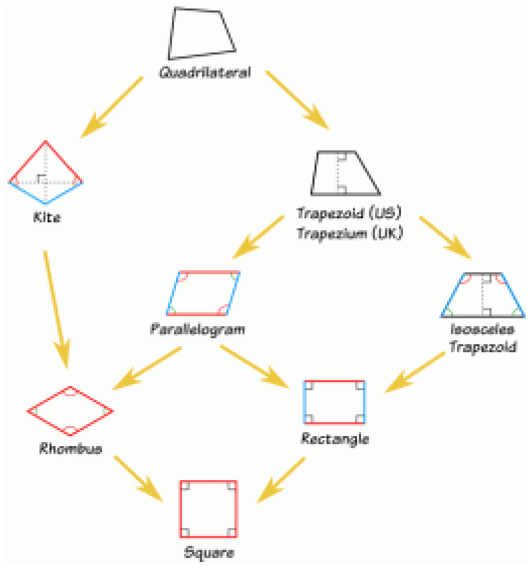
Hey, it looks like a kite. It has two pairs of sides. Each pair is made up of adjacent sides that are equal in length. The angles are equal where the pairs meet. Diagonals (dashed lines) meet at a right angle, and one of the diagonal bisects (cuts equally in half) the other.

Note: The only regular quadrilateral is a square. So all other quadrilaterals are **irregular**.

Relationships between quadrilaterals

The definition shows that squares are special rectangles. Thus all squares are also rectangles. However, some rectangles are not squares.

Parallelograms have two pair sides parallel. This is also true of squares, rectangles and rhombuses. Thus squares, rectangles and rhombuses are special parallelograms. However, there are many parallelograms which are not squares, rectangle or rhombuses.



Exercise

1. Name four shapes which are special **Example**s of trapezium

2. Name three shapes which are special **Example**s of parallelograms

3. Name two shapes which are special **Example**s of kites.

Week 10

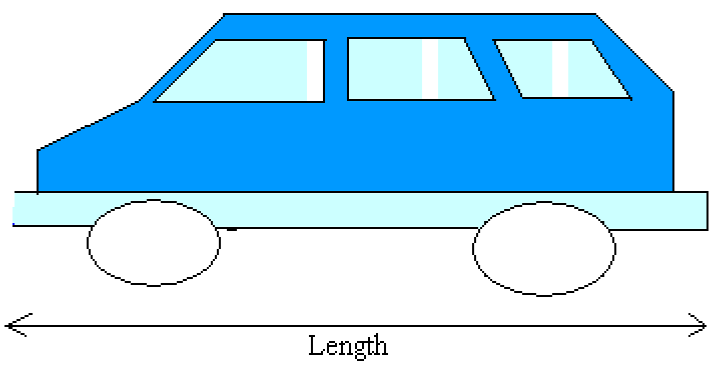
Topic: SCALE DRAWING

A drawing that shows a real object with accurate sizes except they have all been reduced or enlarged by a certain amount (called the scale).

The scale is shown as the length in the drawing, then a colon (":"), then the matching length on the real thing.

Example: this drawing has a scale of "1:10", so anything drawn with the size of "1" would have a size of "10" in the real world, so a measurement of 150mm on the drawing would be 1500mm on the real horse.

Since it is not always possible to draw on paper the actual size of real-life objects such as the real size of a car, an airplane, we need scale drawings to represent the size like the one you see below of a van.



In real-life, the length of this van may measure 240 inches. However, the length of a copy or print paper that you could use to draw this van is a little bit less than 12 inches

Since 240/12 = 20, you will need about 20 sheets of copy paper to draw the length of the actual size of the van

In order to use just one sheet, you could then use 1 inch on your drawing to represent 20 inches on the real-life object

You can write this situation as 1:20 or 1/20 or 1 to 20
Notice that the first number always refers to the length of the drawing on paper and the second number refers to the length of real-life object.

Example

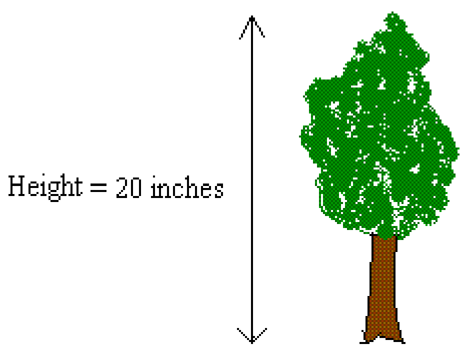
Suppose a problem tells you that the length of a vehicle is drawn to scale. The scale of the drawing is 1:20
If the length of the drawing of the vehicle on paper is 12 inches, how long is the vehicle in real

life?
Set up a proportion that will look like this:

Length of drawing/Real length = 1/20

Do a cross product by multiplying the numerator of one fraction by the denominator of the other fraction
We get :
Length of drawing × 20 = Real length × 1
Since length of drawing = 12, we get:
12 × 20 = Real length × 1
240 inches = Real length

Example
The scale drawing of this tree is 1:500
If the height of the tree on paper is 20 inches, what is the height of the tree in real life?



Set up a proportion like this:

Height of drawing/Real height = 1/500

Do a cross product by multiplying the numerator of one fraction by the denominator of the other fraction
We get :
Height of drawing × 500 = Real height × 1
Since height of drawing = 20, we get:
20 × 500 = Real length × 1
10000 inches = Real height

Scale Drawings

A map cannot be of the same size as the area it represents. So, the measurements are **scaled down** to make the map of a size that can be conveniently used by users such as motorists, cyclists and bushwalkers. A scale drawing of a building (or bridge) has the same shape as the real building (or bridge) that it represents but a different size. Builders use scaled drawings to make buildings and bridges.

A ratio is used in scale drawings of maps and buildings. That is:

The scale of a drawing = Drawing length : Actual length
Likewise, we have:

Map scale = Map distance : Actual distance
A scale is usually expressed in one of two ways:

Using units as in 1 cm to 1 km

Without explicitly mentioning units as in 1 : 100 000.

Note

A scale of 1 : 100 000 means that the real distance is 100 000 times the length of 1 unit on the map or drawing.

Example

Write the scale 1 cm to 1 m in ratio form.

Solution

1 cm to 1m = 1 cm : 1m

= 1 cm : 100 cm

= 1 : 100

Example

Simplify the scale 5 mm : 1 m.

Solution

5 mm : 1 m = 5 mm : 100 cm

= 5 mm : 1000 mm

= 5 : 1000

= 1 : 200

Example

Simplify the scale 5 cm : 2 km.

Solution

5 cm : 2 km = 5 cm : 2000 m

= 5 cm : 200 000 cm

= 5 : 200 000

= 1 : 40 000

Calculating the Actual Distance using the Scale

If the scale is 1 : *x*, then multiply the map distance by *x* to calculate the actual distance.
Example

A particular map shows a scale of 1 : 5000. What is the actual distance if the map distance is 8 cm?

Solution

Scale = 1 : 5000 = 1 cm : 5000 cm

∴ Map distance : Actual distance = 1 : 5000

Map distance = 8 cm

Let the actual distance be *a* cm.

∴ 8 : a = 1: 5000 {Units are in cm}

8/a = 1/5000 {Invert the fractions}

a/8 = 5000/1 {Multiply by 8}

8 X a/8 = 8 X 5000

a = 40 000

∴Actual distance = 40 000 cm

= 40 000/100 m

= 400 m

Alternative Way

Map distance = 8 cm

Scale = 1 : 5000 = 1 cm : 5000 cm

∴ Map distance : Actual distance = 1: 5000

= 1 X 8 : 5000 X 8

= 8 : 40 000

∴ Actual distance = 40 000 cm

=40 000/100 m

= 400 m

Calculating the Scaled Distance using the Actual Distance

If the scale is 1 : *x*, then divide the actual distance by *x* to calculate the map distance.

ASSESSMENT

A particular map shows a scale of 1 cm : 5 km. What would the map distance (in cm) be if the actual distance is 14 km?

ANSWER

Scale = 1 cm : 5 km

∴ Scale factor = 5

Actual distance = 14 km

Map distance = Actual distance/Scale factor

=14/5

= 2.8

So, the map distance is 2.8 cm.

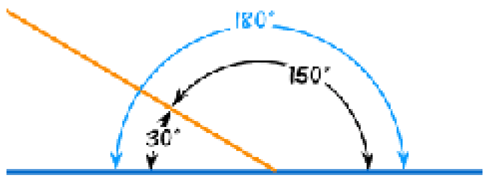
JSS 2
MATHEMATICS
THIRD TERM

Week 1 & 2

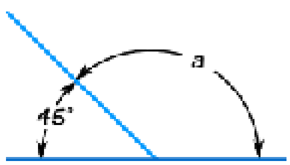
Angles between lines

Angles between lines

If a line is split into 2 and you know one angle you can always find the other one.



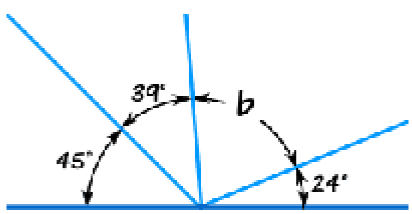
<u>Example</u>: If we know one angle is 45° what is angle "a" ?



Angle a is 180° − 45° = 135°

This method can be used for several angles on one side of a straight line.

<u>Example</u>: What is angle "b" ?



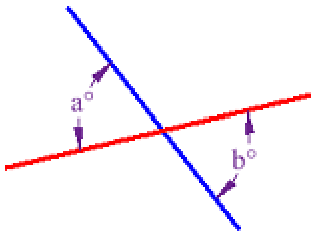
Angle **b** is 180° less the sum of the other angles.

Sum of known angles = 45° + 39° + 24°
Sum of known angles = 108°

Angle **b** = 180° − 108°
Angle **b** = 72°

Vertically opposite angles are equal

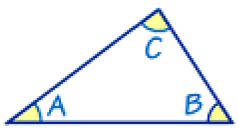
Vertically Opposite Angles are the angles opposite each other when two lines cross



“Vertical” in this case means they share **the same** Vertex (or corner point), not the usual meaning ofup-down.
In this **Example**, a° and b° are vertically opposite angles. The interesting thing here is that vertically opposite angles are equal:

$a^{o} = b^{o}$

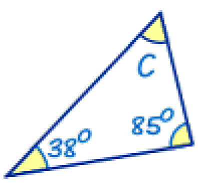
Angles in a triangle



In a triangle, the three interior angles always add to 180°:

A + B + C = 180°

Example: Find the Missing Angle “C”



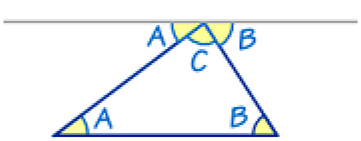
Start with:A + B + C = 180°

Fill in what we know:38° + 85° + C = 180°

Rearrange: C = 180° − 38° − 85°

Calculate: C = 57°

This is a proof that the angles in a triangle equal 180°:



The top line (that touches the top of the triangle) is running parallel to the base of the triangle.

So, angles A are the same, angles B are the same

And you can easily see that A + C + B does a **complete rotation** from one side of the straight line to the other, or **180°**

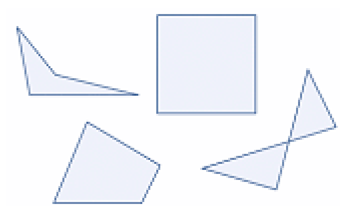
Angles in a quadrilateral

Any quadrilateral can be divided into two triangles by forming its diagonal

Quadrilateral just means "four sides" (*quad* means four, *lateral* means side).

Any four-sided shape is a Quadrilateral.

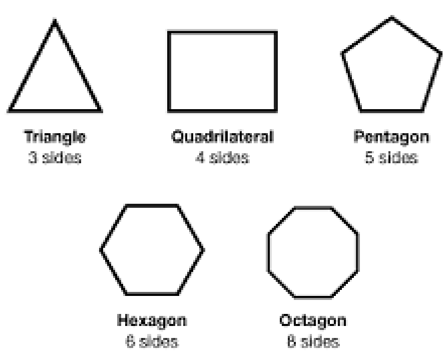
But the sides have to be **straight**, and it has to be **2-dimensional**.



Angles in a polygon

Polygons

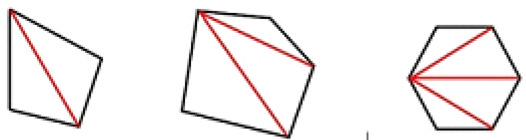
A polygon is any plane figure with straight sides. Thus a triangle is a three-sided polygon and a quadrilateral is a four-sided polygon Polygons are named after the number of sides they have.



triangle	3 sides
quadrilateral	4 sides
pentagon	5 sides
hexagon	6 sides
heptagon	7 sides

octagon	8 sides
nonagon	9 sides
decagon	10 sides

Sum of the interior angles of a polygon



Number of:	Quadrilateral	Pentagon	Hegagon
Sides	4	5	6
Diagonals	1	2	3
Triangles	2	3	4

In each polygon, one vertex is joined to all the other vertices. This divides the polygons into triangles. The number of triangles depends on the number of sides of the polygon.

In each case, the number of triangles is two less than the number of sides. For a polygon with n sides there will be n – 2 triangles. The sum of the angles of a triangle is 180^o.

Thus, in degrees:

the sum of the angles of an n-sided polygon

= (n – 2) x 180^o

and, in right angles:

the sum of the angles of an n – sided polygon

= (n – 2) x 2 right angles

= 2n – 4 right angles

Notice that these formulae are true for the polygons we already know.

In a triangle, n = 3.

Sum of angles = (3 – 2) x 180^o

= 1 x 180^o = 180^o

In a quadrilateral, n = 4.

Sum of angles = 2 x 4 – 4 right angles

= 4 right angles (= 360^o)

Example

Example 1:Find the number of degrees in the sum of the interior angles of an octagon.

An octagon has 8 sides. So $n = 8$. Using the formula from above, 180(n – 2) = 180 (8 – 2) = 180(6) = 1080 degrees.

Example

How many sides does a polygon have if the sum of its interior angles is 720°?

Since, the number of degrees is given, set the formula above equal to 720°, and solve for ***n.***

180(n – 2) = 720
n – 2 = 4
***n* = 6**

Assessment

- How many sides does a polygon have if the sum of its interior angles is 540

Week 3

Topic: ANGLES OF ELEVATION AND DEPRESSION

Any surface which is parallel to the surface of the earth is said to be **horizontal**. For **Example**, the surface of liquid in a container is always horizontal, even if the container is held at an angle.

The floor of your classroom is horizontal. Any line drawn on horizontal surface is will also be horizontal. Any line or surface which is perpendicular to a surface is said to be **vertical**. The walls of your classroom are vertical. A plum-line is a mass which hangs freely on a thread.

1. Say whether the following are horizontal or vertical, or neither:

a. the table top

b. the door

c. the pictures

d. the floor boards

e. the back of the chair

f. the table legs

g. the ruler (on the table)

h. the line where the walls meet

i. the brush handle

j. the top edge of the small

Elevation and depression

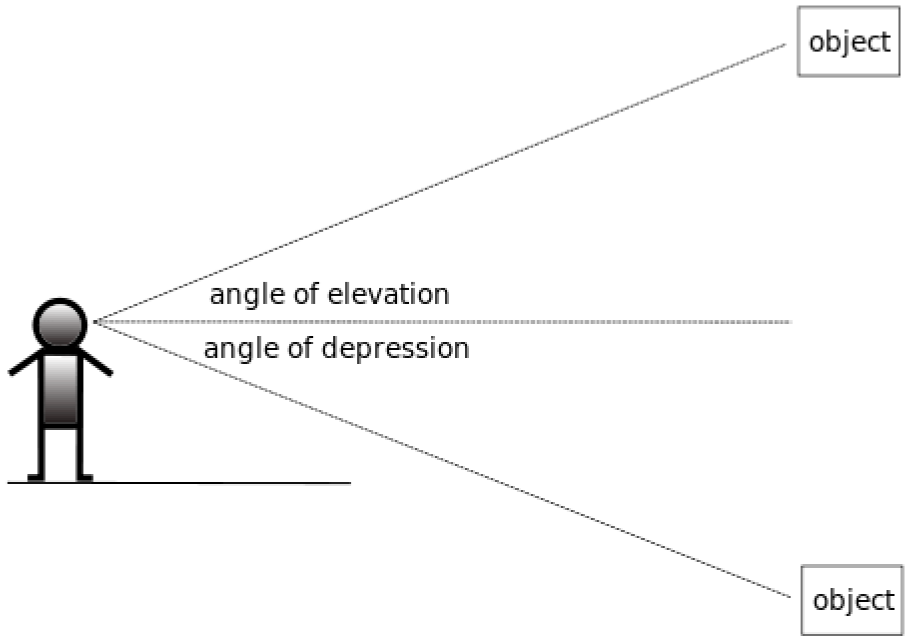
Suppose that you are looking at an object in the distance.

If the object is above you, then the angle of elevation is the angle your eyes look up.

If the object is below you, the angle of depression is the angle your eyes look down.

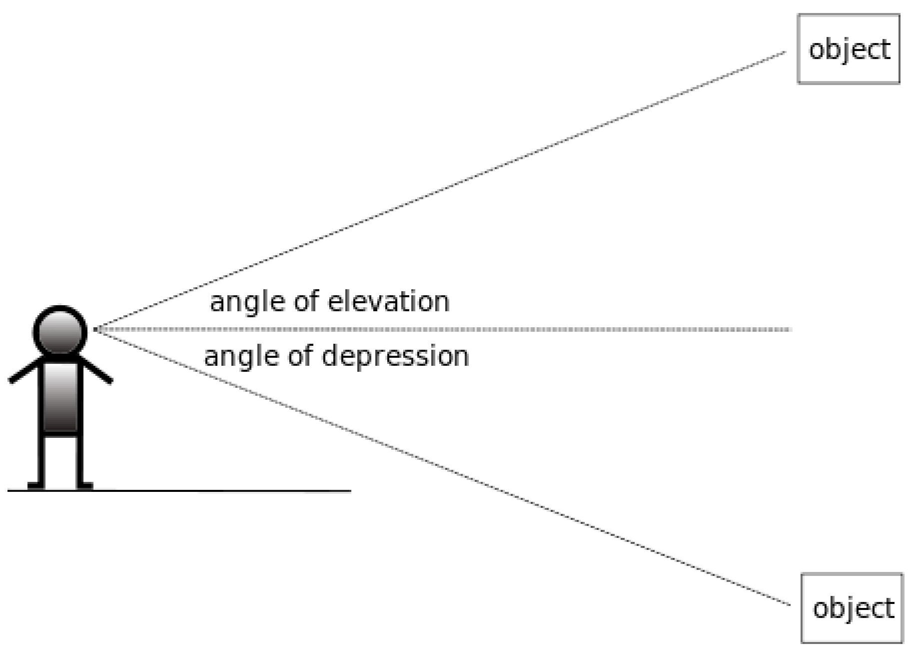
Angles of elevation and depression are measured from the horizontal.

It is common mistake not to measure the angle of depression from the horizontal.



Using the angle of depression or elevation to an object, and knowing how far away the object is, enables us to find the height of the object using trigonometry.

The advantage of doing this is that it is very difficult to measure the height of a mountain or the depth of a canyon directly; it is much easier to measure how far away it is (horizontal distance) and to measure the angle of elevation or depression.



Suppose that we want to find the height of this tree.

We mark point A and measure how far it is from the base of the tree.

Then we measure the angle of elevation from A to the top of the tree.

Now,

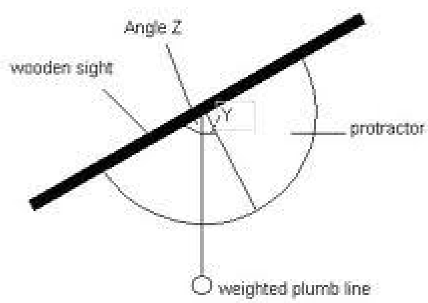
$h/x = \tan(\theta)$

$h = x\tan(\theta)$

we have measured *x* and θ, so we can calculate tan(θ) and thus we can find *h*, which is the height of the tree.

Angles of elevation and depression can be measured with a simple instrument called a **clinometers**.

A plum-line



Example

Solve for x
Solution

Angle of depression = 34°
But ∠ B = ∠ O (alternate angles)
therefore, ∠ B = 34°
From triangle ABO, we have
tan 34° = 40x
=> 0.6745 = 40x
=> 0.6745 x = 40
=> x = 400.6745
=> x = 59.30

ASSESSMENT

Which of these scenarios gives an angle of elevation or angle of depression;

1. A man is looking at a banner at the top of a building.
2. A woman looking at the coin on the floor.
3. A baby looking at an aeroplane in the sky.

Week 4

Topic: STATISTICS 2 – PRESENTATION OF DATA

Types of Presentation

Good **presentation** can make statistical data easy to read, understand and interpret. Therefore it is important to present data clearly.

i. There are two main ways of presenting data: presentation of numbers or values in **lists** and **tables;**

ii. Presentation using **graphs,** i.e. picture. We use the following **Example**s to show the various kinds of presentation.

An English teacher gave an essay
to 15 students.

She graded the essays from A (very good), through B, C.D, E to f (very poor). The grades of the students were:

B, C, A, B, A, D, F, E, C, C, A, B, B, E, B

Lists and tables

Rank and order list

Rank order means in order from highest to lowest. The 15 grades are given in rank order below:

A, A, A, B, B, B, B, C, C, C, E, E, F

Notice that all the grades are put in the list even though most of them appear more than once. The ordered list makes it easier to find the following: the highest and lowest grades; the number of students who got each grades; the most common grade; the number of students above and below each grade; and so on.

Frequency table

Frequency means the number of times something happens. For **Example**, three students got grade A.

The frequency of grade A is three. A **frequency table,** gives the frequency of each grade.

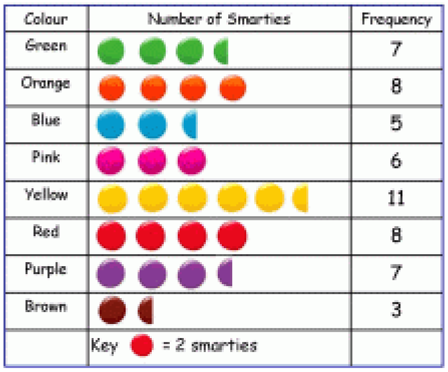
Grade	**A**	**B**	**C**	**D**	**E**	**F**
frequency	3	5	3	**1**	2	1

Graphical Presentation

In most cases, a picture will show the meaning of statistical data more clearly than a list of or table or numbers. The following methods of presentation give the data of the **<u>Example</u>** in picture, or **graph,** form.

Pictogram

A **pictogram** uses pictures or drawings to give a quick and easy meaning to statistical data.



Bar chart

A bar chart represents the data as horizontal or vertical bars. The length of each bar is proportional tothe amount that it represents.

There are 3 main types of bar charts.

Horizontal bar charts, vertical bar chart and double bar charts.

When constructing a bar chart it is important to choose a suitable scale to represent the frequency.

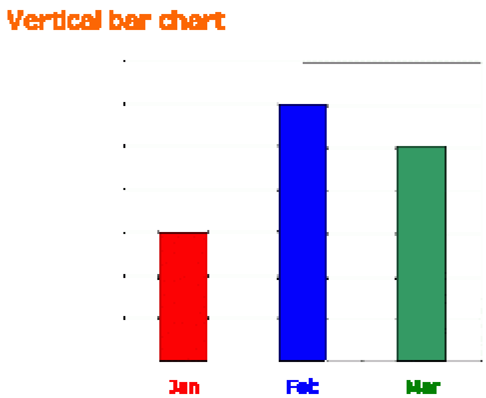
The following table shows the number of visitors to a park for the months January to March.

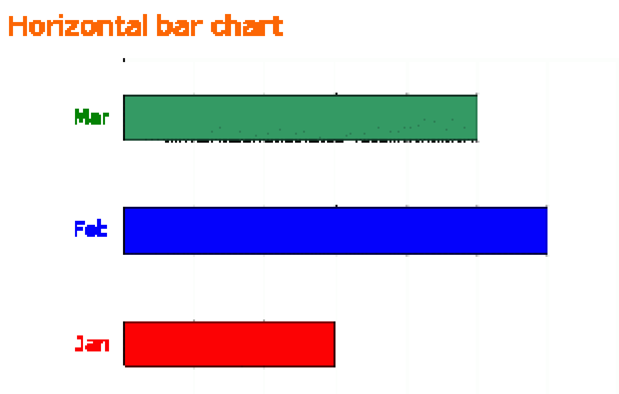
Month	January	February	March
Number of visitors	150	300	250

a) Construct a vertical and a horizontal bar chart for the table.

<u>Solution</u>

a) If we choose a scale of 1:50 for the frequency then the vertical bar chart and horizontal bar chart will be as shown.





Pie Chart

Pie charts are useful to compare different parts of a whole amount. They are often used to present financial information. E.g. A company's expenditure can be shown to be the sum of its parts including different expense categories such as salaries, borrowing interest, taxation and general running costs (i.e. rent, electricity, heating etc).

A pie chart is a circular chart in which the circle is divided into sectors. Each sector visually represents an item in a data set to match the amount of the item as a percentage or fraction of the total data set.

Example

A family's weekly expenditure on its house mortgage, food and fuel is as follows:

Expenses	N
Mortgage	300
Food	225

Fuel 75

Draw a pie chart to display the information.

Solution

The total weekly expenditure = N300 + N225 + N75 = N600

We can find what percentage of the total expenditure each item equals.

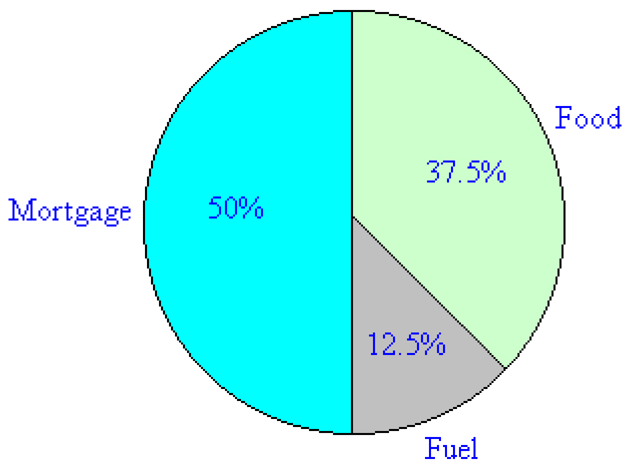
Percentage of weekly expenditure on:

Mortgage = 300/600 X 100% = 50%

Food = 225/600 X 100% = 37.5%

Fuel = 75/600 X 100% = 12.5%

To draw a pie chart, divide the circle into 100 percentage parts. Then allocate the number of percentage parts required for each item.



Note

It is simple to read a pie chart. Just look at the required sector representing an item (or category) and read off the value. For **Example**, the weekly expenditure of the family on food is 37.5% of the total expenditure measured.

A pie chart is used to compare the different parts that make up a whole amount.

Exercise

The following is a rank order list of an exam result: 87, 82, 78, 76, 75, 70, 66, 64, 59, 59, 59, 51, 49, 48, 41.

a. How many students took the exam?

b. What was the highest rank?

c. What was the lowest rank?

d. What is the mark of the student who came 6th?

e. What is the position of the student who got 76 marks?

f. Three students got 59 marks. What is their position?

g. How many students got less than 75 marks?

Week 5

Topic: PROBABILITY

Experimental Probability

A farmer asks, 'Will it rain this month?'. The answer to the farmer's question depends on three things; the months, the place where the farmer is, and what has happened in the past three months in that place. The table below gives some answers to the question for different places and months.

Place	**Month**	**Answer to question**
Sokoto	February	No
Jos	July	Ye
Ibadan	January	Maybe
Port Harcourt	June	yes

Is it possible to give a more accurate answer to a farmer near Ibadan in January? The table below shows that on average, 10mm of rain falls in Ibadan in January. However, this is an average found by keeping records over twelve years. The actual rainfall for Ibadan in January over the 12 years was as follows.

	J	**F**	**M**	**A**	**M**	**J**	**J**	**A**	**S**	**O**	**N**	**D**
Sokoto	0	0	0	10	48	91	155	249	145	15	15	0
Jos	3	3	28	56	203	226	330	292	213	41	3	3
Ibadan	10	23	89	137	150	188	160	84	178	155	46	10
Port Harcourt	66	109	155	262	404	660	531	318	516	460	213	81

18 mm	0 mm	17 mm	9 mm
11 mm	22 mm	14 mm	0 mm
16 mm	0 mm	7 mm	6 mm

From the above data, it can be seen that rain fell in nine of the twelve months on January. If future years follow the pattern of the past, it is likely that in Ibadan, rain will fall in nine out of the next 12 Januaries. We say that the **probability** of rain falling in Ibadan in January is 9/12 (or ¾ or 0.75 or 75%). This probability can never be exact. However, it is the best measure that we can give from the data we have. The number 9/12 is based on the experimental records. We say that it is **experimental probability**.

Example 1

A girl writes down the number of male and female children of her mother and father. She also writes down the number of male and female children of the parents' brothers and sisters. Her resuts are shown in table below.

	Number of male children	**Number of female children**
Mother and father	2	5
Mother's brother	6	8
Mother's sister	4	8
Father's brother	5	8
Father's sister	7	7
Total	**24**	**36**

a. Find the experimental probability that when the girl has children of her own, her first born will be a girl.

b. If the girl eventually has five children, how many are likely to be male?

Solution

In the girl's family there is a total of 60 children. 36 of these are female. If the girl's own children follow the pattern of her family, then the experimental probability that her first born will be a girl is 36/6 = 3/5.

b. Following the family pattern, 3/5 of the girl's children will be female and 2/5 will be male.

Number of male children that the girl is likely to have = 2/5 of 5 =2.

Notice that the results in the above **Example** are based on experimental probability. Thus we are using the past to predict the future. Events can easily turn out differently. The answers in the **Example** above are no more that calculated guesses.

Exercise

1. A woman has four children. They are all males. The children of the rest of her family are equally divided between males and females. What is the woman's next child likely to be, male or female?

2. A rainmaker throws some kola nuts n the ground. From the pattern of kola nut, he says that rain will fall next week. Is this a good method? Does it always work? Compare this method with the use of rainfall records. Can rainfall method always tell us when rain will fall?

Probability as a fraction

Probability is a measure of the likelihood of a **required outcome** happening. It is usually as a fraction:
Probability = number of required outcomes/number of possible outcomes

In **Example** above, the required outcomes were female children and the possible outcomes were both male and female children. Thus probability of having a female child

= number of female children/number of male and female children

= 36/60 = 3/5 = 0.6

If we are completely sure that something will happen, the probability is 1. For **Example**, if today is Tuesday, the probability that tomorrow is Wednesday is 1.

If we are sure that something cannot happen, the probability is 0. For **Example**, the probability of rolling a 7 on a pencil is 0, because there is no number 7 in the pencil. If the probability of something happening is x, then the probability of it not happening is 1 – x. For **Example**, if the probability of it raining month is 9/12, then the probability of it not raining is 3/12.

Example

1. It is known that out of every 1 000 cars, 50 develop a mechanical fault in the first 3 months. What is the probability of buying a car that will develop a mechanical fault within 3 months?

Number of cars developing faults = 50

Number of cars altogether = 1 000

Probability of buying a faulty car = 50/1 000 = 1/20

2. A market trader has 100 hundred oranges for sale. Four of them are bad. What is the probability that an orange chosen at random is good?

'At random means without carefully choosing'.

Either:

Four out of 100 oranges are bad, thus 96 out of 100 oranges are good.

Probability of getting a good orange = 96/100 = 24/25

or:

probability of getting a bad orange = 4/100 = 1/25

thus,

probability of getting a good orange = 1 – 1/25 = 24/25.

The next **Example** shows how to use probability when analyzing statistical data.

Example

City school enters candidates for the WASSCE. The results for the years 2004 – 2008 are given in the table below:

Year	2004	2005	2006	2007	2008
Number of candidates	86	93	102	117	116
Number gaining WASSCE passes	51	56	57	65	70

a. Find the school's success rate at a percentage.

b. What is the approximate probability of a student at City School getting a WASSCE pass?

Solution

a. Total number of passes

= 51 + 56 + 57 + 65 + 70

= 299

Total number of candidates

= 86 + 93 + 102 + 117 + 116

= 514

Success rate as a percentage

= 299/514

=0.58 to 2.s.f.

Success rate as a percentage = 0.58 X 100% = 58%

b. The probability of a student getting a WASSCE pass = 0.58 = 0.6 to 1 s.f.

In part **b** it is assumed that the student's chances of success are the same as the school's success rate.

Exercise

a. The probability of passing an examination is 0.8. What is the probability of failing the examination?

b. The probability that a girl wins a race is 0.6. What is the probability that she loses?

c. The probability that a pen does not write is 0.05. What is the probability that it writes.

Answers

a. 0.2 b. 0.4 c. 0.9

Week 6

Topic: SOLVING EQUATIONS

Solving Equations (1)

2x – 9 = 15 is an **equation** in x. x is the **unknown** in the equation. 2x – 9 is on the left-hand side (LHS) of the equals sign and 15 is on the right-hand side (RHS) of the equal sign.

To **solve an equation** means to find the value of the unknown that makes the equation true.

The balance method (revision)

Think of the two sides of an equation as forming a balance. Keep the balance by doing the same operation to both sides of the equation.

Example

Solve 3x = 12

3x = 12

Divide both the LHS and RHS by 3, the coefficient of the unknown. This keeps the balance of the equation.

3x/3 = 12/3

x = 4

x = 4 is the solution of the equation 3x = 12

check: when x = 4, LHS = 3 X 4 = 12 = RHS

Example

Solve 2x – 9 = 15.

2x – 9 = 15

a. The LHS contains the unknown. Add 9 to 2x – 9. This leaves 2x. 9 must also be added to the RHS to keep the balance of the equation.

2x – 9 = 15

Add 9 to both sides (+9 is the additive inverse of -9)

Simplify 2x = 24

b. The equation is now simpler. Divide the LHS by 2 to leave x. The RHS must also be divided by 2to keep the balance of the equation.

2x = 24

Divide both sides by 2.

2x/2 = 24/2

x = 12

x = 12 is the solution of the equation 2x − 9 = 15.

Check: when x = 12, LHS = 2 x 12 − 9 = 24 − 9 = 15 RHS.

Exercise

Use the balance method to solve the following:

a. 3x − 8 = 10

b. 4x − 1 = 1

c. 27 =10x − 3

Solving Equations (2)

Using directed numbers

It is possible to use operations with directed numbers when solving equations.

Example

Solve 25 − 9x = 2

25 − 9x = 2

Subtract 25 from both sides.

25 − 25 − 9 = 2 − 25

− 9x = − 23

Divide both sides by -9.

− 9x/−9 = -23/−9

x = 23/9 = **2** 5/9

check: when x = 23/9,

LHS = 25 − 9 X 23/9 = 25 − 23 = 2 = RHS

Unknowns on both sides

If an equation has unknown terms on both sides of the equal sign, collect the unknown terms on one side and the number terms on the side.

Example

Solve 5x − 4 = 2x + 11

5x − 4 = 2x + 11 (1)

Subtract 2x from both sides of (1).

$5z - 2x - 4 = 2x - 2x + 11$

$3x - 4 = 11$ (2)

Add 4 to both sides of (2).

$3x - 4 + 4 = 11 + 4$

$3x = 15$

Divide both sides of (3) by 3. (3)

$x = 5$

Check: $x = 5$,

LHS = 5 x 5 -4 25 – 4 =21

RHS = 2 x 5 + 11 = 10 + 11 = 21 = LHS

Note that equations (1), (2), and (3) are still equivalent.

Exercise

a. $13 - 6 = 1$

b. $4b + 24 = 0$

c. $12 + 5a = 23$

Equations with brackets

Always remove brackets before collecting terms.

Solve $3(3x - 1) = 4(x + 3)$

$3(3x - 1) = 4(x + 3)$ (1)

Remove brackets.

$9x - 3 = 4x + 12$ (2)

Subtarct 4x from both sides and add 3 to both sides.

$9x - 4x -3 + 3 = 4x - 4x + 12 + 3$

$5x = 15$ (3)

Divides both sides by 5.

$x = 3$

Check: when $x = 3$,

LHS = 3(3 x 3 -1) = 3(9 − 1) = 3 X 8 = 24

RHS = 4(3 + 3) = 4 X 6 = 24 = LHS

Example

Solve 5(x + 11) + 2(2x − 5) = O.

5(x + 11) + 2(2x − 5) = O.   (1)

5(x + 11) + 2(2x − 5) = O.

Remove brackets.

5x + 55 + 4x − 1O = O   (2)

Collect like terms.

9x + 5 = O   (3)

Subtract 45 from both sides.

9x = -45   (4)

Divide both sides by 9.

x = -5

Check: when x = -5

LHS = 5(-5 + 11) + 2(2 X (-5) -5)

= 5 X 6 + 2(-1O -5)

= 3O + 2 X (-15) = 3O − 3O = O = RHS

Exercise

a. 5(x − 4) − 4(x + 1) = O

b. 3(2x + 3) − 7(x + 2) = O

c. 2(x + 5) = 18

Equations with fractions

Always clear fractions before collecting terms. To clear fractions multiply both sides of the equation by the LCM of the denominators of the fractions.

Example

Solve the equation 4m/5 – 2m/3 = 4.

4m/5 – 2m/3 = 4

The LCM of 5 and 3 is 15.

Multiply both sides of the equations by 15, i.e. multiply every term by 15.

15 X (4m/5) – 15 X (2m/3) = 15 X 4

3 X 4m – 5 X 2m = 15 X 4

12 – 10m = 60

2m = 60

Divide both sides by 2.

m =30

check: when m = 30,

LHS = 4 X 30/5 – 3 X 30/3 = 120/5 – 60/3

= 24 – 20 = 4 = LHS

Example

Solve the equation 3x – 2/6 – 2x + 7/9 = 0.

The LCM of 6 and 9 is `8.

18(3x – 2)/6 – 18(2x + 7)/9 = 18 X 0

3(3x – 2) – 2(2x + 7) = 0

Clear brackets.

9x – 6 – 4x – 14 = 0

Collect like terms.

5x – 20 = 0

Add 20 to both sides.

5x = 20

Divide both sides by 5

x = 4

Check: when x = 4

LHS = 3 X 4 – 2/6 – 2 X 4 + 7/9

= 12 – 2/6 – 8 + 7/9

= 10/6 – 15/9 = 5/3 -5/3 = 0 = RHS

<u>Exercise</u>

a. x/3 = 5

b. x/5 = ½

c. 4/3 = 2z/15

d. x – 2/3 = 4

Word Problems

We can use equations to solve **word problems,** i.e. problems using everyday language instead of just numbers or algebra. There is always an **unknown** in a word problem. For **<u>Example</u>**, if a question says *what is the length of the room?.* Then length is the unknown and the task is to find its numerical value.

From words to algebra

When solving a word problem:

1. Choose a letter for the unknown

2. Write down the information of the question in algebra form.

3. Make and equation.

4. Solve the equation

5. Give the answer in written form

6. Check the result against the information given in the question.

<u>Example</u>

I think of a number. I multiply it by 5. I add 15. The result is 100. What is the number I thought of.

Let the number be n

I multiply n by 5: 5n

I add 15: 5n + 15

The result is 100; 5n + 15 = 100 (1)

Subtract 15 from both sides of (1).

$5n + 15 - 15 = 100 - 15$

$5n = 85$ (2)

Divides both sides of (2) by 5.

$5n/5 = 85/5$

$n = 17$

The number is 17.

Check: 17 X 5 = 85; 85 + 15 = 100

Example

When 6 is added to four times a number, the result is 50. Find the number.

Step 1: What are we trying to find?

A number.

Step 2: Assign a variable for the number.

Let's call it *n*.

Step 3: Write down what the variable represents.

Let n = a number

Step 4: Write an equation.

We are told 6 is added to 4 times a number. Since n represents the number, four times the number would be $4n$. If 6 is added to that, we get $6 + 4n$. We know that answer is 50, so now we have an equation $6 + 4n = 50$

Step 5: Solve the equation.

$6 + 4n = 50$

$4n = 44$

$n = 11$

Step 6: Answer the question in the problem

The problem asks us to find a number. We decided that n would be the number, so we have $n = 11$. The number we are looking for is 11.

Step 7: Check the answer.

The answer makes sense and checks in our equation from Step 4.

$6 + 4(11) = 6 + 44 = 50$

Exercise

1. John thinks of a number. He doubles it. His result is 58. What numbe did John think of?

2. Six boys each have the same number of sweets. The total number of sweets is 78. How many sweets did each boy have?

3. A number is multiplied by 6 and then 4 is added. The result is 34. Find the first number.

Word problems with brackets

Example

1. If fish cost £2 and chips cost £1 and you went into the shop and asked for 2 fish and chips would you be expecting to pay £5 or £6?

Well it all depends on what you actually wanted.

Was it?

2 x (fish and chips) = 2 x (£2 + £1) = 2 x £3 = £6

or

(2 x fish) + chips = (2 x £2) + £1 = £4 + £1 = £5

2. I bought 3 boxes of eggs in the market. Each box contained 12 eggs. When I got home I found that 5 were broken and had to be thrown away. How many eggs did I have left?

(3 x 12) – 4 = 32

36 – 4 = 32

If I had not done the calculation in brackets first, I could have got 24 as an answer

3 x 12 – 4 = 24

3 x 8 = 24

and that would have been the wrong answer.

Exercise

1. The farmer has four chicken runs. In each run there are 67 brown and fourteen black hens. How many chicken are there altogether?

Hint:(67+14) x 4 = 224

2. 124 cakes were bought, but there wasn't enough so they decided to buy 4 times more. Then there were too many so they took 10 away. How many did they have in the end?

Hint: (124×4) – 10 = 486

Word Problems with fractions

I add 55 to a certain number and then divide the sum by 3. The result is four times the first number. Find the number.

Let the number n.

I add 55 to n: this gives $n + 55$

I divide the sum by 3: this gives $n + 55/3$

The result is 4n.

So, $n + 55/3 = 4n$ (1)

Multiply both sides by 3.

$3(n + 55)/3 = 3 \times 4n$ (2)

$n + 55 = 12n$ (3)

Collect terms.

$55 = 12n - n$

$55 = 11n$ (4)$2x - 9 = 15$ is an **equation** in x. x is the **unknown** in the equation. $2x - 9$ is on the left-hand side (LHS) of the equals sign and 15 is on the right-hand side (RHS) of the equal sign.

So,

$n = 5$, the number is 5.

Assessment

1. I think of a number. I double it. I divide the result by 5. My answer is 6. What number did I think of?

2. I subtract 17 from a certain number and then divide the result by 5. My final answer is 3. What was the original number?

3. I add 9 to a certain number and then divide the sum by 16. Find the number if my final answer is 1.

WEEK 7

Topic: USING CALCULATORS AND TABLES

Power

Most calculators get their power from a solar cell. This powers the calculator so long as light is available (daylight, electric bulb or even candle-light).

Display

The display shows the answers. The digits in the display are usually made of small line segments.

Keyboard

The keyboard has four main set of keys or buttons:

1. **Number keys**

Press these keys: “0”, “1”, “2”, “3”, “4”, “5”, “6” ,”7” , “8”, “9” and the decimal point key (usually shown as a dot .) to enter number into the calculator.

2. Basic calculation keys

Press these keys: “+”,” −“, “X”, “÷”, “%”, “√” and “=” to operate on the numbers you have entered, and to display answers.

3. **Clearing keys**

The “c” key clears the last number you entered. Press “c” if you entered a wrong number by mistake. On some calculator “CE” is written instead of “C”. The “AC” key clears the whole calculation that you are working on. Use this if you want to start from the beginning again. Often the “AC” key is linked to the calculator’s ON key and is written as “ON/AC” or just as “AC”. Press “AC” before starting any calculation and 0 is shown on the display.

4. **Memory keys**

a. Memory plus key

Press M+ to store the displayed number in the memory of the calculator. If there is any previous number in the number, it adds the displayed number to it.

b. Memory minus key

Press “M –“ to subtract the displayed number from the number in the. The answer obtained from the addition or subtraction will be the new number in the memory.

c. Memory recall key

If there is a number in the memory, the calculator usually shows a small M in the corner of the display.

Press “MR” to display the number in the memory.

d. Memory clear key

Press “MC” to clear the number stored in the memory.

Exercise

1. Display on your calculator:

a. The highest possible number,

b. the lowest possible number.

2. To find what a snail lives in:

a. calculate 5 X 31 X 499;

b. turn your calculator upside down and read the display.

3. Use your calculator to complete the table below

Power of 7	**value**
7^1	7
7^2	49
7^3	343
7^4	
7^5	
7^6	
7^7	
7^8	
7^8	

b. Look at the final digits of the values displayed in the table above. Is there a pattern? If so, what is it?

c. Is there a recognizable pattern in the final two digits?

Using Tables

If calculators are not available, then calculations can be done by using tables such as the one below.

Table of squares can be used to convert 2-digit numbers to squares of those numbers.

Tables of squares and square roots (Squares from 1.0 to 9.9).

	.0	.1	.2	.3	.4	.5	.6	.7	.8	.9
1	1.00	1.21	1.44	1.69	1.96	2.25	2.56	2.89	3.24	3.61
2	4.00	4.41	4.48	5.29	5.76	6.25	6.76	7.29	7.84	8.41
3	9.00	9.61	10.24	10.89	11.56	12.25	12.96	13.69	14.44	15.21
4	16.00	16.81	17.64	18.49	19.36	20.25	21.16	22.09	23.04	24.01
5	25.00	26.01	27.04	28.09	29.16	30.25	31.36	32.49	33.64	34.81
6	36.00	37.21	38.44	39.69	40.96	42.25	43.56	44.89	46.24	47.61
7	49.00	50.41	51.84	53.29	45.76	56.25	57.76	59.29	60.84	62.41
8	64.00	65.61	67.24	68.89	70.56	72.25	73.96	75.69	77.44	79.21
9	81.00	82.81	84.64	86.49	88.36	90.25	92.16	94.09	96.04	98.01

Approximate Square roots

$5^2 = 5 \times 5 = 25$

Thus $\sqrt{25} = 5$.

Numbers that have exact square roots are said to be **perfect square.** We can find the approximate square root of a number by knowing the perfect squares immediately before and after it.

Example

Between what whole numbers do the square roots of the following numbers lie?

a. 3.6 b. 9.4 c. 40 d. 78

Solution

a. $\sqrt{3.6}$

3.6 lies between 1 and 4

thus $\sqrt{3.6}$ lies between $\sqrt{1}$ and $\sqrt{4}$

i.e. $\sqrt{3.6}$ lies between 1 and 2

b. $\sqrt{9.4}$

9.4 lies between 9 and 16

thus $\sqrt{9.4}$ lies between $\sqrt{9}$ and $\sqrt{16}$

i.e. $\sqrt{9.4}$ lies between 3 and 4

c. $\sqrt{40}$

40 lies between 36 and 49

Thus $\sqrt{40}$ lies between $\sqrt{36}$ and $\sqrt{49}$

i.e. 40 lies between 6 and 7

d. $\sqrt{78}$

78 lies between 64 and 81

Thus $\sqrt{78}$ lies between $\sqrt{64}$ and $\sqrt{81}$

i.e. $\sqrt{78}$ lies between 8 and 9

Exercise

1. Find the value of the following

a. 1.4^2 b. 2.3^2 c. 6.8^2 d. 7.2^2 e. 4.9^2 f. 8.5^2

Week 8

Topic: PYTHAGORAS' THEOREM

Pythagorean theorem which states the special relationship between the sides of a right triangle is perhaps the most popular and most applied theorem in Geometry. The algebraic statement of the Pythagorean theorem is used to derive the distance formula in coordinate Geometry and to prove the Pythagorean identities in Trigonometry. In fact, the fundamentals of Trigonometry are taught using the ratios of the sides of a right triangle.

Right triangles and Pythagorean theorem are not only used to solve real life problems, but often used in solving many advanced problems in Mathematics and Physical Sciences.

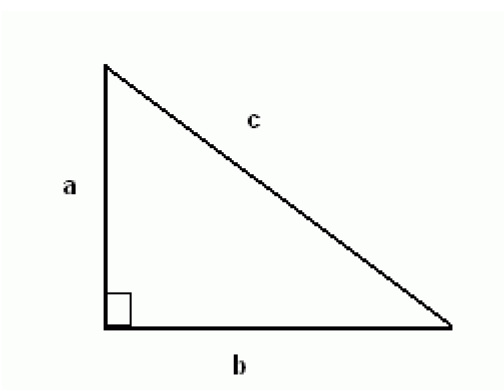
Euclid used squares drawn on the sides of the right angles and showed the area of the square drawn on the hypotenuse is equal to the sum of the areas of the squares drawn on the legs of a right triangle.

The algebraic form of the statement of Pythagoras theorem $c^2 = a^2 + b^2$ is used in solving right triangles.

The statement of the Pythagorean theorem is as follows:

In a right triangle the sum of the squares of the lengths of the legs is equal to the square of the length of the hypotenuse.

If Δ ABC is a right triangle,



then the hypotenuse is the side opposite to the right angle (AB) and the legs (BC and CA) are the sides containing the right angle.

Then according to Pythagorean theorem $BC^2 + CA^2 = AB^2$ or $a^2 + b^2 = c^2$.

Pythagoras' Theorem Formula

The algebraic form of the Pythagorean theorem
$c^2 = a^2 + b^2$.
is used as a formula to solve for the third side of a right triangle if the lengths of any two sides are given.
If the lengths of the legs are given, the formula used to find the length of the hypotenuse is $c = \sqrt{a^2 + b^2}$.

When the lengths of hypotenuse and one of the legs is known, we use one of the following formula to solve for the second leg.

$a = \sqrt{c^2 - b^2}$ or

$b = \sqrt{c^2 - a^2}$

We will be using these formulas when solving few **Example** problems on Pythagorean theorem.

Example

Finding the Length of the Hypotenuse:

Find the length x.

x is the length of the hypotenuse corresponding to the value 'c' in the formula

$c^2 = a^2 + b^2$.

$c^2 = 84^2 + 13^2$.

= 7056 + 169

= 7225

C = √7225 = 85

Hence the length of the hypotenuse = 85cm.

Example

Find the length of the unknown leg in the adjoining diagram.

We can take a = x, b = 48 and c = 50.
$a^2 = c^2 - b^2$.
$x^2 = 50^2 - 48^2$.
= 2500 – 2304 = 196
x = √196 = 14
Hence the length of the leg = 14″.

Pythagoras in real life

Pythagorean theorem is used to find the lengths, distances and heights using right triangles which model real life situations.

When fire occurs in high raise buildings, the fire fighting men cannot use the regular stairs or lifts. They can reach some floors using ladders. In order to determine the ladder length, they apply the Pythagorean theorem as they can estimate the height of the floor affected and the horizontal distance they can use to keep the ladder in position.

Example

A ladder of length 16 ft is placed against a wall. If the foot of the ladder is 7 ft from the wall find the height the ladder reaches on the wall nearest to the tenth of a foot.

Solution

The situation is described using the right triangle ABC, where AB represents the wall, BC the floor and AC the ladder.
It is required to find the measure h, which is the length of a leg in right triangle ABC. Using Pythagorean theorem,

$h^2 = AC^2 - BC^2$.
$= 16^2 - 7^2 = 256 - 49 = 207$.

$h = \sqrt{207} = 14.4$ ft (rounded to the tenth of a foot).

Assessment

A ladder of length 20 ft is placed against a wall. If the foot of the ladder is 10 ft from the wall find the height the ladder reaches on the wall nearest to the tenth of a foot.

Week 9

Topic: TABLES, TIMES TABLES AND CHARTS

Reading tabulated data

Tabulated data means information given in a table. Tabulated data is often in numerical form. Numerical tables are a neat way of storing a lot of information. They have a wide range of uses. When reading a numerical table, always look first at its title, its column headings and its row headings. These show what the table is all about. Look at the headings in the table below, it's a monthly rainfall chart. The column heading show the letters J, F, M, ... These are **abbreviations** for January, February, March, ...

	J	**F**	**M**	**A**	**M**	**J**	**J**	**A**	**S**	**O**	**N**	**D**
Sokoto	0	0	0	10	48	91	155	249	145	15	15	0
Jos	3	3	28	56	203	226	330	292	213	41	3	3
Ibadan	10	23	89	137	150	188	160	84	178	155	46	10
Port Harcourt	66	109	155	262	404	660	531	318	516	460	213	81

Abbreviations are often used in tables. They save space. We often have to use common sense when deciding what the abbreviations are short for. The row headings give the names of four towns for any month.

We read the table by making a **cross-reference.** For **Example**, to find the rainfall for Ibadan and down from S. Where the two directions corss, gives the information. On average, Ibadan gets 178 mm of rain in September.

Refer to the table above for the **Exercise**

1. Use the table to find the average rain fall for the following:

a. Sokoto in January

b. Port Harcourt in December

c. Jos in May

d. Ibadan in March

e. Port Harcourt in June

f. Sokoto in August

g. Ibadan in July

h. Jos in October

i. Sokoto in February

j. Jos in November

i. Port Harcout in September

2. For each town, name the month which has the highest rainfall.

3. For each town, name the month(s) with the lowest rainfall.

Social and environmental issues

The **Exercise** below gives practice in interpreting data based on important social and environmental issue. Discuss this question with your classmates and teacher. Supplement the **Exercise** with relevant articles and data from newspaper.

Exercise

The table below shows the gender gaps in primary school enrolment in six Local Government Authorities (LGA) of a state for the years 2006/07 to 2008/09. [A note on gender gap: if a school population is 54% boys and 46% girls, then the gender gap is 8 percentage points].

Average gender gap

LGA	**2006/07**	**2007/08**	**2008/09**
LGA 1	25.0	13.1	9.3
LGA 2	30.8	11.8	7.2
LGA 3	11.5	3.7	-4.7
LGA 4	35.6	20.9	15.5
LGA 5	33.9	21.6	22.7
LGA 6	28.6	25.2	19.2
Average	**27.6**	**16.0**	**11.5**

a. Did the gender gaps generally increase or decrease over the period?

b. Which LGA do you think had the best record?

c. Which LGA showed the greatest improvement?

d. Look at the LGA 3 in 2008/09. What does a negative gender gap mean?

Timetables and charts

Railway timetables

Railway timetables give the times or arrival and departure of trains for various towns.

The table below is a timetable for train services between Ibadan and Offa.

IBADAN – OFFA

		201 UP		202 DN
Ibadan	**D**	6.00 pm	**A**	11.30 am
Oshogbo	**A**	9.32 pm	**D**	7.58 am
	D	9.42 pm	**A**	7.48 am
Offa	**A**	11.45 pm	**D**	6.00 am

In the figure above, UP and DN means *up* and *down*. In Nigeria, UP trains travel in a direction away from the coast. DN trains travel towards the coast. 201 and 202 are the numbers of the train service.

Read the timetable in figure above.

1. What do the letters D and A stand for?

2. What time does 201 UP train **a.** depart from Ibadan, **b**. arrive Offa?

3. How long does the 'up journey' take from Ibadan to Offa?

4. How long does 201 UP train stay at Oshogbo?

5. What time does 202 DN train **a**. depart from Offa **b**. arrive Ibadan?

ASSESSMENT

Express the following times in terms of the 24-hour clock?

a. 5.40 am **b**. 2.10 pm **c**. 10.10 pm **d**. 9.26 am **e**. 7.25 pm **f**. 11.55 am

Week 10

Topic: CYLINDERS AND CONES

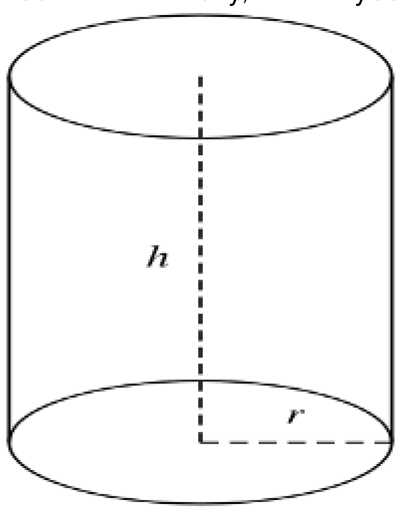
Shows that the net of a cylinder is made up of two circles and a rectangle. The total surface area of the cylinder will be the total area of the two circles and the rectangle.

Then, fold the rectangle until you make an **open cylinder** with it. An open cylinder is a cylinder that has no bases. A good real life **Example** of an open cylinder is a pipe used to flow water if you have seen one before

Next, using the two circles as bases for the cylinder, put one on top of the cylinder and put one beneath it.

obviously, the two circles will have the exact same size or the same diameter as the circles obtained by folding the rectangle

You Finally, you end up with your cylinder!



Now, what did we go through so much trouble? Well if you can make the cylinder with the rectangle and the two circles, you can use them to derive the surface area of the cylinder. Does that make sense?

The area of the two circles is straightforward. The area of one circle is $\pi \times r^2$, so for two circles, you get $2 \times \pi \times r^2$

To find the area of the rectangle is a little bit tricky and subtle!

Let us take a closer look at our rectangle again.

Thus, the longest side or folded side of the rectangle must be equal to $2 \times \pi \times r$, which is the circumference of the circle

To get the area of the rectangle, multiply h by $2 \times \pi \times r$ and that is equal to $2 \times \pi \times r \times h$

Therefore, the total surface area of the cylinder, call it SA is:

$SA = 2 \times \pi \times r^2 + 2 \times \pi \times r \times h$

Example

A cylindrical cup has a circular base of radius 7 cm and height of 10 cm. taking the value of π to be 22/7, calculate **a**. its curved surface area, **b**. the area of its circular base.

Solution

a. The area of the curved surface of a cylinder of radius r and height h is s $2\pi rh$.

The curved surface area of the cup

$= 2 X 22/7 X 7 X 10 cm^2$

$= 440 cm^2$

b. The area of the circular base of the cup

$= 22/7 X 7 X 7cm^2$

$= 154cm^2$

Example

Find the surface area of a cylinder with a radius of 2 cm, and a height of 1 cm

$SA = 2 \times \pi \times r^2 + 2 \times \pi \times r \times h$

$SA = 2 \times 3.14 \times 2^2 + 2 \times 3.14 \times 2 \times 1$

$SA = 6.28 \times 4 + 6.28 \times 2$

SA = 25.12 + 12.56

Surface area = 37.68 cm^2

<u>Example</u>:

Find the surface area of a cylinder with a radius of 4 cm, and a height of 3 cm

SA = $2 \times \pi \times r^2 + 2 \times \pi \times r \times h$

SA = $2 \times 3.14 \times 4^2 + 2 \times 3.14 \times 4 \times 3$

SA = $6.28 \times 16 + 6.28 \times 12$

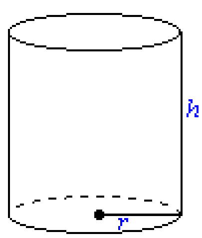
SA = 100.48 + 75.36

Surface area = 175.84 cm^2

Volume of a cylinder

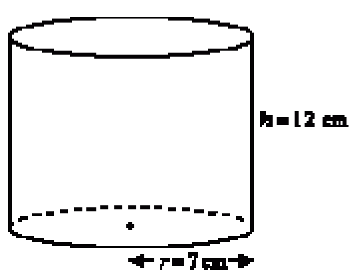
A cylinder with radius *r* units and height *h* units has a volume of *V* cubic units given by

$v = \pi r^2 h$



<u>Example</u>

Find the volume of a cylindrical canister with radius 7 cm and height 12 cm.



$v = \pi r^2 h$

= $3.142 \times 7^2 \times 12$

= 1847.50

So, the volume of the canister is 1847.50 cm^2

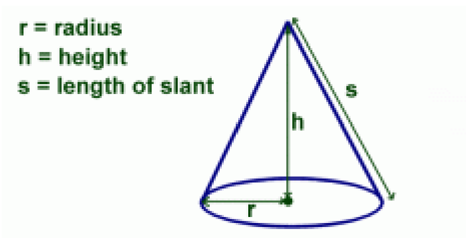
Cones

Surface area of a cone

What is a cone?

A cone is a type of geometric shape. There are different kinds of cones. They all have a flat surface on one side that tapers to a point on the other side.

We will be discussing a right circular cone on this page. This is a cone with a circle for a flat surface that tapers to a point that is 90 degrees from the center of the circle.



Terms of a Cone

In order to calculate the surface area and volume of a cone we first need to understand a few terms:

Radius – The radius is the distance from the center to the edge of the circle at the end.

Height – The height is the distance from the center of the circle to the tip of the cone.

Slant – The slant is the length from the edge of the circle to the tip of the cone.

Pi – Pi is a special number used with circles. We will use an abbreviated version where Pi = 3.14. We also use the symbol π to refer to the number pi in formulas.

Surface Area of a Cone

The surface area of a cone is the surface area of the outside of the cone plus the surface area of the circle at the end. There is a special formula used to figure this out.

Surface area = $\pi rs + \pi r^2$

r = radius
s = slant
π = 3.14

This is the same as saying (3.14 x radius x slant) + (3.14 x radius x radius)

<u>Example</u>

What is the surface area of a cone with radius 4 cm and slant 8 cm?

Surface area = $\pi rs + \pi r^2$
= (3.14x4x8) + (3.14x4x4)
= 100.48 + 50.24
= 150.72 cm^2

Volume of a Cone

There is special formula for finding the volume of a cone. The volume is how much space takes up the inside of a cone. The answer to a volume question is always in cubic units. **Volume = $1/3\pi r^2h$**

This is the same as 3.14 x radius x radius x height ÷ 3

Example

Find the volume of a cone with radius 4 cm and height 7 cm?

Volume = $1/3\pi r^2h$
= 3.14 x 4 x 4 x 7 ÷ 3
= 117.23 cm^3

Things to Remember

Surface area of a cone = $\pi rs + \pi r^2$

Volume of a cone = $1/3\pi r^2h$

The slant of a right circle cone can be figured out using the Pythagorean Theorem if you have the height and the radius.

Answers for volume problems should always be in cubic units.

Answers for surface area problems should always be in square units.

Mensuration and estimation

Mensuration is the use of measurement, formulae and calculation to find lengths, areas and volumes of shapes. The use of approximate values in mensuration formulae gives results that are sufficiently accurate for most purposes. This section provides practice in using estimation techniques in mensuration problems.

Assessment

- What is the surface area of a cone with radius 8 cm and slant 12 cm?
- Find the volume of a cone with radius 2 cm and height 9 cm?

Week 11

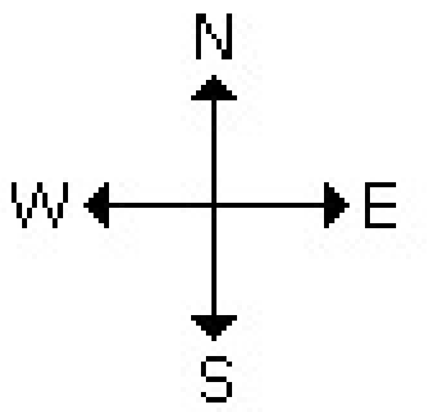
Topic: BEARING AND DISTANCE

Compass directions

There are four main directions: north (N), south (S), east (E), and west (W). The sun rises in the east and sets in the west. If you face east and turn through an angle of 90^{0} to your left, you will face north.

If you face north and turn an angle of 90^{0} to the left, you will face west.

To face south, face east, then turn an angle of 90^{0} to the right. See the figure below.

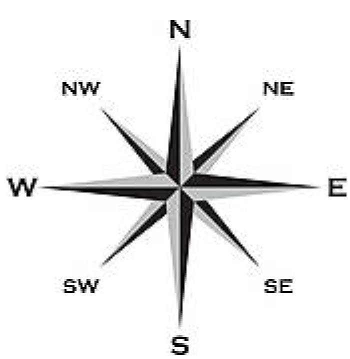


The magnetic compass

The magnetic compass is used for finding **direction.** It has a magnetic needle which always points in the North direction.

Points of the compass

The figure below shows the main **points** of the compass.



Apart from the four main points, or directions: north (N), south (S), east (E) and west (W), there are also four secondary directions: north-west (NE), south-east (SE), south-west (SW) and north-west (NW). The angle between the direction N and E is 90^{0}. NE is the direction mid-way between N and E. Thus the angle between N and E is 45^{0}.

Bearings

If student A is standing 5m north of student B, we say that A is north of B. We can also say that the direction of A from B is north. We use the word **bearing** for directions between two points.

In other words the bearing of A from B is north. Similarly, the bearing of B from A is south. This section explains the three main ways of giving bearing.

Compass bearings

Bearings are simply a way of giving directions (a bit more accurately than just relying on North, South, East and West!).

If they are given with a distance as well, then you have an exact **position.**

Note:

1. Bearings are always measured from **North,** which is 000^{0} (or 360^{0}). **Always draw a North line at each point** to give a guide for your protractor.

2. They are always measured **clockwise** and must have **three** figures. So East is 090^{0}, South is 180^{0} and West is 270^{0}.

Finding a position

If you are given the bearing and position of something **from a point** and asked to mark it in (after which you may be asked to measure something else) then here's what you do:

1. Draw a North line at the point you are measuring from.

2. Put your protractor on this point and line up 0^{0} with the North line. Always lean right over the paper when you're doing this!

3. Read round the protractor **clockwise** until you reach the angle you want and mark it with a pencil.

4. Draw a pencil line from the point you are measuring from right through your angle point.

5. Check your scale (if there is one) and measure the distance you need along this line. Put a cross to show your position.

Don't rub any of your pencil lines out. These help to show what you've done.

Here's the kind of diagram you might have for a ship which is 8 km away from a lighthouse on a bearing of 125^{0} using a scale of 1 cm = 1 km.
Acute-angle bearings

Bearing can also be given as **acute angles** measured from the north (N) or south (S).

Exercise

Do the following and state the final direction that you face.

1. Face east, turn through an angle of 180^{0}.
2. Face east, turn through 45^{0} to your right.
3. Face east, turn through 90^{0} to your right.
4. Face east turn through angle 90^{0} to your left.
5. What is the angle between the following directions? a.N and S b. W and E c. NE and S d. N and W e. E and SW f. SW and W g. N and NW h. NE and SW.