MATHEMATICS

FOR

Junior Secondary School

1

AKADALEARN

JSS 1

MATHEMATICS

FIRST TERM

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WEEK 1 & 2

Topic: Development of Number Systems

It is most likely that mathematics began when people started to count and measure. Counting and measuring are part of everyday life. Nearly every language in the world contains words for numbers and measures.

People have always used their fingers to help them when counting. This led to collect numbers in groups: sometimes fives (fingers of one hand), sometimes tens (both hands) and even in groups of twenty (hands and feet). For example, someone with twenty three sheep might say, ‘I have four five and three’ sheep or one twenty and three’ sheep. It will depend on local custom and language. In every case, the number of sheep would be the same.

When people group numbers in fives we say that they are using a **base five** method of counting. Most people use **base ten** when counting. For this reason base ten is used internationally.

The table 1.1.below gives the words for the number 1 to 20 in the Hausa, Igbo and Yoruba languages.

	Hausa	Igbo	Yoruba
1	Daya	Out	Ookan
2	Biyu	Abuo	Eeji
3	Uku	Ato	Eeta
4	Hudu	Ano	Eerin
5	Biyar	Ise	Aarun-un
6	Shida	Isii	Eefa
7	Bakwai	Asaa	Eeje
8	Takwas	Asato	Eejo
9	Tara	Iteghete	Eesan
10	Goma	Iri	Eewa

11	Goma sh daya	Iri na out	Ookanla
12	Goma sha biyu	Iri na abuo	Eejila
13	Goma sha uku	Iri na ato	Eetala
14	Goma sha hudu	Iri na ano	Eerinla
15	Goma sha biyar	Iri na ise	Eedogun
16	Goma sha shida	Iri na isii	Eerindinlogun
17	Goma sha bakwai	Iri na asaa	Eetadinlogun
18	Goma sha takwas	Iri na asato	Eejidinlogun
19	Goma sha tara	Iri na iteghete	Ookandinlogun
20	Ashirin	Iri abuo	OOgun

Other bases of counting: Seven and Sixty

There are seven days in a week. Suppose that a baby is two weeks and 5 days old. This means that it is two lots of seven days and 5 days old, 19 days altogether.

Example 1

Find the total of 1 week 5 days, 6 days and 3 weeks 4 days. Give the

a. in weeks and days b. in days.

Solution

wk **d** Method in days column:

1 5 5 + 6 + 4 = 15 days

0 6 = 2 x 7 days + 1 day

3 4 = 2 weeks + 1 day

————

6 1 Write down 1 day and carry 2 weeks

Answer:

a. 6 weeks and 1 day,

b. 6weeks 1 day = 6 x 7 days + 1 day

= 42 days + 1 day = 43 days.

Example 2

Find the number of seconds in 3 min 49 s.

Number of seconds in 3 min = 3 x 60 s = 180 s

Number of seconds in 3 min 49 s = 180 s + 49 s

= 229 s

Symbol for Numbers

As civilization developed, spoken languages were written down using **symbols.** Symbols are letters and marks which represent sounds and ideas. Thus the words on this page are symbols for spoken words. Numbers were also written down. We use the words **numerals** for number symbols.

The first numerals were probably tally marks. People who looked after cattle made tally marks to represent the number of animals they had. The tally marks were scratched on stones or sometimes cut on sticks.

We still use the tally system; it is very useful when counting a large of objects.

We usually group tally marks in fives; thus 𝍸 𝍸 𝍸 III mean three fives and two, or seventeen. Notice that in each group of five, the fifth tally is marked across the other four: IIII = 4; 𝍸 = 5.

Roman system

There are many ancient methods of writing numbers. The Roman system is still used today. The Romans used capital letters of the alphabets for numerals. In the Roman system I's stand for units, X's stands for tens ad C's stands for hundreds. Other letters stand for 5's, 50's and 500's. Table 1.2 below shows how the letters were used.

1	I	20	XX
2	II	40	XL
3	III	50	L
4	IIII or IV	60	LX
5	V	90	XC
6	VI	00	C
7	VII	400	CD
8	VIII	500	D
9	IX	900	CM
10	X	1000	M

Roman numerals were first used about 2 500 years ago. They are still in use today. You sometimes find Roman numerals on clockfaces and as chapter number in books.

Example

What number does MDCLXXVIII represent?

Work from the left:

M = 1000

D = 500

C = 100

L = 50

(two tens) XX = 20

V = 5

(three units) III = 3

Addings: MDCLXXVIII = 1678

A simple code

The Romans used letters of the alphabet to stand for numbers. We can use numbers to stand for letters of the alphabet. This gives a simple code shown in Table 1.3 beow.

A B C D E F G H I J K L M N O P Q R S T U V W X Y Z

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 26 26

ASSESSMENT

1. What does (6, 1, 20)(2, 15, 25) mean in the code in table 1.3 above?

2. Find the total of 2 weeks 6 days, 5days and 6 weeks 5 days. Give the

(a) in weeks and days (b) in days.

3. What number does CCXC represent

ANSWER

1. From the table,

6 = F, 1 = A, 20 = T

(6, 1, 20) = FAT

2 = B, 15 = O, 25 = Y

(2, 15, 25) = BOY

Thus (6, 1, 20)(2, 15, 25) means FAT BOY.

2.

wk.	**d**	Method in days column:
2	6	6+ 5 + 5 = 16 days
0	5	= 2 x 7 days + 2 days
6	5	= 2 weeks + 2 days

————

10 2 Write down 2 days and carry 2 weeks

(a) 10 weeks and 2 days

(b) 10 weeks and 2 days = 10 x 7 = 70 days

70 days+ 2 days = 72 days

3. CC = 200

XC = 90

Adding: CCXC = 290

Week 3

Topic: LARGE AND SMALL NUMBERS

Large Numbers

There is no such thing as 'the biggest numbers in the world'. It is always possible to count higher. Science and economics use very large numbers. Thus we need special names for large numbers.

Table 2.1 gives the names and values of some large numbers.

Name	Value
Thousand	1000
Million	1000 thousand = 1 000 000
Billion	1000 million =1 000 000 000
Trillion	1000 billion = 1 000 000 000 000

How big is a million?

The following examples may give you some idea of the size of a million.

1. A 1 cm by 1 cm square of 1mm graph paper contains 100 small 1mm x 1mm squares.

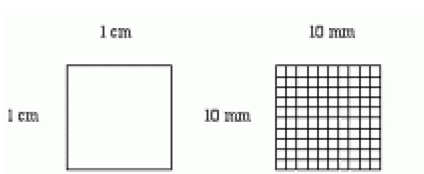


Fig 2.2 100 small squares

A 1 m by 1 m square of the same graph paper contains 1 million of these small squares.

1 m = 1000 mm

1 square
metre

Fig 2.2 1000 mm x 1000 mm = 1 000 000 mm^2

2. A cubic metre measures 100 cm by 100 cm by 100 cm.

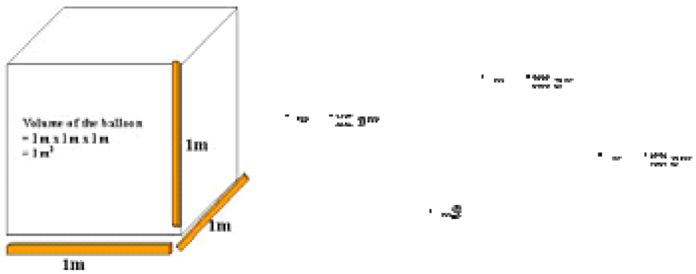


Fig 2.3 1 cubic metre

Volume of cubic metre

= 100 cm x 100 cm x 100 cm

= 1 000 000 cm^3

Thus 1 million cubic centimeters, cm^3, will exactly fill a cubic metre box.

Reading and Writing Large Numbers

Grouping digits

Read the numbers 31556926 out aloud. Was it easy to do? It may have been quite difficult. You had to decide, ‘Is the number bigger than a million or not?, ‘Does it begin with 3 million, 31 million or 315 million?’

It is necessary to write large numbers in a helpful way. It is usually to group the digits of large numbers in threes from the decimal point. A small group is left between each group.

315556929 should be written as 31 556 826. Now it is easy to see that the number begins with 31 million. In words, we would say or write this number as

thirty-one million

five hundred and fifty-six thousand

nine hundred and twenty-six.

Note: we no longer use commas between the groups of digits. Many countries use a comma as a decimal point; thus, to avoid confusion do not use commas for grouping the digits.

Digits and words

Editors of newspapers know that large numbers sometimes confuse readers. They often use a mixture of digits and words when writing large numbers.

Example

What do the numbers in the following headlines stand for?

a. FOOD IMPORT RISE TO N1 BILLION

b. OIL PRODUCTION NOW 2.3 MILLION BARRELS DAILY

c. FLOODS IN INDIA- 0.6 MILLION HOMELESS

d. NEW ROAD TO COST N2 ¼ TRILLION

a. N1 billion is short for N1 000 000 000

b. 2.3 million = 2.3 x 1 000 000

= 2 300 000

c. 0.6 million = 0.6 x 1 000 000

= 600 000

d. N 2 ¼ trillion = N2.25 trillion

= N2 250 000 000 000

Small Numbers

Decimal fractions

Decimal fractions also have names

8 tenths = 0.8

8 hundredths = 0.08

8 thousandths = 0.008

8 tens thousandths = 0.0008

8 hundred thousandths = 0.000 08

Notice that digits are grouped in threes from the decimal point as before.

Example

Write the following as decimal fractions.

a. 28 thousandths b. 865/100 000

Solution

28 thousandths = 1 thousandths x 28

= 0.001 x 28

= 0.028

865/100 000 = 0.00865

There are five zeros in the denominator.

The decimal fraction is obtained by moving the digits in the numerator five places to the right.

ASSESSMENT

Write the following as decimal fractions.

1. 6 hundredths
2. 9 tenths
3. 4 thousandths
4. 4 then thousandths
5. 9/100 000

ANSWER

1. 0.06
2. 0.9
3. 0.004
4. 0.0004

5. 0.00009

Week 4 & 5

Topic: FACTORS AND MULTIPLES – LCM and HCF

Factors

40 ÷ b = 5 and 40 ÷ 5 = 8.

We say that 8 and 5 are factors of 40.

If we can divide a whole number by another whole number without remainder, the second number is a **factor** of the first.

The numbers 1, 2, 4, 5, 8, 10, 20 and 40 all divide into 40. They are all factors of 40. We can write 40 as a product of two factors in eight ways:

40 = 1 x 40 = 2 x 20 = 4 x 10 = 5 x 8

= 8 x 5 = 10 x 4 = 20 x 2 = 40 x 1

Prime numbers

A prime number has only two factors, itself and 1.

2, 3, 5, 7, 11, 13, ... are prime numbers. 1 has only one factor, itself; 1 is not a prime number.

Prime factors

The prime factors of a number are the factors of the number that is prime. It is possible to write every non-prime number as a product of prime factors. For example:

15 = 3 x 5

24 = 2 x 2 x 2 x 3

42 = 2 x 3 x 7

To find the prime factors of a number:

a. Start with the lowest prime number, 2.

Find out if this will divide into the number. If it will not divide, try the next prime number, 3. And so on, trying 5, 7, 11, 13, ... in turn.

b. If a prime number will divide, check if it will divide again before moving on to the next prime.

Example

Express 15 288 as a product of prime factors.

15 288 = 2 x 7644

= 2 x 2 x 3822

= 2 x 2 x 2 x 1911

= 2 x 2 x 2 x 3 x 637

= 2 x 2 x 2 x 3 x 7 x 91

= 2 x 2 x 2 x 3 x 7 x 7 x 13

The working can be set out as a continued division as follows:

2 | 15 288

2 | 7 644

2 | 3 822

3 | 1 911

7 | 637

7 | 91

13 | 13

....

1 1

Thus 15 288 = 2 x 2 x 2 x 3 x 7 x 7 x 13

Index form

In the previous example we saw that

15 288 = 2 x 2 x 2 x 3 x 7 x 7 x 13.

It is possible to write the product in a shorter way. 2^3 is a short way of writing 2 x 2 x 2. The number 3 in 2^3 is called the **index** or **power.** It shows that three 2's are to be multiplied together.

In the same way, 7^2 is a short way of writing 7 x 7. The index two shows that two 7's are to be multiplied together.

In index form,

15 288 = 2^3 x 3 x 7^2 x 13.

The use of index form saves space and can help to prevent errors in counting and copying. It would be easy to make a mistake when copying the products of factors of 256.

256 = 2 x 2 x 2 x 2 x 2 x 2 x 2 x 2

In index form, 256 = s^8. This saves space and clearly shows that eight 2's are to be multiplied together.

We say 2^8 as 'two to the power of eight', or 'two to the eighth power' or 'two raised to the power of eight' or, often, just 'two to the eighth'.

The plural of index is indices. The indices 2 and 3are usually said in a special: 7^2 as '**seven** squared', 2^3 as '**two** cubed'.

Common factors

The number 12, 21 and 33 are all divisible by 3. We say that 3 is a **common factor** Of 12, 21 33.

There may be more than one common factor of a set of numbers. For example, both 2 and 7 are common factors of 28, 42 and 70. Since 2 and 7 are common factors and are both prime numbers, then 14 (= 2 x 7) must also be a common factor of the set of numbers.

1 is a common factor of all numbers.

Highest Common Factor (HCF)

2, 7 and 14 are common factors of 28, 42 and 70; 14 is the greatest of three common factors. We say that 14 is the **highest common factor** of 28, 42 and 70.

To find the HCF of a set of numbers:

Express the number as a product of prime factors;

b. Find the common prime factors

c. Multiply the current prime factor together to give the HCF.

Example

Find the HCF of 18, 24 and 42.

18 = 2 x 3 x 3

24 = 2 x 2 x 2 3

42 = 2 x 3 x 7

The common prime factors are 2 and 3.

The HCF = 2 x 3 = 6.

Find the HCF of 216

2 | 216

2 | 108

2 | 54

3 | 27

3 | 9

3 | 3

....

1 1

2 | 288

2 | 144

2 | 72

2 | 36

2 | 18

3 | 9

3 | 3

....

1   1

In index notation

$216 = 2^3 \times 3^3$

$288 = 2^5 \times 3^2$

2^3 is the lowest power of two contained in the two numbers. Thus the HCF contains 2^3.

3^2 is the lowest power of 3 contained in the tow numbers. The HCF contains 3^2.

$216 = (2^3 \times 3^3) \times 3$

$288 = (2^2 \times 3^3) \times 2^2$

The HCF = $2^2 \times 3^3 = 8 \times 6 = 72$.

Multiples

Common multiples

6 is a factor of 12, 18, 24, 30, ...We say that 12, 18, 24, 30, ..., etc are all multiples of 6. In the same way, 8, 12, 16, 20, ..., etc. are all multiples of 4.

Example

Give three common multiples of

a. 3 and 11

Solution

33 is a common multiple of 3 and 11.

Likewise, 66 and 99.

Lowest Common Multiples (LCM)

The least common multiple, or LCM, is another number that's useful in solving many math problems. Let's find the LCM of 30 and 45. One way to find the least common multiple of two numbers is to first list the prime factors of each number.

$30 = 2 \times 3 \times 5$
$45 = 3 \times 3 \times 5$

Then multiply each factor the greatest number of times it occurs in either number. If the same factor occurs more than once in both numbers, you multiply the factor the greatest number of times it occurs.

Then multiply each factor the greatest number of times it occurs in either number. If the same factor occurs more than once in both numbers, you multiply the factor the greatest number of times it occurs.

2: one occurrence
3: two occurrences
5: one occurrence
$2 \times 3 \times 3 \times 5 = 90$ *<— **LCM***

After you've calculated a least common multiple, always check to be sure your answer can be divided evenly by both numbers.

EXAMPLES

Find the LCM of these sets of numbers.

3, 9, 21
Solution: List the prime factors of each.
3: 3
9: 3×3
21: 3×7
Multiply each factor the greatest number of times it occurs in any of the numbers. 9 has two 3s, and 21 has one 7, so we multiply 3 two times, and 7 once. This gives us 63, the smallest number that can be divided evenly by 3, 9, and 21. We check our work by verifying that 63 can be divided evenly by 3, 9, and 21.

12, 80
Solution: List the prime factors of each.
12: $2 \times 2 \times 3$

80: 2 × 2 × 2 × 2 × 5 = 80
Multiply each factor the greatest number of times it occurs in either number. 12 has one 3, and 80 has four 2's and one 5, so we multiply 2 four times, 3 once, and five once. This gives us 240, the smallest number that can be divided by both 12 and 80. We check our work by verifying that 240 can be divided by both 12 and 80.

30, 60, 90 are all common multiples of 6, 10 and 15. 30 is the lowest number that 6, 10 and 15 will divide into. We say that 30 is the lowest common multiple of 6, 10 and 15.

The LCM of 4, 5 and 6 is 60 (not 120).

a. Express the number as a product of prime factors;

b. Find the lowest product of factors which contains all the prime factors of the numbers.

Example

Find the LCM of 8, 9 and 12.

8 = 2 x 2 x 2

Any multiple of 8 must contain 2 x 2 x 2.

9 = 3 x 3

Any multiple of 9 must contain 3 x 3.

12 = 2 x 2 x 3

Any multiple of 12 must contain 2 x 2 x 3.

The lowest product containing all the three is

2 x 2 x 2 x 3 x 3

The LCM of 8, 9and 12 is 2 x 2 x 2 x 3 x 3 = 72

ASSESSMENT

Find the LCM of the following:

1. 4 and 6
2. 6 and 8
3. 8, 10 and 15

Express the following as products of prime factors

4. 12
5. 18

ANSWER

1. 12
2. 24
3. 120
4. 2 x 2 x 3
5. 2 x 3 x 3

Week 6 & 7

Topic: FRACTIONS 1 – Improper and Proper Fractions

Common fractions

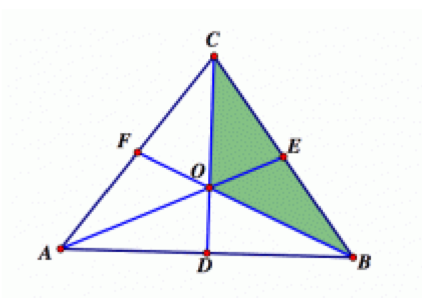
It is not always possible to use whole numbers to describe quantities.

We use **fractions** to describe parts of quantities. We write the fractions like this:

one-third 1/3

three-fifths 3/5

four-ninths 4/9



The figure above has two-sixths of the triangle has been shaded.

The number below the line is called the denominator. The denominator shows the number of equal parts the whole has been divided into. The number above the line is called the numerator. The numerator shows the number of parts in the fraction.

Some quantities need whole numbers and fractions to describe them

The figure above has 'Two and a half oranges'.

The two and a half, written 3½, is a **mixed number**. A mixed number has a whole number part and a fractional part: 3½ = 3 + ½ .

It is possible to express a mixed number as a single fraction:

3½ = 3 + ½ = 3/1 + ½ = 3 x 2/1 x 2 + ½

= 6/2 + ½ = 6 + ½ = 7/2

3½ = 7/2

The numerator of the fraction 7/2 is greater than the denominator. This is an example of **improper fraction.** If the numerator is less than the denominator, the fraction is a **proper fraction**. For example, 2/7 is a proper fraction.

Example

Express **4** 5/6 as an improper fraction.

4 5/6 = 4 + 5/6 = 24/6 + 5/6 = 24 + 5/6 + 29/6

Or, more quickly,

4 5/6 = 4 x 6 + 5/6 = 24 + 5/6 = **29/6**

Express 19/8 as a mixed number.

19/8 = 16 + 3/8 = 16/8 + 3/8 = 2 + 3/8 = **2** 3/8

Or, more quickly,

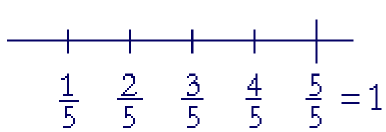
19/8 = 19 ÷ 8 = 2 with remainder 3,

But the remainder is also divided by 8:

19 ÷ 8 = 2 + 3/8 = **2** 3/8

We can recognize an improper fraction when the numerator is greater than or equal to the denominator.

In fact, when the numerator is equal to the denominator,



then the fraction is equal to 1.

$\frac{5}{5} = \frac{6}{6} = \frac{7}{7} = \frac{8}{8} = 1.$

We say that those fractions also are improper.

We will see in the next Lesson, Example 4, that we can write 1 equal to a fraction with *any* denominator.

Problem 1. Which of these fractions are less than 1, equal to 1, or greater than 1?

$\frac{2}{3}, \frac{3}{2}, \frac{8}{5}, \frac{8}{8}, \frac{8}{9}, \frac{9}{9}, \frac{10}{9}.!$

Less than 1: $\frac{2}{3}$ $\frac{8}{9}$

Equal to 1: $\frac{8}{8}$ $\frac{9}{9}$

Greater than 1: $\frac{3}{2}$ $\frac{8}{5}$ $\frac{10}{9}$

*

We will now see explicitly why we use the division bar to signify a fraction.

How do we change an improper fraction to a mixed number or a whole number?

$$\frac{9}{2} = 4\frac{1}{2}$$

Divide the numerator by the denominator. Write the quotient (4), and write the remainder (1) as the numerator of the fraction; do not change the denominator.

When we change an improper fraction to a mixed number, we say that we are extracting, or taking out, the whole number.

Example 9. Extract the whole number from $\frac{53}{8}$.

Solution. $\frac{53}{8} = 6\frac{5}{8}$.

"8 goes into 53 six (6) times (48) with 5 left over."

We have extracted the whole number 6.

As for the remainder, it is the number we must *add* to 48 to get 53.

Compare Lesson 11.

(Obviously, the student must know the multiplication table.)

Example 10. $\frac{31}{9} = 3\frac{4}{9}$.

"9 goes into 31 three (3) times (27) with remainder 4."

27 plus 4 is 31.

Again: The remainder is what we must *add* to 27 to get 31.

Example 11. $\frac{28}{4} = 7$

"4 goes into 28 seven (7) times exactly."

Problem 2. Answer with a mixed number or with a whole number and a remainder, whichever makes sense.

a) How many basketball teams — 5 on a team — could you make from
a) 23 students?

23 ÷ 5 = 4 R 3.

You could make 4 teams. 3 students will be left out.

($4\frac{3}{5}$ teams makes no sense.)

b) You are going on a trip of 23 miles, and you have gone a fifth of the
b) distance. How far have you gone?

$4\frac{3}{5}$ miles.

To calculate the fifth part of a number, that is, to divde it into five equal parts, divide by 5

$\frac{23}{5} = 4\frac{3}{5}$.

1 mile, which is continuous, will have *any* part.

Lowest Terms

The value of a fraction stays the same if both the numerator and denominator have no common factor, we say that the factor is in its **lowest term**s, or in its **simplest form**. Thus 16/24 in its lowest term is 2/3, ¼ is the simplest form of 15/60.

To express a fraction in its lowest terms:

a. look for common factors of the numerator and denominator

b. divide the numerator and denominator by their common factors;

c. repeat until there are no more common factors.

Example

Express **a.** 42/70 and **b.** 26/78 in their lowest terms.

a. By inspection:

42/70 = 42 ÷ 7/70 ÷7 = 6/10 = 6 ÷ 2/10 ÷ 2 = 3/5

or by using prime factors:

42/70 = 2 x 3 x 7/2 x 5 x 7 =2 ÷ 2 x 3 x 7 ÷ 7/ 2 ÷ 2 x 5 x 7 ÷ 7

= 1 x 3 x 1/1 x 5 x 1 = 3/5

b. By inspection:

26/78 = 26 ÷ 2/78 ÷ 2 = 13/39 = 13 ÷ 13/39 ÷ 13 = 1/3

After practice, you will be able to leave out many of the steps shown in the above example.

Basic operations on fractions

Addition and subtraction

We can only add or subtract fractions if they have the same denominators.

Example

2/9 + 5/9 = 2 + 5/9 = 7/9

If fractions have different denominators:

a. find a common denominator (preferably the lowest);

b. express each fraction as an equivalent fraction, using that denominator;

c. add or subtract as above.

Example

Simplify 5/5 + 3/8

The LCM of 6 and 8 is 24.

5/6 + 3/8 = 5 x 4/6 x 4 + 3 x 3/8 x 3 = 20/24 + 9/24

= 20 + 9/24 = 29/24 = **1** 5/24

Multiplication

Whole number x fraction

4 x 1/3 means 4 lots of something.

4 x 1/3 = 1/3 + 1/3 + 1/3 + 1/3 = 4/3 = **1** 1/3

Or 4 x 1/3 = 4/3

Fraction x whole number

2/5 x 3 means 2/5 of 3 objects. Let the objects be squares (fig below)

I	I	I

The 3 squares can be divided into fifths. As shown below

I	I	I
II	II	II
III	III	III
IIII	IIII	IIII
IIIII	IIIII	IIIII

Fraction x fraction

2/3 x 5/7 means 2/3 of 5/7 of something.

Simplify 3/8 of **2** 2/9 x **1** 3/5.

3/8 of **2** 2/9 x **1** 3/5 = 3/8 x 20/9 x 8/5 = 3x20x8/8x9x5 = 4/3 = **1** 1/3

Divide the numerator and the denominator by the common factors 3, 5 and 8 to simplify.

Division

Look at the following working very carefully.

2 ÷ 5/8 = 2**/** 5/8 = 2 x 8/5 **/** 5/8 x 8/5 = 2 x 8/5 **/** 1 = 2 x 8/5

Thus, 2 ÷ 5/8 = 2 x 8/5.

8/5 is the reciprocal of 5/8; i.e. it is the same fraction turned upside down.

To divide by a fraction, simply multiply by its reciprocal.

Example

Find the value of **2**¼ ÷ 3/7.

2¼ ÷ 3/7 = **2**¼ x 7/3 = 9/4 x 7/3 = 9×7/4×3 = 21/4 = 5¼.

Notice that this example shows that it is possible to get a result which is greater than either of the given numbers.

We saw before that the value of a fraction is unchanged if we multiply the numerator and denominator by the same number. This gives another method of dividing by fractions.

2¼ ÷ 3/7 = 9/4 **/** 3/7 = 9/4 x 28' **/** 3/7 x 28 = 9×7**/**4×3 = 21/4 = 5¼

'28 is the LCM of the denominators of the two fractions.

Percentages

20% means 20/100. The symbol % is short for per cent. **Per cent** means hundredths. A fraction in the form 20% is called a **percentage.**

100% means 100/100·100/100 = 1. So, when we say we want 100% of something, we mean we want all of it.

To express a percentage as a fraction:

A write the percentage as a fraction of 100;

B reduce the fraction to its lowest terms.

Example

Express 15% as a fraction in its lowest terms.

Solution

15% = 15/100 = 3×5/20×5 = 3/20

Express 7 min 30 sec as a fraction of 1 hour

7 min 30 s/1 h = 450s'/3600s = 9 x 50/72 x 50 = 9/72 = 1/8h

Or

7 min 30 s/1 h = 7½ min'/60 min = 15/120 = 1 x 15/8 x 15 = 1/8h

Notice that both quantities must be in the same units before fractions can be reduced.

ASSESSMENT

Express the following percentages as fractions in their lowest terms

1. 5%

2. 50%

3. 25%

Express the following fractions as percentages

4. 3/5

5. 9/25

ANSWER

1. 1/20

2. 1/2

3. 1/4

4. 60%

5.36%

Week 8 & 9

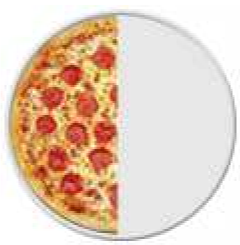
Topic: FRACTIONS 2: DECIMALS AND PERCENTAGES

Decimal Fractions

We have seen in previous lessons how to extend the place value system to include decimal fractions. The number 3.549 is a way of writing 3 units + 5/10 + 4/100 + 9/1000, or simply 3 units + 549/1000:

units	decimal	tenth	hundredths	thousandths
↓	↓	↓	↓	↓
3	**.**	**5**	**4**	**9**

The decimal point acts as a place-holder between the whole-number part and the fractional part of the number.

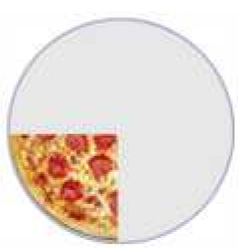


A Half can be written...

As a fraction:½

As a decimal:0.5

As a percentage:50%



A Quarter can be written...

As a fraction:1/4

As a decimal:0.25

As a percentage:25%

Addition and Subtraction

Be very careful to set out your work correctly. Units must be under units, decimal points under decimal points, … and so on. For example, 24.8 + 6.5 is set out as

```
  24.8          24.8
+  6.5   not   6.5
———           ———
  31.3
```

After you have set out your work correctly, add and subtract in the same way as you do for whole numbers, but remember to write down the decimal point when you come it.

ASSESSMENT

a. 0.2 + 0.6

b. 0.6 – 0.5

c. 1.3 + 0.8

```
d.   15.86
  +   5.15
 ———–

e.   0.56
  –  0.18
  ———–
```

Positive and Negative Numbers

ADDITION:

If the signs are the same then you add the two numbers and keep the

Ex. 6 + 2 = 8 or −6 + −2 = −8

If the signs are different, subtract the two numbers and take the sign

of the larger number.

Ex. −6 + 2 = −4 or 6 + −2 = 4

SUBTRACTION:

Change the sign of the second number, then add the two numbers using the rules for addition, above.

Ex. 6 − 2 = 6 + (−2) = 4

Ex. −6 − −2 = −6 + (+2) = −4

Ex. −6 − 2 = −6 + (−2) = −8

Ex. 6 − −2 = 6 + (+2) = 8

Exercise

Week 10

Topic: Fractions 3 – Multiplication and Division

Multiplication and Division

Multiplication and division by powers of 10

The table below shows what happens when 3.07 is multiplied by increasing powers of 10.

3.07 X 1 = 3.07

3.07 X 10 = 3.07 X 10^1 = 30.7

3.07 X 100 = 3.07 X 10^2 = 307.

3.07 X 1 000 = 3.07 X 10^3 3070.

3.07 X 10 000 = 3.07 X 10^4 = 30700.

Notice that when multiplying by powers of 10:

a. as the power of 10 increases it is as if the decimal point stays where it is and the digits in the number move to the left;

b. the digits move as many places to the left as the power of 10 (or as the number zeros in the multiplier);

c. as each place to the right of the digits becomes empty we fill it with a zero to act as a placeholder;

d. if the fraction to the right of the decimal point becomes zero there is no need to write anything after the point.

Similarly, the table below shows what happens when 3.07 is divided by increasing powers of 10.

3.07 ÷ 1 = 3.07

3.07 ÷ 10 = 3.07 ÷ 10^1 = 0.307

3.07 ÷ 100 = 3.07 ÷ 10^2 = 0.0307

3.07 ÷ 1000 =3.07 ÷ 10^3 = 0.00307

$3.07 \div 10000 = 3.07 \div 10^4 = 0.000307$

Notice that when dividing by powers of 10:

a. as the power of ten increases it is as if the decimal point says where it is and the digits in the number move to the right.;

b. the digits move as many places to the right as the power of 10 (or as the number zeros in the divisor);

c. as each place to the left of the digits becomes empty we fill it with zero to act as a placeholder;

d. if the number to the left of the decimal point becomes zero it is usual to write a zero there.

Example

Write the following as decimal numbers.

a. 0.036 x 10 000

b. 23/1000

c. 120÷100 000

Solution

$0.036 \times 10\,000 = 0.036 \times 10^4 = 360$

b. $23/1000 = 23 \div 1\,000 = 23 \div 10^3 = 0.023$

c. $120 \div 100\,000 = 120 \div 10^5 = 0.001\,20 = 0.001\,2$

It is not necessary to write zeros to the right of a decimal fraction. For example, o.200 000 is just the same as 0.2.

Example

Multiplication of decimals

Find the product of 38.6 and 1.64

38.6 X 1.64 = 386/10 X 164/100 = 386 X 164/1 000

Use long multiplication to find the numerator:

386

X 164

———-

1 544

23 160

38 600

———-

63 304

38.6 X 1.64 = 63 304/ 1 000 = 63.304

The method in the example above can be made shorter as follows

a. Multiply the given numbers without decimal points.

b. Counts the digits after the decimal points in the numbers being multiplied.

c. Place the decimal point so that the product has the same number of digits after the point.

Example

Calculate 0.25 X 0.009

25 X 9 = 225

There are five digits after the decimal points in the given numbers. So,

0.25 X 0.009 = 0.002 25

MULTIPLICATION AND DIVISION:

Do the multiplication or division of the two numbers and then determine the sign by the following:

If the signs of the two numbers are the same the answer is positive.

Ex. 6 X 2 = 12   or -6 X -2 = 12

Ex. 6/2 = 3   or -6/-2 = 3

If the signs of the two numbers are different the answer is negative.

1. -6 X 2 = -12 or 6 X -2 = -12

-6/2 = -3 or 6/-2 = -3

Solution

1. When you're working with squaring and cubing negative numbers, just remember that squaring and cubing is like multiplying. For example,

2^2 = 2 X 2 (The 2^2 means 2 raised to the power two)

3^4 = 3 X 3 X 3 X 3

So $(-3)^2$ = (-3) X (-3)

Now just use the rules of multiplying two negative numbers together to get the answer.

If you're cubing a number, it's just as if you've multiplied it together three times.

Ex. $(-4)^3$ = (-4) X (-4) X (-4)

Using the rules of multiplication, multiply the first two together to get 16. Then the problem becomes:

(16) X (-4)

and you can use the regular multiplication rules again.

ASSESSMENT

Give the following as decimal numbers

1. 0.36 x 100

2. 52/1000

3. 60/100

Find the following products

4. 0.7 x 8

5. 0.2 x 4

ANSWER

1. 36

2. 0.052

3. 0.6

4. 5.6

5. 0.8

<u>Week 11</u>

<u>Topic: FRACTIONS (Continued)</u>

Addition and Subtraction of Fractions

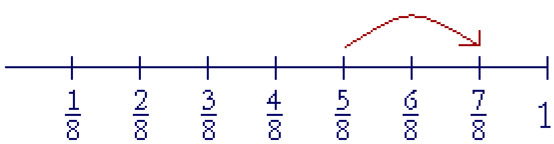
1. How do we add or subtract fractions?

$$\frac{2}{9} + \frac{3}{9} = \frac{5}{9}$$

The names of what we are adding or subtracting — the denominators — must be the same. Add o
the numerators, and keep that same denominator.

Example 1. $\frac{5}{8} + \frac{2}{8} = \frac{7}{8}$.

"5 eighths + 2 eighths = 7 eighths."



The denominator of a fraction has but one function, which is to *name* what we are counting. In this
counting *eighths*.

Example 2. $\frac{5}{8} - \frac{2}{8} = \frac{3}{8}$.

Fractions with different denominators

To add or subtract fractions, the denominators *must* be the same. Before continuing, then, the stuc
how to convert one fraction
to an equivalent one, by multiplying the numerator and the denominator.

2. How do we add fractions with different denominators?

$\frac{2}{3} + \frac{1}{4}$

Convert each fraction to an equivalent fraction with the *same* denominator.

3. What number should we choose as the common denominator?

Choose a common multiple of the original denominators. Choose their *lowest* common multiple.

We choose a common multiple of the denominators because we change a denominator by multiply

Example 3. $\frac{2}{3} + \frac{1}{4}$.

Solution.

The lowest common multiple of 3 and 4 is their product, 12.

We will convert each fraction to an equivalent fraction with denominator 12.

$$\frac{2}{3} + \frac{1}{4} = \frac{8}{12} + \frac{3}{12}$$

$$= \frac{11}{12}.$$

We converted $\frac{2}{3}$ to $\frac{8}{12}$ by saying, "3 goes into

(is contained in) 12 *four* times. Four times 2 is 8."

(In that way, we multiplied both 2 and 3 by the same number, namely 4. See Lesson 22, Question 3.

We converted $\frac{1}{4}$ to $\frac{3}{12}$ by saying, "4 goes into 12 *three*

times. Three times 1 is 3." (We multiplied both 1 and 4 by 3.)

The fact that we *say* what we do shows again that arithmetic is a spoken skill.

In practice, it is necessary to write the common denominator only once:

$$\frac{2}{3} + \frac{1}{4} = \frac{8 + 3}{12} = \frac{11}{12}.$$

Example 4. $\frac{4}{5} + \frac{2}{15}$

Solution.

The LCM of 5 and 15 is 15. Therefore,

$$\frac{4}{5}+\frac{2}{15}=\frac{12+2}{15}=\frac{14}{15}.$$

We changed $\frac{4}{5}$ to $\frac{12}{15}$ by saying, "5 goes into 15 *three* times. Three times 4 is 12."

We did not change $\frac{2}{15}$, because we are not changing the denominator 15.

Example 5. $\frac{2}{3}+\frac{1}{6}+\frac{7}{12}$

Solution. The LCM of 3, 6, and 12 is 12.

$$\frac{2}{3}+\frac{1}{6}+\frac{7}{12}=\frac{8+2+7}{12}$$

$$\frac{2}{3}+\frac{1}{6}+\frac{7}{12}=\frac{17}{12}$$

$$\frac{2}{3}+\frac{1}{6}+\frac{7}{12}=1\frac{5}{12}.$$

We converted $\frac{2}{3}$ to $\frac{8}{12}$ by saying, "3 goes into 12 *four* Times. Four times 2 is 8."

We converted $\frac{1}{6}$ to $\frac{2}{12}$ by saying, "6 goes into 12 *two* Times. Two times 1 is 2."

We did not change $\frac{7}{12}$, because we are not changing the

Denominator 12.

Finally, we changed the improper fraction $\frac{17}{12}$ to $1\frac{5}{12}$ by

Dividing 17 by 12.

"12 goes into 17 one (1) time with remainder 5."

Example 6. $\frac{5}{6} + \frac{7}{9}$

Solution.

The LCM of 6 and 9 is 18.

$$\frac{5}{6} + \frac{7}{9} = \frac{15 + 14}{18} = \frac{29}{18} = 1\frac{11}{18}.$$

We changed $\frac{5}{6}$ to $\frac{15}{18}$ by multiplying both terms by 3.

We changed $\frac{7}{9}$ to $\frac{14}{18}$ by multiplying both terms by 2.

Example 7. Add mentally $\frac{1}{2} + \frac{1}{4}$.

Answer. $\frac{1}{2}$ is how many $\frac{1}{4}$'s?

$$\frac{1}{2} = \frac{2}{4}.$$

Just as 1 is half of 2, so 2 is half of 4. Therefore,

$$\frac{1}{2} + \frac{1}{4} = \frac{3}{4}.$$

The student should not have to write any problem in which one of

the fractions is $\frac{1}{2}$, and the denominator of the other is even.

For example,

$$\frac{1}{2} + \frac{2}{10} = \frac{7}{10}$$

— because $\frac{1}{2} = \frac{5}{10}$.

Example 8. In a recent exam, one-eighth of the students got A, two-fifths got B, and the rest got C
got C?

Solution.

Let 1 represent the whole number of students. Then the question is:

$$\frac{1}{8} + \frac{2}{5} + ? = 1.$$

Now,

$$\frac{1}{8} + \frac{2}{5} = \frac{5 + 16}{40} = \frac{21}{40}.$$

The rest, the fraction that got C, is the complement of $\frac{21}{40}$.

It is $\frac{19}{40}$.

4. How do we add mixed numbers?

$$4\frac{3}{8} + 2\frac{2}{8}$$

Add the whole numbers and add the fractions separately.

Example 9. $4\frac{3}{8} + 2\frac{2}{8} = 6\frac{5}{8}$.

Example 10. $3\frac{2}{5} + 1\frac{4}{5} = 4\frac{6}{5}$.

But $\frac{6}{5}$ is improper, we must change it to a mixed number:

$\frac{46}{5} = 1\frac{1}{5}$.

Therefore,

$4\frac{6}{5} = 4 + 1\frac{1}{5} = 5\frac{1}{5}$.

Example 11.

$$\begin{array}{r} 6\frac{3}{4} \\ +\ 3\frac{5}{8} \\ \hline \end{array}$$

Solution.

When the denominators are different, we may arrange the work vertically; although that is not nece

To add the fractions, the denominators must be the same. The LCM

of 4 and 8 is 8. We will change $\frac{3}{4}$ to $\frac{6}{8}$ — by multiplying

both terms by 2:

$$\begin{array}{l} \ \ 6\frac{3}{4} = 6\frac{6}{8} \\ +\ 3\frac{5}{8} = 3\frac{5}{8} \\ \hline \qquad\quad 9\frac{11}{8} = 9 + 1\frac{3}{8} \\ \qquad\qquad = 10\frac{3}{8}. \end{array}$$

We added $6 + 3 = 9$. $\frac{6}{8} + \frac{5}{8} = \frac{11}{8} = 1\frac{3}{8}$.

$9 + 1\frac{3}{8} = 10\frac{3}{8}$.

Subtraction of Fractions

As you know, fractions represent parts of the whole. So, when these parts are from the whole broken into same number of parts it is easy to subtract them.

For Type 1 problems, we just need to subtract the top parts (numerator) of the fractions and leave the denominator as such. So, to solve the problem in the above example the solution will be:

$$\frac{3}{7}-\frac{1}{7}=\frac{3-1}{7}=\frac{2}{7}$$

For Type 2 problems where the denominator is different, we can not subtract these fractions by simply subtracting the numerators. In order to solve these problems first, we will need to make into fractions with the same denominator. See how this is done in examples below.

Example 1

Solve: $\frac{4}{5}-\frac{1}{2}$

changing both fractions to a common denominator (see comments on the right)

$$=\frac{4}{5}-\frac{1}{2}$$
$$=\frac{4\times 2}{5\times 2}-\frac{1\times 5}{2\times 5}$$
$$=\frac{8}{10}-\frac{5}{10}$$
$$=\frac{8-5}{10}$$
$$=\frac{3}{10}$$

Example 2

Solve $3\frac{2}{3}-2\frac{4}{5}$

Solution

Changing them to simple fractions we find:

$$= \frac{(3\times3+2)}{3} - \frac{(5\times2+4)}{5}$$
$$= \frac{11}{3} - \frac{15}{5}$$

Now, changing both fractions to a common denominator (see comments on the right)we can solve it as:

$$= \frac{11\times5}{3\times5} - \frac{15\times3}{5\times3}$$
$$= \frac{55}{15} - \frac{45}{15}$$
$$= \frac{10}{15}$$

This fraction can be further simplified as:

$$\frac{10}{15} = \frac{10\div5}{15\div5} = \frac{2}{3}$$

ASSESSMENT

Simplify the following

1. 3/4 + 2/3

2. 5/6 – 2/9

3. 7/12 – 3/8

4. 11/15 + 1/5

5. 3/4 – 2/3

ANSWER

1. 17/12

2. 11/18

3.5/24

4. 14/15

5. 1/12

Week 12

Topic: ESTIMATION

Estimation

There are many advantages in being able to estimate quantities and distances. A quick estimate can prevent errors.

Common Measures

The most common units for length are millimetre(mm), centimetre(cm), metre (m), and kilometre(km). We use the lower units mm and cm for short lengths and m and km for larger distances.

The most common units of mass are gramme (g), kilogramme (kg), tonne (t).

The common units of capacity are millilitre (ml), centilitre (cl) and litre (l).

ASSESSMENT

State the units of the following lengths

1. the height of a desk
2. height of yourself
3. distance from Lagos to Kano
4. the thickness of a coin
5. your waist

ANSWER

1. cm
2. cm
3. km
4. mm
5. cm

Week 13

Topic: REVIEW OF THE FIRST HALF TERM'S WORK AND PERIODIC TEST

JSS1

MATHEMATICS

SECOND TERM

TABLE OF CONTENT

Week 1

Topic: REVISION OF FIRST TERM'S WORK

Week 2 & 3

Topic: APPROXIMATION

ROUNDING OFF

To 'round off' or 'approximate' a number to a desired degree of accuracy, we round the number *up* if the next digit is 5 or more round the number *down* if the next digit is less than 5.

We represent approximately equal to as and approximately as '~'

Examples

73 is close to 70 if approximating to or rounding in "tens". So

73 ~ 70 (Read as 73 is approximately equal to 70)

86 is close to 90 when rounded off to the nearest "tens", or approximate to the nearest "tens"

86 ~ 90

650 ~ 700 when rounded off to the nearest hundreds

26432 rounded off to nearest 100 is 264 / 32 ~ 26400 nearest 10,000 is 2 / 6432 ~ 30000

Strategy to round off numbers

Put a line where you want to round off. In the above example 3 to round off 650, put a big line after 6, because 6 is in the hundreds place, and you want to round to the nearest hundred.

6/50

The digit before the big line (6 in this case) will go up by 1, and the rest of the digits after the line will become 0, since the number after the line is 5. So the answer is 700.

Approximation

Approximation is the process of using rounded off numbers to estimate the outcome of calculations. As with the estimation of quantities, the ability to find the approximate results is very useful. Consider the following

A farmer is thinking of buying 220 week old chickens at N170 each He does a rough calculation first

N170 x 220 ≅ N200 x 220 = N40, 000.

The symbol ≅ means approximately equal to. The approximation is not correct but it gives the farmer a good idea of the true cost of the chickes. With this he knows how much to budget and then he does the real calculationN170 x 220 = N37,400

N37,400 ≅ N40,000

Estimation

Sometimes, we may have to guess or estimate numbers as close as possible to their real values. This process of guessing the numbers sensibly is called estimation. Generally you can use estimation to work out whether your calculation or the answer you are working towards is right or wrong.

We use a simple strategy to estimate a number for calculations – we use the first digit position for rounding off. This first digit is also called the leading figure.

To estimate the answer of a calculation using leading figure estimation:

round each number to its nearest leading figure

evaluate the resulting expression

Let’s look at some examples to demonstrate this strategy.

Note that the sign ~, means approximation.

234 + 57 ~ 200 + 60 = 260

Here 234 is close to 200, and 57 is closer to 60 – the leading figures here are 2 and 5 respectively. So the answer to this equation should be close to 260.

The correct answer for 234 + 57 = 291, which is a bit further away from the estimated answer of 260.

So how do we get a better estimation?

We need to change the leading figure for 234 – earlier, we considered 2 to be the leading figure, but now we should consider 3 to be the leading figure. So

234 + 57 = 230 + 60 = 290, which is very close to 291 (the correct answer).

287 x 34

287 ~ 300, and 34 ~ 30 (leading figures being 2 and 3 respectively)

So 287 x 34 ~ 300 x 30 = 9000

The correct answer of 287 x 34 is 9758.

If 523 people pass through the gates of a stadium every two minutes, estimate how many people attending the game if the gates were open for only an hour.

523 people in 2 minutes ~ 500 people in 2 minutes

In 60 minutes, there are 30 groups of 2 minutes. So

523 x 30 ~ 500 x 30 = 15000 people

Round off each number to one significant figure, then approximate the answers.

Example

a. 47 + 31

b. 291 + 603

Solution

a. 47 + 31 ~ 50 + 30 = 80

b. 291 + 603 ~ 300 + 600 = 900

Exercises

Round off each number to one significant figure. Then approximate each answer

1. 23 + 19

a. 40 b. 30 c. 80 d. 50

2. 24 X 37

a. 940 b. 800 c. 700 d. 650

3. 73 – 48

a. 24 b. 34 c. 20 d. 30

Round off each number to the nearest whole number. Then find the approximate answer.

4. 6.2 + 3.7

a. 9 b. 6 c. 7 d. 10

5. 3.4 X 5.8

a. 18 b. 24 c. 19 d. 20

Answers

1. A 2. B 3. C 4. D 5. A

Week 4 & 5

Topic: BASE TWO ARITHMETIC

Converting between different number bases is actually fairly simple, but the thinking behind it can seem a bit confusing at first. And while the topic of different bases may seem somewhat pointless to you, the rise of computers and computer graphics has increased the need for knowledge of how to work with different (non-decimal) base systems, particularly binary systems (ones and zeroes) and hexadecimal systems (the numbers zero through nine, followed by the letters A through F).

In our customary base-ten system, we have digits for the numbers zero through nine. We do not have a single-digit numeral for "ten". Yes, we write "10", but this stands for "1 ten and 0 ones". This is two digits; we have no single solitary digit that stands for "ten".

Instead, when we need to count to one more than nine, we zero out the ones column and add one to the tens column. When we get too big in the tens column — when we need one more than nine tens and nine ones ("99"), we zero out the tens and ones columns, and add one to the ten-times-ten, or hundreds, column. The next column is the ten-times-ten-times-ten, or thousands, column. And so forth, with each bigger column being ten times larger than the one before. We place digits in each column, telling us how many copies of that power of ten we need.

The only reason base-ten math seems "natural" and the other bases don't is that you've been doing base-ten since you were a child. And (nearly) every civilization has used base-ten math probably for the simple reason that we have ten fingers. If instead we lived in a cartoon world, where we would have only four fingers on each hand (count them next time you're watching TV or reading the comics), then the "natural" base system would likely have been base-eight, or "octal".

Binary

Let's look at base-two, or binary, numbers. How would you write, for instance, 12_{10} ("twelve, base ten") as a binary number? You would have to convert to base-two columns, the analogue of base-ten columns. In base ten, you have columns or "places" for $10^0 = 1$, $10^1 = 10$, $10^2 = 100$, $10^3 = 1000$, and so forth. Similarly in base two, you have columns or "places" for $2^0 = 1$, $2^1 = 2$, $2^2 = 4$, $2^3 = 8$, $2^4 = 16$, and so forth.

The first column in base-two math is the units column. But only "0" or "1" can go in the units column. When you get to "two", you find that there is no single solitary digit that stands for "two" in base-two math. Instead, you put a "1" in the twos column and a "0" in the units column, indicating "1 two and 0 ones". The base-ten "two" (2_{10}) is written in binary as 10_2.

A “three” in base two is actually “1 two and 1 one”, so it is written as 11_2. “Four” is actually two-times-two, so we zero out the twos column and the units column, and put a “1” in the fours column; 4_{10} is written in binary form as 100_2. Here is a listing of the first few numbers:

decimal (base 10)	binary (base 2)	
0	0	0 ones
1	1	1 one
2	10	1 two and zero ones
3	11	1 two and 1 one
4	100	1 four, 0 twos, and 0 ones
5	101	1 four, 0 twos, and 1 one
6	110	1 four, 1 two, and 0 ones
7	111	1 four, 1 two, and 1 one
8	1000	1 eight, 0 fours, 0 twos, and 0 ones
9	1001	1 eight, 0 fours, 0 twos, and 1 one
10	1010	1 eight, 0 fours, 1 two, and 0 ones
11	1011	1 eight, 0 fours, 1 two, and 1 one
12	1100	1 eight, 1 four, 0 twos, and 0 ones
13	1101	1 eight, 1 four, 0 twos, and 1 one
14	1110	1 eight, 1 four, 1 two, and 0 ones
15	1111	1 eight, 1 four, 1 two, and 1 one
16	10000	1 sixteen, 0 eights, 0 fours, 0 twos, and 0 ones

Converting between binary and decimal numbers is fairly simple, as long as you remember that each digit in the binary number represents a power of two.

Convert 101100101_2 to the corresponding base-ten number.

I will list the digits in order, and count them off from the RIGHT, starting with zero:

The first row above (labelled “digits”) contains the digits from the binary number; the second row (labelled ” numbering”) contains the power of 2 (the base) corresponding to each digits. I will use this listing to convert each digit to the power of two that it represents:

$1\times2^8 + 0\times2^7 + 1\times2^6 + 1\times2^5 + 0\times2^4 + 0\times2^3 + 1\times2^2 + 0\times2^1 + 1\times2^0$

$= 1\times256 + 0\times128 + 1\times64 + 1\times32 + 0\times16 + 0\times8 + 1\times4 + 0\times2 + 1\times1$

$= 256 + 64 + 32 + 4 + 1$

= 357

Then 101100101_2 converts to 357_{10}.

Operations with binary numbers

We can add, subtract and multiply binary numbers in much the same ways as we operate with base ten numbers. The main things to remember in base two are:

Addition:

0 + 0 = 0 1 + 0 = 1

0 + 1 = 1 1 + 1 = 10

Multiplication:

0 X 0 = 0 1 X 0 = 0

0 X 1 = 0 1 X 1 = 1

Add the following

 1011

\+ 1101

———

 11000

———

Note: 1st column: 1 + 1 = 10, write down 0 and carry 1

2nd and 3rd columns: as above

4th column: 1 + 1 + 1 (carried) = 11. Write down 1 and carry 1

Let's look at base-two, or binary, numbers. How would you write, for instance, 12_{10} ("twelve, base ten") as a binary number? You would have to convert to base-two columns, the analogue of base-ten columns. In base ten, you have columns or "places" for $10^0 = 1$, $10^1 = 10$, $10^2 = 100$, $10^3 = 1000$, and so forth. Similarly in base two, you have columns or "places" for $2^0 = 1$, $2^1 = 2$, $2^2 = 4$, $2^3 = 8$, $2^4 = 16$, and so forth.

The first column in base-two math is the units column. But only “0” or “1” can go in the units column. When you get to “two”, you find that there is no single solitary digit that stands for “two” in base-two math. Instead, you put a “1” in the twos column and a “0” in the units column, indicating “1 two and 0 ones”. The base-ten “two” (2_{10}) is written in binary as 10_2.

A “three” in base two is actually “1 two and 1 one”, so it is written as 11_2. “Four” is actually two-times-two, so we zero out the twos column and the units column, and put a “1” in the fours column; 4_{10} is written in binary form as 100_2. Here is a listing of the first few numbers:

decimal (base 10)	binary (base 2)	
0	0	0 ones
1	1	1 one
2	10	1 two and zero ones
3	11	1 two and 1 one
4	100	1 four, 0 twos, and 0 ones
5	101	1 four, 0 twos, and 1 one
6	110	1 four, 1 two, and 0 ones
7	111	1 four, 1 two, and 1 one
8	1000	1 eight, 0 fours, 0 twos, and 0 ones
9	1001	1 eight, 0 fours, 0 twos, and 1 one
10	1010	1 eight, 0 fours, 1 two, and 0 ones
11	1011	1 eight, 0 fours, 1 two, and 1 one
12	1100	1 eight, 1 four, 0 twos, and 0 ones
13	1101	1 eight, 1 four, 0 twos, and 1 one
14	1110	1 eight, 1 four, 1 two, and 0 ones
15	1111	1 eight, 1 four, 1 two, and 1 one
16	10000	1 sixteen, 0 eights, 0 fours, 0 twos, and 0 ones

Converting between binary and decimal numbers is fairly simple, as long as you remember that each digit in the binary number represents a power of two.

- **Convert 101100101_2 to the corresponding base-ten number.**

I will list the digits in order, and count them off from the RIGHT, starting with zero:

digits:	1	0	1	1	0	0	1	0	1
numbering:	8	7	6	5	4	3	2	1	0

The first row above (labelled “digits”) contains the digits from the binary number; the second row (labelled ” numbering”) contains the power of 2 (the base) corresponding to each digits. I will use this listing to convert each digit to the power of two that it represents:

$1\times2^8 + 0\times2^7 + 1\times2^6 + 1\times2^5 + 0\times2^4 + 0\times2^3 + 1\times2^2 + 0\times2^1 + 1\times2^0$

$= 1\times256 + 0\times128 + 1\times64 + 1\times32 + 0\times16 + 0\times8 + 1\times4 + 0\times2 + 1\times1$

$= 256 + 64 + 32 + 4 + 1$

$= 357$

Converting between binary and decimal numbers is fairly simple, as long as you remember that each digit in the binary number represents a power of two.

DECIMAL	BINARY
9	1001
10	1010
11	1011
12	1100
13	1101
14	1110
15	1111
100	0100100
512	1000000000

= 19

DECIMAL	BINARY
0	0000
1	0001
2	0010
3	0011
4	0100
5	0101
6	0110
7	0111
8	1000

ASSESSMENT

Converting between binary and decimal numbers is fairly simple, as long as you remember that each digit in the binary number represents a power of two.

- Convert 100101101_2 to the corresponding base-ten number.

Week 6

Topic: SIMPLE EQUATIONS

Do you go blank when you see x, y and z in mathematics? Well, this is your abc to solving equations.

Solving simple equations

In an equation, letters stand for a missing number. To solve an equation, find the values of missing numbers. A typical exam question is:

Solve the equation **$2a + 3 = 7$**

This means we need to find the value of *a*. The answer is **$a = 2$**

There are two methods you can use when solving this type of problem:

Trial and improvement

Using inverses

Trial and improvement

This method involves trying different values until we find one that works.

Look at the equation **$2a + 3 = 7$**

To solve it:

Write down the equation: $2a +3 = 7$

Then, choose a value for **'a'** that looks about right and work out the equation. Try '3'.

$a = 3$, so $2 \times 3 + 3 = 9$.

Using '3' to represent **'a'** makes the calculation more than 7, so choose a smaller number for **'a'**.

Try $a = 2$

$2 \times 2 + 3 = 7$

Which gives the right answer. So $a = 2$

Be systematic in your approach:

1. choose a number
2. work it out
3. then move the number up or down

However, sometimes the answers are negatives or decimal fractions, and the trialand improvement method will take a long time. Luckily, there is a better method.

Using inverses

The best way to solve an equation is by using 'inverses', or undoing what the equation is doing.

To use this method to solve equations remember that:

Adding and subtracting are the inverse (or opposite) of each other.

Multiplying and dividing are the inverse of each other.

This method is explained in the following pages. But for now, here is how to solve the question in the above example using inverses:

First, write down the expression:

$2a + 3 = 7$

Then, undo the + 3 by subtracting 3. Remember, you need to do it to BOTH sides!

$2a + 3 - 3 = 7 - 3$,

so $2a = 4$.

Undo the multiply by 2 by dividing by 2, again on both sides:

$2a \div 2 = 4 \div 2$

The answer is: **a = 2**

Solution of an equation

It is usually possible to find the value of the unknown which makes an equation true. We call this value the **solution** of the equation.

$x = 6$ is the solution of $3x = 18$.

To **solve an equation** means to find its solution.

Example

Solve the equation 18 – x = 7

The problem is to find a number which when taken from 18 gives 7. The number is 11.

x = 11 is the solution

Find the solution of 3x = 15.

Which number multiplied by 3 gives 15? The number is 5. Thus

x = 5.

Exercise

Solve the following equations

1. 20 + x = 28

2. 14 – x = 11

3. x – 2 = 15

4. 4x = 20

5. 3x + 4 = 17

Week 7

Topic: REVIEW OF FIRST HALF TERM'S WORK AND PERIODIC TEST

Week 8

Topic: BASIC OPERATIONS (Continued)

Addition and subtraction of negative and positive integers

INTEGERS

Whole numbers, which are figures that do not have fractions or decimals, are also called integers. They can have one of two values: positive or negative.

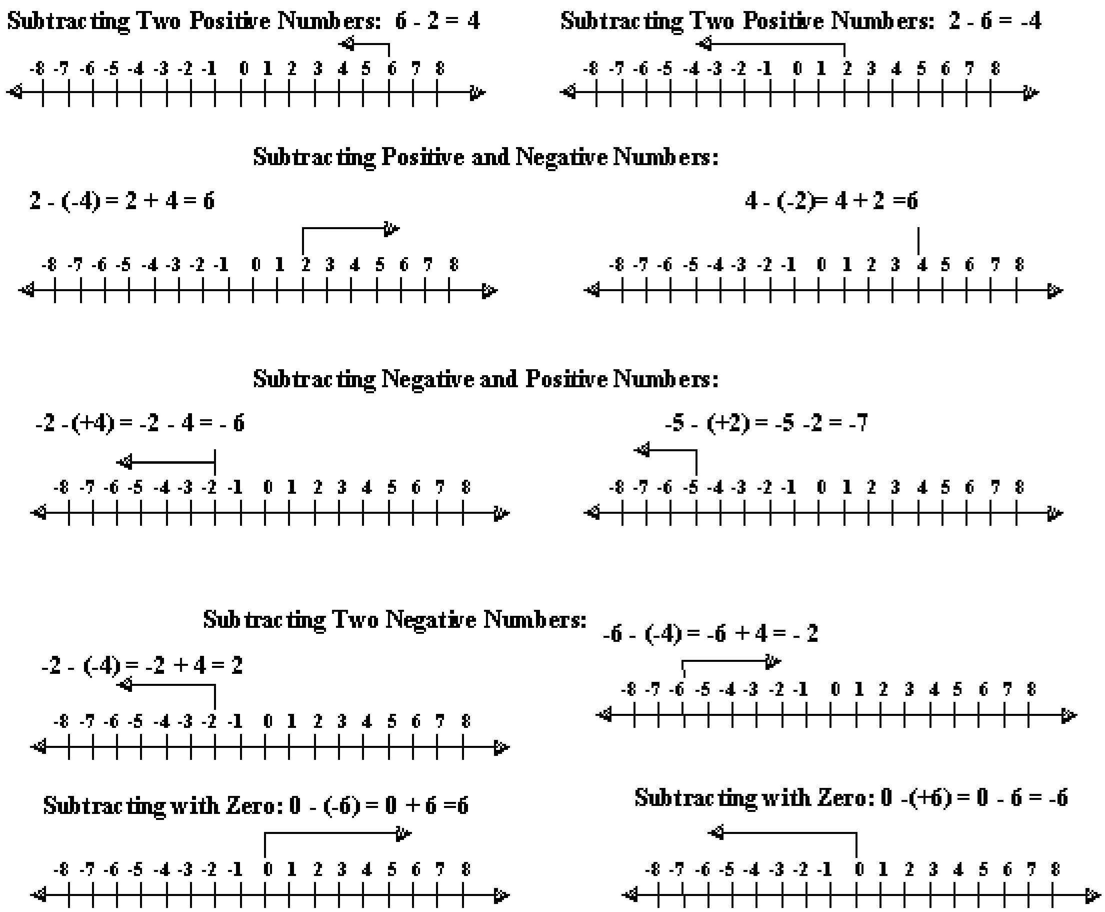
- **Positive integers** have values greater than zero.

- **Negative integers** have values less than zero.
- **Zero** is neither positive nor negative.

The rules of how to work with positive and negative numbers are important because you'll encounter them in daily life, such as in balancing a bank account, calculating weight, or preparing recipes.

- **Subtracting two positive numbers:** The operation is subtraction and the result is either a positive or a negative number.
- **Subtracting a positive and a negative number:** The operation is addition and the result is always positive.
- **Subtracting a negative and a positive number:** The operation is addition and the result is always negative.
- **Subtracting two negative numbers:** The operation is subtraction and the result is a positive or negative number.

Examples:



ADDITION

Whether you're adding positives or negatives, this is the simplest calculation you can do with integers. In both cases, you're simply calculating the sum of the numbers. For example, if you're adding two positive integers, it looks like this:

- 5 + 4 = 9

If you're calculating the sum of two negative integers, it looks like this:

- (−7) + (−2) = −9

To get the sum of a negative and a positive number, use the sign of the larger number and subtract. For example:

- (−7) + 4 = −3
- 6 + (−9) = −3
- (−3) + 7 = 4
- 5 + (−3) = 2

The sign will be that of the larger number. Remember that adding a negative number is the same as subtracting a positive one.

SUBTRACTION

The rules for subtraction are similar to those for addition. If you've got two positive integers, you would subtract the smaller number from the larger one. The result will always be a positive integer:

- 5 − 3 = 2

Likewise, if you were to subtract a positive integer from a negative one, the calculation becomes a matter of addition (with the addition of a negative value):

- (−5) − 3 = −5 + (−3) = −8

If you're subtracting negatives from positives, the two negatives cancel out and it becomes addition:

- 5 − (−3) = 5 + 3 = 8

If you're subtracting a negative from another negative integer, use the sign of the larger number and subtract:

- (−5) − (−3) = (−5) + 3 = −2
- (−3) − (−5) = (−3) + 5 = 2

If you get confused, it often helps to write a positive number in an equation first and then the negative number. This can make it easier to see whether a sign change occurs.

ASSESSMENT

1. 4−(−2)=
   a. 5 b. 4 c. 2 d 6
2. 3+5=
   a 6. b 7 c. 8 d. 10
3. (−4) −(−3)=
   a. −1 b. 1 c. −7 d. 2
4. (−3) −(−4)=
   a. 1 b. −1 c. 2 d. −2
5. (−6)+(−1)
   a. 7 b. −7 c. −5 d 5

ANSWERS

1. d
2. c
3. a
4. a
5. b

Week 9 & 10

Topic: USE OF SYMBOLS 1: LETTERS FOR NUMBERS

Open sentences

14 + □ = 17. What number in the box will make this true? You may have seen problems like this before. 14 + □ = 17 will be true if 3 goes in the box: 14 + 3 = 17 is true,

We say 14 + □ = 17 is an **open sentence**. Any value can go in the box, but usually, only one value can make an open sentence true.

Exercises

1. 3 + 2 = □

2. 8 – 7 = □

3. 7 + 7 = □

Letters for numbers

In mathematics we use letters of the alphabets to stand for numbers instead of boxes. We write 14 + x instead of 14 + □. Any letter can be used. For example, 14 + *a* would be just as good as 14 + x. Capital letters are not used; only small letters are used.

When using a letter instead of a number, the letter can stand for any number in general. Thus the value of 14 + x depends on the value of x.

For example,

If x stands for 2, 14 + x has the value 16;

If x stands for 12, 14 + x has the value 26;

If x stands for 5, 14 + x has the value 19.

When letters and numbers are used together in this way, the mathematics is called **generalized arithmetic** or **algebra** The word algebra comes to us from an important book written around AD830 by Mohammed Musa al Khowarizmi, a noted mathematician from Baghdad. The title of the book was Al-jabr wa'l Muqabalah.

The statement 14 + x = 17 is called an **algebraic sentence**. It means 14 plus a number x makes 17, or 14 plus x equals 17.

In the next exercise, each letter stands for a number. The number that the letter stands for shOuld make the sentence true. For example, 14 + x = 17 will be true if x stands for 3.

We write this:

If 14 + x = 17

Then x = 3

Exercise

Each sentence is true. Find the number that each letter stands for.

1. x = 2 + 7

2. x = 3 – 1O

3. y = 9 + 5

4. z = 18 + 11

5. z = 15 – 8

What is the value of z + 6? The value of x + 6 depends on what x stands for.

If x = 3, then x + 6 = **3** + 6 = 9

If x = 8, then x + **8** + 6 = 14

Examples of algebra word problems are numerous. The goal of this unit is to give you the skills that you need to solve a variety of these algebra word problems

Example 1:

A football team lost 5 yards and then gained 9. What is the team's progress?

Solution

For lost, use negative. For gain, use positive.

Progress = $-5 + 9 = 4$ yards

Example 2:

Use distributive property to solve the problem below:

Maria bought 10 notebooks and 5 pens costing 2 dollars each. How much did Maria pay?

Solution

$2 \times (10 + 5) = 2 \times 10 + 2 \times 5 = 20 + 10 = 30$ dollars

Example 3:

A customer pays 50 dollars for a coffee maker after a discount of 20 dollars

What is the original price of the coffee maker?

Solution

Let x be the original price.

$x - 20 = 50$

$x - 20 + 20 = 50 + 20$

$x + 0 = 70$

$x = 70$

Example 4:

Half a number plus 5 is 11.What is the number?

Solution

Let x be the number. Always replace “is” with an equal sign

$(1/2)x + 5 = 11$

$(1/2)x + 5 - 5 = 11 - 5$

$(1/2)x = 6$

$2 \times (1/2)x = 6 \times 2$

x = 12

Example 5:

The sum of two consecutive even integers is 26. What are the two numbers?

Solution

Let 2n be the first even integer and let 2n + 2 be the second integer

2n + 2n + 2 = 26

4n + 2 = 26

4n + 2 – 2 = 26 – 2

4n = 24

n = 6

So the first even integer is 2n = 2 × 6 = 12 and the second is 12 + 2 = 14

Below are more complicated algebra word problems

Example 6:

The ratio of two numbers is 5 to 1. The sum is 18. What are the two numbers?

Solution

Let x be the first number. Let y be the second number

x / y = 5 / 1

x + y = 18

Using x / y = 5 / 1, we get x = 5y after doing cross multiplication

Replacing x = 5y into x + y = 18, we get 5y + y = 18

6y = 18

y = 3

x = 5y = 5 × 3 = 15

As you can see, 15/3 = 5, so ratio is correct and 3 + 15 = 18, so the sum is correct.

Example 7: Algebra word problems can be as complicated as example #7. Study it carefully!

Peter has six times as many dimes as quarters in her piggy bank. She has 21 coins in her piggy bank totaling $2.55

How many of each type of coin does she have?

Solution

Let x be the number of quarters. Let 6x be the number of dimes

Since one quarter equals 25 cents, x quarters equals x × 25 cents or 25x cents

Since one dime equals 10 cents, 6x dimes equals 6x × 10 cents or 60x cents

Since one 1 dollar equals 100 cents, 2.55 dollars equals 2.55 × 100 = 255 cents

Putting it all together, 25x cents + 60x cents = 255 cents

85x cents = 255 cents

85x cents / 85 cents = 255 cents / 85 cents

x = 3

6x = 6 × 3 = 18

Therefore Peter has 3 quarters and 18 dimes

Example 8:

The area of a rectangle is $x^2 + 4x - 12$. What are the dimensions of the rectangle (length and width)?

Solution

The main idea is to factor $x^2 + 4x - 12$

Since −12 = **−2** × **6** and **−2** + **6** = 4

$x^2 + 4x - 12 = (x + -2) \times (x + 6)$

Since the length is usually longer, length = x + 6 and width = x + −2

Example 9: A must know how when solving algebra word problems

The area of a rectangle is 24 cm^2. The width is two less than the length. What is the length and width of the rectangle?

Solution

Let x be the length and let x – 2 be the width

Area = length × width = $x \times (x - 2) = 24$

$x \times (x - 2) = 24$

$x^2 + -2x = 24$

$x^2 + -2x - 24 = 0$

Since $-24 = 4 \times -6$ and $4 + -6 = -2$, we get:

$(x + 4) \times (x + -6) = 0$

This leads to two equations to solve:

$x + 4 = 0$ and $x + -6 = 0$

$x + 4 = 0$ gives $x = -4$. Reject this value since a dimension cannot be negative

$x + -6 = 0$ gives $x = 6$

Therefore, length = 6 and width = $x - 2 = 6 - 2 = 4$

Example 10:

The sum of two numbers is 16. The difference is 4. What are the two numbers?

Let x be the first number. Let y be the second number

$x + y = 16$

$x - y = 4$

Solution

Let x be the first number. Let y be the second number

$x + y = 16$

$x - y = 4$

Solve the system of equations by elimination

Adding the left sides and the right sides gives:

$x + x + y + -y = 16 + 4$

$2x = 20$

$x = 10$

Since $x + y = 16$, $10 + y = 16$

$10 + y = 16$

$10 - 10 + y = 16 - 10$

$y = 6$

The numbers are 10 and 6

The algebra word problems I solved above are typical questions. You will encounter them a lot in algebra. Hope you had fun solving these algebra word problems?

Assessment

1. The sum of two numbers is 36. The difference is 8. What are the two numbers?
2. The area of a square is 16 cm^2. What is the length of a side?
3. The ratio of two numbers is 3 to 2. The sum is 25. What are the two numbers?
4. The sum of two integers is 10 and their product is 24. What are the two numbers?
5. Half a number plus 10 is 12.What is the number?
6. Deji bought 10 apples and 5 oranges costing 8 Naira each. How much did Maria pay?

Week 11

Topic: ALGEBRAIC SIMPLIFICATION 2: BRACKETS

Multiplying and Dividing Algebraic Terms

a. Just as 5a is a short for 5 X a, so ab is short for a X b.

b. Just as 5 X 3 = 3 X 5,

so a X b = b X a.

It is usually to write the letters in alphabetical order, but would be just as correct as ab.

c. Just as 5^2 is short for 5 X 5, so a^2 is short for a X a and x^3 is shorth for x X x X .

d. 4x + 4x + 4x = 12x

3 X 4x = 12x

and 3x + 3x + 3x + 3x = 12x

4 X 3x = 12x

Thus: 3 X 4x = 4 X 3x = 12x

The term 3, 4 and x can be multiplied in any order.

3 X 4x = 4 X 3x = 3x X 4 = 4x X 3

= 4 X x X 3 = x X 3 X 4 = 12x

It is usual to write the numbers before the letters.

Example

We can multiply two algebraic terms to get a product, which is also an algebraic term.

Example :

Evaluate $3pq^3 \times 4qr$

Solution:

$3pq^3 \times 4qr$
$=3 \times p \times q \times q \times q \times 4 \times q \times r$
$=3 \times 4 \times p \times q \times q \times q \times q \times r$
$=12 \times p \times q4 \times r$
$= 12pq4r$

Example :

Evaluate $-2a^3b \times 3ab^2c$

Solution:

$-2a^3b \times 3ab^2c$
$=-2 \times 3 \times a^3 \times a \times b \times b^2 \times c$
$= -6 \times a^4 \times b^3 \times c$
$= -6a^4b^3c$

Example

Multiply the following terms.

$2x^2y \cdot 6x^2y$

$3ab \cdot 5xy^2$

Answers

First multiply the numerical coefficients, then use the exponent rules.

$2x^2y \cdot 6x^2y = (2 \cdot 6)(x^2y \cdot x^2y) = 12x^{2+2}y^{1+1} = 12x^4y^2$

$3ab \cdot 5xy^2 = (3 \cdot 5)abxy^2 = 15abxy^2$

Division

In algebra, letters stand for numbers. Just as fractions can be reduced to their lowest terms by equal division of the numerator and denominator so a letter can be divided by the same letter.

Example

$x \div x = 1$, just as $3 \div 3 = 1$.

Example

Simplify	Working	result
14a/7	= 7 X 2a/7 =1 X 2a/1	= 2a
1/3 of 36x	= 36 X x/3 = 3 X 12x/3	
	= 1 X 12x/1	= 12x
1/5 of y	does not simplify	=1/5y or y/5

Example

Simplify	working	result
5ab ÷ a	= 5 X a X b/a	
	= 5 X 1 X b/1 = 5 X b	= 5b
6xy/2y	= 6 X x X y/2 X y	
	= 3 X x X 1/1 X 1 = 3 X x	= 3x
X^2 ÷ x	= X x X/X = 1 X x/1	= x
$24x^2y$ ÷ 3xy	= 24 X x^2 X y/3 X x X y	
	= 3 X 8 X x X 1/3 X 1 X 1	= 8x
	= 8 X x	

Order of Operation

What is the value of 17 – 5 X 2? It is possible to get two different answers:

a. (17 – 5) X 2 = 12 X 2 = 24

b. 17 – (5 X 2) = 17 – 10 = 7

The answer depends on whether we do the subtraction first or the multiplication first. To avoid confusion, remember the following rules.

a. If there are no brackets do multiplication or division before addition or multiplication.

b. If there are brackets do the operations inside the brackets first.

Usually, operations involving multiplication and division are enclosed in brackets and done before addition and subtraction.

Use the word BODMAS to remember the correct order: **B**rackets, **O**f, **D**ivision, **M**ultiplication, **A**ddition, **S**ubtraction.

Example

Find the value of 16 X 2 – 3 + 14 ÷ 7.

16 X 2 – 3 + 14 ÷ 7

= (16 X 2) – 3 + (14 ÷ 7)

= 32 – 2 + 2

= 34 – 3

= 31

Example

Simplify

Simplify 7 X 3a – (3a + 5a) X 2

7 X 3a – (3a + 5a) X 2 = 7 X 3a – 8a X 2

= (7 X 3a) – (8a X 2)

=21a – 16a = 5a

Removing Brackets

Always try to simplify the terms inside brackets first. If they will not simplify, remove the brackets.

Positive sign before a bracket

a. 9 + (5 + 2) = 9 + 7 = 16

also 9 + 5 + 2 = 14 + 2 = 16

thus 9 + (5 + 2) = 9 + 5 + 2

Similarly with letters,

$a + (b + c) = a + b + c$

b. 9 + (5 − 2) = 9 + 3 = 12

also 9 + 5 − 2 = 14 −2 = 12

thus 9 + (5 − 2) = 9 + 5 −2

Similarly with letters,

$a + (b - c) = a + b - c$

when there is a positive sign before a bracket:

the signs of the terms inside the bracket stay the same when it is removed.

Example

Simplify $7g + (3g + 4h)$.

$7g + (3g + 4h) = 7g + 3g + 4h = 10g + 4h$

Example

Simplify $13p + (6p - 3q)$.

$13p + (6p - 3q) = 13p + 6p - 3q = 19p - 3q$

Negative sign before a bracket

a. 9 − (5 + 2) = 9 − 7 = 2

The result is the same as first taking away 5, then taking away 2:

i.e. 9 − 5 − 2 = 4 − 2 = 2

thus 9 − (5 + 2) = 9 − 5 − 2 = 2.

Similarly with letters,

$a - (b + c) = a - b - c$.

b. $9 - (5 - 2) = 9 - 3 = 6$

The result is the same as first taking away 5 and then adding 2:

i.e. $9 - 5 + 2 = 4 + 2 = 6$

thus $9 - (5 - 2) = 9 - 5 + 2$.

Similarly with letters,

$a - (b - c) = a - b + c$.

When there is a negative sign before the bracket:

The signs of the terms inside the bracket are changed when the bracket is removed.

Example

Simplify $5a - (2a + 8)$.

$5a - (2a + 8) = 5a - 2a - 8 = 3a - 8$

Example

Simplify $10d - (9c - 4d)$

$10d - (9c - 4d) = 10d - 9c + 4d = 14d - 9c$

Exercise

Simplify the following in two ways. The first example shows you how to do this

1. $9 - (7 - 3)$

a. $9 - (7 - 3) = 9 - 4 = 5$

b. $9 - (7 - 3) = 9 - 7 + 3 = 2 + 3 = 5$

2. $9 - (5 + 2)$

3. $16 - (4 - 1)$

4. 24 – (15 + 8)

5. 12 – (9 – 5)

JSS 1

MATHEMATICS

THIRD TERM

TABLE OF CONTENT

Week 1

Topic: REVISION OF SECOND TERM'S WORK

ASSESSMENT

Summary of last terms work

Week 2

Topic: SIMPLE EQUATIONS

1. Read the problem carefully and figure out what it is asking you to find.

Usually, but not always, you can find this information at the end of the problem.

2. Assign a variable to the quantity you are trying to find.

Most people choose to use x, but feel free to use any variable you like. For example, if you are being asked to find a number, some students like to use the variable n. It is your choice.

3. Write down what the variable represents.

At the time you decide what the variable will represent, you may think there is no need to write that down in words. However, by the time you read the problem several more times and solve the equation, it is easy to forget where you started.

4. Re-read the problem and write an equation for the quantities given in the problem.

This is where most students feel they have the most trouble. The only way to truly master this step is through lots of practice. Be prepared to do a lot of problems.

5. Solve the equation.

The examples done in this lesson will be linear equations. Solutions will be shown, but may not be as detailed as you would like. If you need to see additional examples of linear equations worked out completely, click here. (link to linear equations solving.doc)

6. Answer the question in the problem.

Just because you found an answer to your equation does not necessarily mean you are finished with the problem. Many times you will need to take the answer you get from the equation and use it in some other way to answer the question originally given in the problem.

7. Check your solution.

Your answer should not only make sense logically, but it should also make the equation true. If you are asked for a time value and end up with a negative number, this should indicate that you've made an error somewhere. If you are asked how fast a person is running and give an answer of 700 miles per hour, again you should be worried that there is an error. If you substitute these unreasonable answers into the equation you used in step 4 and it makes the equation true, then you should re-think the validity of your equation.

Let's Practice:

1. When 6 is added to four times a number, the result is 50. Find the number.

Step 1: What are we trying to find?

A number.

Step 2: Assign a variable for the number.

Let's call it n.

Step 3: Write down what the variable represents.

Let n = a number

Step 4: Write an equation.

We are told 6 is added to 4 times a number. Since n represents the number, four times the number would be $4n$. If 6 is added to that, we get **6 + 4n**. We know that answer is 50, so now we have an equation **6 + 4n = 50**

Step 5: Solve the equation.

6 + 4n = 50

4n = 44

n = 44

Step 6: Answer the question in the problem

The problem asks us to find a number. We decided that n would be the number, so we have n = 11. The number we are looking for is 11.

Step 7: Check the answer.

The answer makes sense and checks in our equation from Step 4.

$6 + 4(11) = 6 + 44 = 50$

2. The sum of a number and 9 is multiplied by -2 and the answer is -8. Find the number.

Step 1: What are we trying to find?

A number.

Step 2: Assign a variable for the number.

Let's call it *n*.

Step 3: Write down what the variable represents.

Let n = a number

Step 4: Write an equation.

We know that we have the sum of a number and 9 which will give us $n + 9$. We are then told to multiply that by -2, so we have $-2n\ (n+9)$. Be very careful with your parentheses here. The way this is worded indicates that we find the sum first and then multiply. We also know the answer is -8. So we will solve $-2(n+9) = -8$

Step 5: Solve the equation.

$-2(n+9) = -8$

$-2n - 18 = -8$

$-2n=10$

$n=-5$

Step 6: Answer the question in the problem

The problem asks us to find a number. We decided that *n* would be the number, so we have $n = -5$. The number we are looking for is -5.

Step 7: Check the answer.

The answer makes sense and checks in our equation from Step 4.

$-2(n+9)=-2(-5+9)=-2(4)=-8$

1. **On an algebra test, the highest grade was 42 points higher than the lowest grade. The sum of the two grades was 138. Find the lowest grade.**

Step 1: What are we trying to find?

The lowest grade on an algebra test.

Step 2: Assign a variable for the lowest test grade.

Let's call it l.

Step 3: Write down what the variable represents.

Let l= the lowest grade

Step 4: Write an equation.

Whatever the lowest grade is, we are told that the highest grade is 42 points higher than that. That means we need to add 42 to the lowest grade. This tells us the highest grade is l+42. We also know that the highest grade added to the lowest grade is 138. So, (highest grade) + (lowest grade) = 142. In terms of our variable, (l+42)+(l)=138

Step 5: Solve the equation.

(l+42)+(l)=138

2l+42=138

2l=96

l=48

Step 6: Answer the question in the problem

The problem asks us to find the lowest grade. We decided that l would be the number, so we have l= 48. The lowest grade on the algebra test was 48.

Step 7: Check the answer.

The answer makes sense and checks in our equation from Step 4.

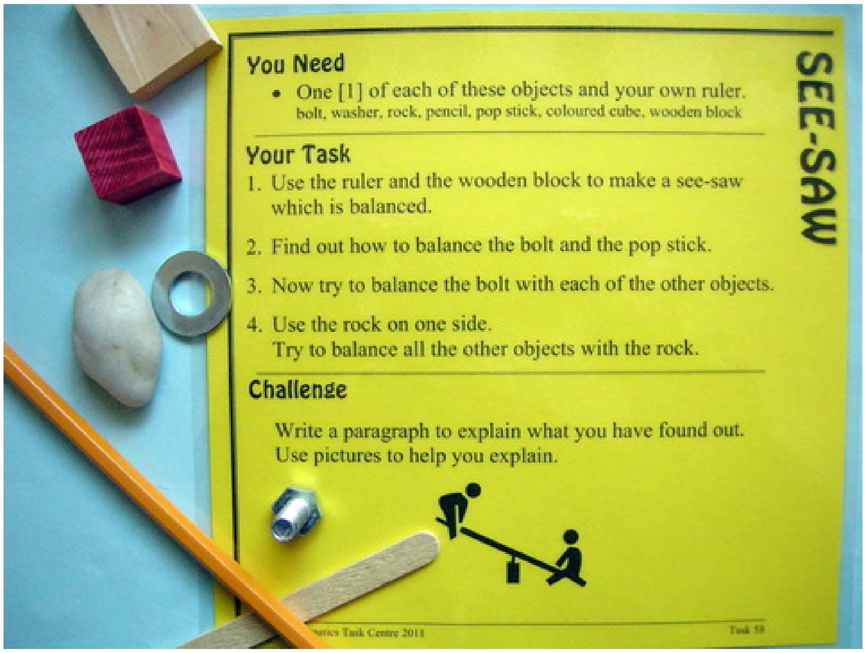
(48+42)+(48)=90+48=138

Materials

- Your own ruler and a block to make a see-saw
- Objects to balance such as a washer, bolt, pencil, nut, stone, pop-stick, wooden cube

Content

- informal investigation of weight and mass
- concept of balance/equality
- basic arithmetic calculations
- reporting an investigation



The task is quite an open-ended exploration. Who knows what will be discovered?

There is a relationship behind what happens and it is expressed mathematically as M = Fd, where M Moment (or Torque), F = Force at a distance d from the fulcrum (or balance point). The force in this downward and described as the weight of the object. Balancing happens when the Moment of one equal to the Moment of the other.

ASSESSMENT
If one object is exerting a force of 2 units at a distance of 6 units from the fulcrum, and the other o

a weight of 4 units, then it would have to be placed 3 units from the fulcrum (and on the other side
a balance.

No one is expecting Year 2 students to express their understanding in this way, but an explanation

Light things can balance heavy things if the heavy things move in and the light things move out.
or
If two things are the same you have to balance them the same distance out.

displays understanding of the essence of the system. It is also the principle behind the Number Eq
many classrooms – a balance on which students hang masses to explore equations.

The objective of the task is to encourage students to explain – orally, then in written word and pictu
they have discovered. Questions which might encourage this are:

1. Tell me what you have found out so far?
2. How did you discover that?
3. If you had a real see-saw and a big fat kid and a little skinny kid, where would you place them
balance?
4. Suppose I gave you two wooden cubes that where exactly the same, where would you place
get a balance.
5. Suppose I gave you three wooden cubes that were exactly the same, where would you place
get a balance?
6. The wooden cubes are over there. Experiment with them to see what you can find out.
7. Do the measurement on the ruler help?
8. What could you do to make your experiment more accurate?

Week 3 & 4

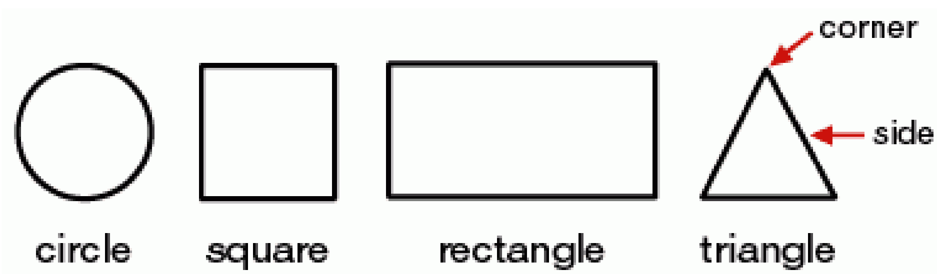
Topic: Geometry

Plane Shapes

Plane shapes in mathematics are any closed, flat, 2-dimensional shapes. A closed, two-dimensional or flat figure is called a **plane shape.** Different plane shapes have different attributes, such as the numbers of **sides** or **corners.** A side is a straight line that makes part of the shape, and a corner is where two sides meet. Let's take a look at some shapes:

As you can see, the plane shapes in the top row include a triangle, rectangle, diamond, and star. The plane shapes in the bottom row include a pentagon (or a 5-

Sided shape), circle, and square.

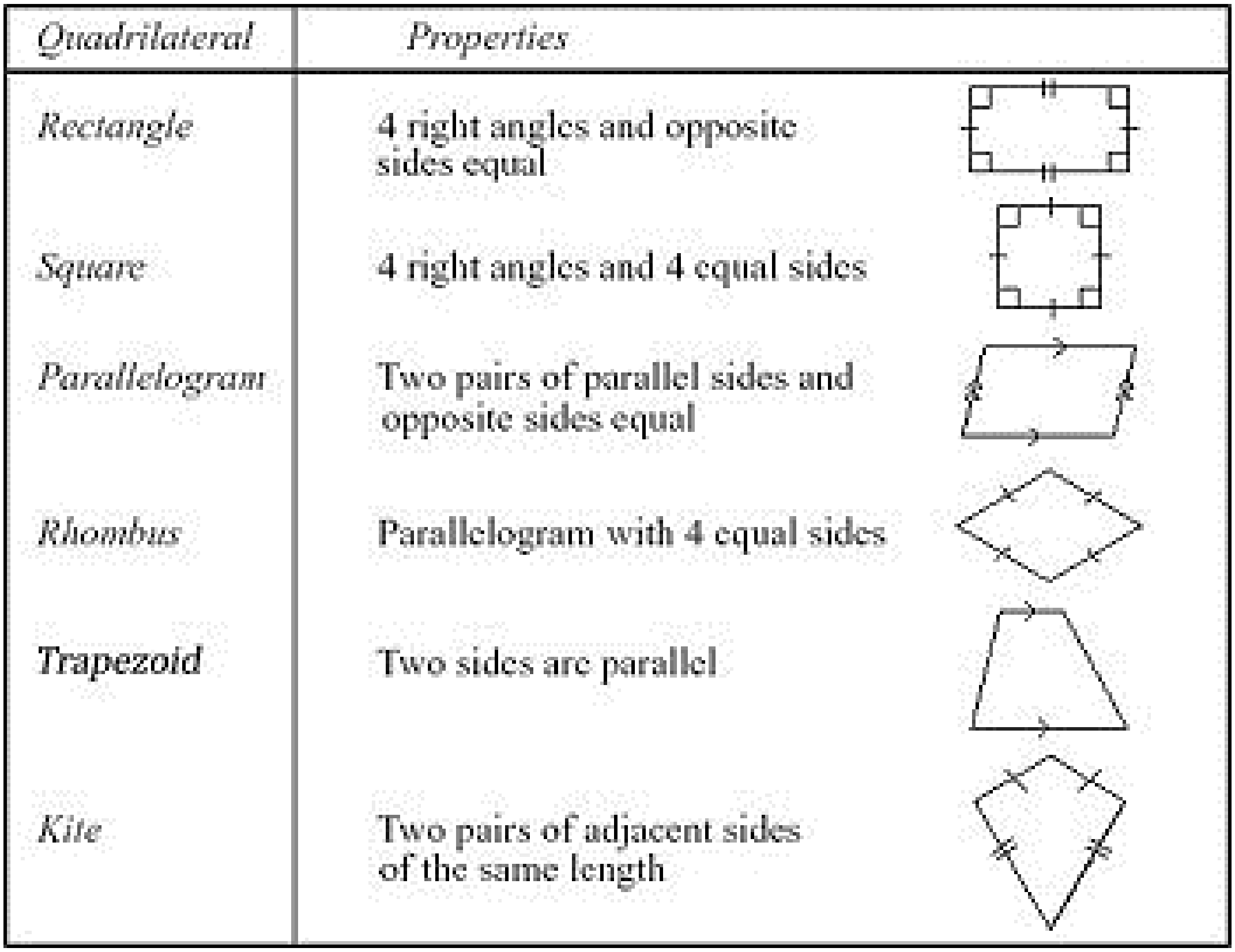


Types of Plane Shapes

There are different types of plane shapes? For instance, many plane shapes are **polygons**, or any 2-dimensional shape with straight sides or lines that is closed and has no open sides. Let's go back and take another look at the image of the plane shapes and see if any of these shapes are polygons.

That's right; the triangle, rectangle, diamond, star, pentagon, and square are all polygons.

Another type of plane shape is known as a **quadrilateral**, or a 2-dimensional shape with 4 straight sides that is closed and has no open sides. Let's look at some examples of quadrilaterals:



Quadrilateral	Properties
Rectangle	4 right angles and opposite sides equal
Square	4 right angles and 4 equal sides
Parallelogram	Two pairs of parallel sides and opposite sides equal
Rhombus	Parallelogram with 4 equal sides
Trapezoid	Two sides are parallel
Kite	Two pairs of adjacent sides of the same length

Quadrilaterals Chart

Plane shapes are both polygons and quadrilaterals.

A **triangle** is a shape with three sides and three corners. A **rectangle** is a shape with four sides and four corners. They may notice that opposite sides are the same length. A **square** is a rectangle in which all four sides are of equal length. A **circle** is a round shape that has no sides or corners.

Properties of Plane Shapes

Plane shapes have properties including sides, corners, and faces.

Triangles

A triangle is a shape with three sides. It can be classified according to its sides or angles, with three kinds each. Here they are:

- **Equilateral triangles**, which are also **equiangular triangles**, have three sides equal and three angles equal. Their angles are always 60°.
- **Isosceles triangles** are triangles in which two of the sides are equal. The non-included angles of the sides are also equal.
- **Scalene triangles** have no equivalence in any way.
- **Right triangles** are triangles with a right angle. The longest side of such triangles is called a hypotenuse.
- **Obtuse triangles** are triangles with an obtuse angle.
- **Acute triangles** are triangles with no right or obtuse angle.

It is interesting to note that the interior angles of triangles must add up to 180°. This is commonly used in proofs and other problems. Imagine a triangle whose points are marked A, B and C, angle A is 60 degrees, and angle B is 70 degrees:

Usually, when drawing a triangle, we draw one side horizontally. This side is usually called the **base**. There is nothing special about the base. By turning your paper you can make any side into the base. There is no mathematical reason to call one side a base; we do it to make talking about the triangle easier. When you have a triangle and think of one of the sides as the base, then there is one corner of the triangle that is not on the base and this point is the furthest point on the triangle from the base. The **height** of the triangle is the line that is perpendicular to the base and goes through that furthest point. Sometimes instead of being called the **height** it is called the **altitude** of the triangle.

Main properties of triangles

1) A sum of triangle angles a + b + g = 180°.

2) Angles lying opposite the equal sides are also equal, and inversely.

All angles in an equilateral triangle are also equal. It follows, that each angle in an equilateral triangle is equal to 60 degrees.

3) In any triangle, if one side is extended, the exterior angle is equal to a sum of interior angles, not supplementary.

The sum of exterior angles is 360°.

4) Any side of a triangle is less than a sum of two other sides and greater than their difference.

5) An angle lying opposite the greatest side, is also the greatest angle, and inversely.

Proof: By turning the side BC, of the scalene triangle ABC below, around the vertex C by the angle g into direction of the side AC, obtained is the isosceles triangle BCD with equal angles on the base BD.

Quadilaterals

A quadrilateral is a shape with four sides. You will spend a lot of time with these. They can be classified into many different categories:

- **Parallelograms** are shapes where opposite sides and angles are equal. The opposite sides are parallel, hence the name.
  - **Rectangles** are parallelograms where the angles are all 90°. Its width or breadth refers to the shorter sides, while its length refers to its longer ones.
  - **Rhombuses** are parallelograms where all the sides are equal, and opposite angles are equal.
  - **Squares** are parallelograms that are both rectangles and rhombuses, i.e. all angles are right and all sides are equal.
- **Trapeziums**, called **trapezoids** in American English, have two opposite sides that are parallel. The parallel sides are sometimes called the upper and lower bases.
  - **Right-angles trapeziums** are trapeziums with a right angle.
  - **Isosceles trapeziums** are trapeziums where the laterals sides are equal but not parallel.
  - **Scalene trapeziums** are trapeziums that fall into neither category.
- **Kites** are quadrilaterals where two pairs of adjacent sides are equal and one pair of opposite angles is equal.
- **Irregular quadrilaterals** are any quadrilaterals that do not fit into one of the groups above.

Rectangles, Squares and Rhombus

- **The rhombus has the following properties:**
  - All the properties of a parallelogram apply (the ones that matter here are parallel sides, opposite angles are congruent, and consecutive angles are supplementary).
  - All sides are congruent by definition.
  - The diagonals bisect the angles.
  - The diagonals are perpendicular bisectors of each other.

Rectangle – is a quadrilateral in which two opposite sides are equal and all angles is right.

- **The rectangle has the following properties:**
  - All the properties of a parallelogram apply (the ones that matter here are parallel sides, opposite sides are congruent, and diagonals bisect each other).
  - All angles are right angles by definition.
  - The diagonals are congruent.

Rectangle can be a parallelogram, rhombus or square in which all the angles right.

1. An opposite sides of the rectangle are the same length, i.e. they are equal:

$AB = CD, \ \ BC = AD$

2. An opposite sides of the rectangle are parallel:

$AB || CD, \ \ BC || AD$

3. An adjacent sides of the rectangle are always perpendicular:

$AB \perp BC, \ \ BC \perp CD, \ \ CD \perp AD, \ \ AD \perp AB$

4. All four angles of the rectangle is right:

$\angle ABC = \angle BCD = \angle CDA = \angle DAB = 90°$

5. The sum all of the angles of a rectangle is equal to 360 degrees:

∠ABC + ∠BCD + ∠CDA + ∠DAB = 360°

6. A diagonals of the rectangle are equal:

AC = BD

7. The sum of the squares two diagonals is equal to the sum of the squares of the sides:

2d2 = 2a2 + 2b2

8. The each diagonal divides the rectangle into two equal shape, namely a right triangle.

9. A diagonal of a rectangle in half divides each other

AO = BO = CO = DO = d2

10. Intersection point of the diagonals is called the center of the rectangle and also a center of the circumcircle (in center).

11. Diagonal of a rectangle is the diameter of the circumcircle.

12. Around the rectangle can always describe a circle, because the sum of the opposite angles is 180 degrees:

∠ABC = ∠CDA = 180° ∠BCD = ∠DAB = 180°

13. In rectangle with the different size of sides never enter the in circle.

Square is a regular quadrilateral in which all four sides and angles are equal. Squares differ only in sides length but all four angles is right angles.

- **The square has the following properties:**
  - All the properties of a rhombus apply (the ones that matter here are parallel sides, diagonals are perpendicular bisectors of each other, and diagonals bisect the angles).
  - All the properties of a rectangle apply (the only one that matters here is diagonals are congruent).
  - All sides are congruent by definition.
  - All angles are right angles by definition

1. All four sides of a square are same length, they are equal:

AB = BC = CD = AD:

AB = BC = CD = AD

2. Opposite side of a square are parallel:

AB||CD, BC||AD

3. All four angles of a square are right angles:

∠ABC = ∠BCD = ∠CDA = ∠DAB = 90°

4. Sum of the angles of a square are equal to 360 degrees:

∠ABC + ∠BCD + ∠CDA + ∠DAB = 360°

5. Diagonal of a square are same length:

AC = BD

6. Each diagonal of a square divides its into two equal symmetrical area
7. Diagonals of a square intersect its right angles, and share each other half:

AO = BO = CO = DO = d2

8. Intersection point of the diagonals is called the center of the square and also the circumcenter of the inscribed circle and circumscribed circle
9. Each diagonal divides the angle of the square in half, meaning they are bisectors of the angles of the square:

ΔABC = ΔADC = ΔBAD = ΔBCD

∠ACB = ∠ACD = ∠BDC = ∠BDA = ∠CAB = ∠CAD = ∠DBC = ∠DBA = 45°

10. Both diagonals divide the square into four equal triangle besides these triangles are both isosceles and rectangular:

ΔAOB = ΔBOC = ΔCOD = ΔDOA

Polygons

- **Pentagons** have five sides.
- **Hexagons** have six sides.

- **Heptagons** or septagons have seven sides.
- **Octagons** have eight sides.
- **Nonagons** have nine sides.
- **Decagons** have ten sides.

And here are two more extras:

- **Hendecagons** (also known as **undecagons**) have eleven sides.
- **Dodecagons** have twelve sides.

Plane Shapes 2: PERIMETER

Measuring perimeters

Perimeter is the boundary or edge of a plane shape. For example, the boundary fence of your school compound is its perimeter. We also use the word perimeter to mean the length of the boundary. For example, if you take 240 paces to walk your school boundary, you could say its perimeter is 240 paces.

Perimeter of regular and irregular shapes

The simplest way to find the perimeter of any **regular** or **irregular shape** is to measure it directly with a ruler or tape measure.

Perimeter
The distance around a polygon is called the perimeter. Although there are shortcuts which can be usedto find perimeter, the basic process involves adding all of the lengths of all of the sides together.
The word "all" has been highlighted because, even though this is a fairly easy process, the one mistake many students make is forgetting to add all of the lengths. Make sure that you include all of the sides!!

6ft. + 3ft. + 6ft. + 3ft. = 18ft.
The perimeter is 18 ft

Let us start with an easy example

A new playground is being built in the neighborhood. It is to be in the shape of a pentagon. If each of the sides is 45 feet in length, how many feet of fencing is needed to totally enclose the area?

Wait a minute, I thought this was supposed to be a “perimeter” problem!
Oh! I see what’s going on, since the fence is going to go all the way around the playground,when we find out the number of feet of fencing, we are also finding the “perimeter” of the shape!
Pretty sneaky!
Since a pentagon has 5 sides, all we have to do is add 45 to itself 5 times:
45 + 45 + 45 + 45 + 45 =
or….5(45) =
Both of these processes will work…
In either case the answer is 225 feet.
We use a capital “P” to represent perimeter,
P = 225′, they will need 225′ of fencing!

Example

First, find the value of “A” and “B” and then fin d the perimeter of this shape.

First to find the length of “A” we need to see that if we add the length of “A” to 19 it would be the same total length as the other side of the shape (32″)
In other words,
“A” + 19 = 32
Solving for “A”:
“A” = 13!
Repeat the process for “B”
“B” + 25 = 56
“B” = 31

Now that we have all the pieces, finding the perimeter is easy!
P = 32 + 56 + 19 + (31) + (13) +25
P = 176″

Example

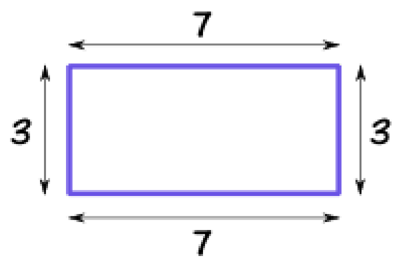
Each of the equal sides of each these regular hexagons is 7.2cm. Find the perimeter, remember, where the shapes are joined is not part of the perimeter

In this problem, what we all have to do is make sure that we include only the sides that actually make up the boundary of this weird shape. One suggestion is to take your pencil and cross out each side as it is counted. Whatever method you use, just

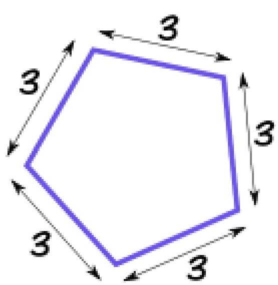
make sure to include all of them, but remember the note in the box above, the sides that are connected are not a part of the perimeter!
Did you come up with 18 sides??
I hope so...
Since all of the sides are 7.2 cm, all we have
to do is multiply:
P = 18 (7.2)
P = 129.6 cm.

Example

The perimeter of this rectangle is 7 + 3 + 7 + 3 = 20



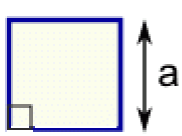
The perimeter of this regular pentagon is 3 + 3 + 3 + 3 + 3 = 5 X 3 = 15



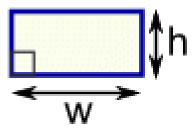
Perimeter Formulas

Triangle

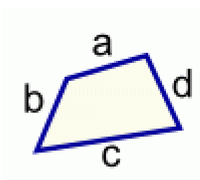
Perimeter = a + b + c



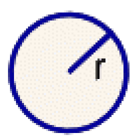
Square
Perimeter = 4 × a
a = length of side



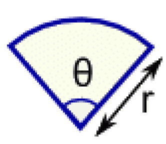
Rectangle
Perimeter = 2 × (w + h)
w = width
h = height



Quadrilateral
Perimeter = a + b + c + d



Circle
Circumference = $2\pi r$
r = radius



Sector
Perimeter = $r(\theta+2)$
r = radius
θ = angle in **radians**

Copy and complete the table of rectangles

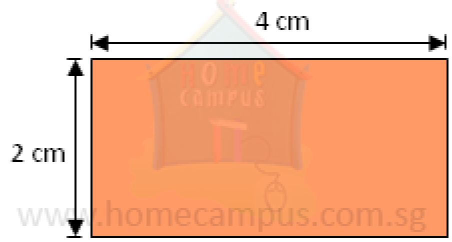
	length	breath	perimeter
a	3cm	2cm	
b	5cm	4m	
c	16cm	10cm	
d	2½ m	1m	
e	6m	2½ m	
f	8 ½ cm	4½ cm	
g		3.2cm	16.6cm
h	4.3cm		10.2cm
i	7.35cm	7.15cm	
j	9m	0.9m	
k		8m	36m
l	14cm		52cm
m	17cm		60cm
n		7.5m	45m
o		6m	33m

PLANE SHAPES 3: AREA

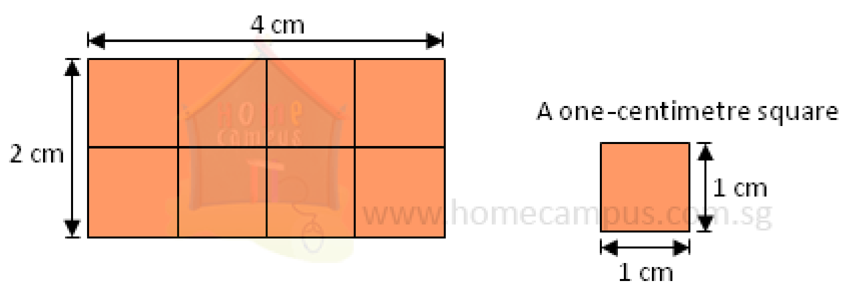
Area

The **Area** of a shape is a measure of its surface. The square is used as the shape for the basic unit of area. A square of side 1m covers an area of **1 square metre** or **1m^2.** A square of side 1 cm covers an area of **1 square centimeter** or **1 cm^2.**

A rectangle is 4 cm long and 2 cm wide. What is its area?



To find the area of the rectangle, we find out how many one-centimetre squares we can fit into the triangle.

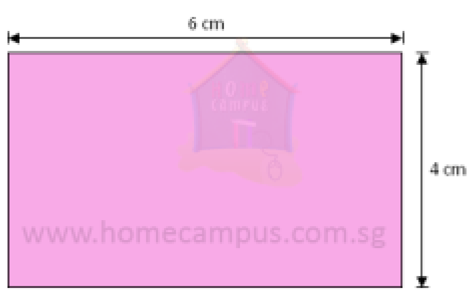


Area of the rectangle = 4 X 2 = 8 cm^2

Area of rectangle = Length × Breadth Area of square = Length × Length
We need 8 one-centimetre squares to make a rectangle 4 cm long and 2 cm wide.
The area of the rectangle is 8 cm^2.

Example

Find the area and perimeter of a rectangle with a length of 6 cm and a breadth of 4 cm.



Area = length X breath

4 cm X 6 cm

24 cm^2

Area of Parallelogram

A parallelogram is a 4-sided shape formed by two pairs of parallel lines. Opposite sides are equal in length and opposite angles are equal in measure. To find the area of a parallelogram, multiply the base by the height. The formula is:

A = b.h where b is the base, h is the height, and · means multiply.

The base and height of a parallelogram must be perpendicular. However, the lateral sides of a parallelogram are not perpendicular to the base. Thus, a dotted line is drawn to represent the height. Let's look at some examples involving the area of a parallelogram.

Find the area of a parallelogram with a base of 12cm and a height of 5cm

Solution

A = b.h

A = 12 cm X 5 cm = 60 cm^2

Example

Find the area of a parallelogram with a base of 7 inches and a height of 10 inches

Solution

A = b.h

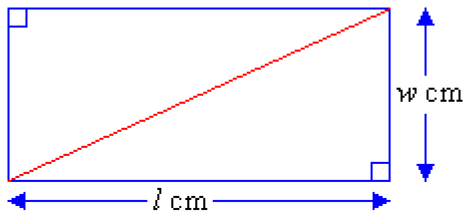
7 inches X 10 inches

70 in^2

Area of Triangles and Trapeziums

Right-angled triangle

Consider a rectangle of length *l* cm and width *w* cm.



Draw a diagonal and cut out the rectangle. Then cut along the diagonal to form two right-angled triangles.

By arranging one triangle over the other, we find that the triangles are **congruent**. In other words, the triangles are the same size and thus, equal in area. This suggests that the area of a triangle is equal to half the area of a rectangle around it. Therefore:

Area of Triangle = ½ X Area or rectangle around it

= ½ X length and width

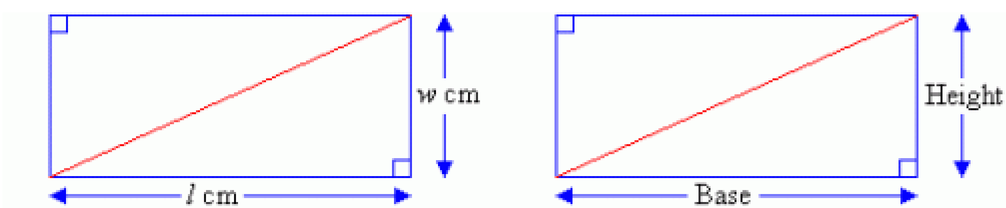
= ½ lw .... (i)

In the diagram, we notice that the length of the rectangle is one side of the triangle. This is said to be the **base** of the triangle. So:

Base of the triangle = Length of the rectangle

The distance from the top of the triangle to the base is called the **height** of the triangle. Therefore:

Height of the triangle = Width of the rectangle



Replacing *l* and *w* with the Base and Height in equation (1), we obtain:

Area of triangles = ½ Base X Height

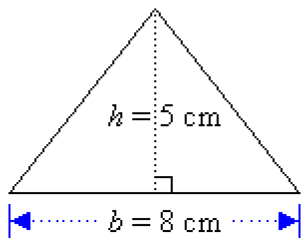
Using the pronumerals A for area, b for base and h for height, we can write the formula for the area of a right-angled triangle as:

A = ½ bh

Example

Find the area of a triangle with base 8 cm and height 5 cm.

Solution



A = ½ bh

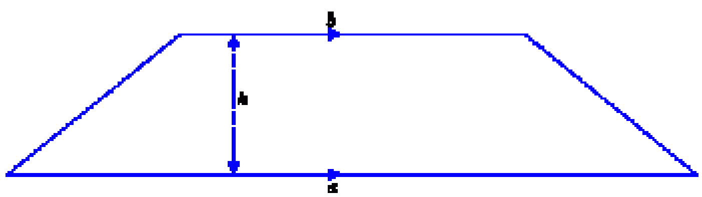
½ X 8 X 5

= 20, so the area is 20 cm^2

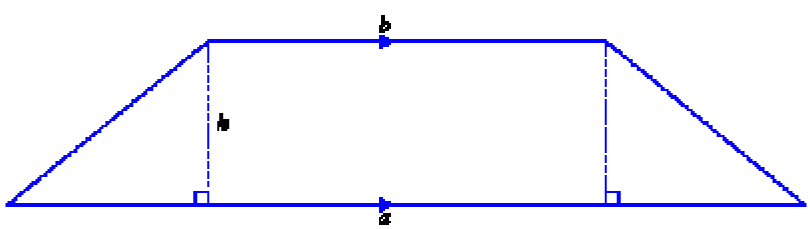
Area of Trapezium

A trapezium is a quadrilateral that has only one pair of parallel sides.

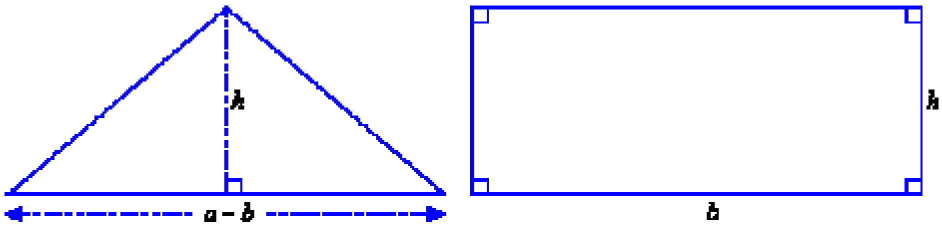
Consider the area of the following trapezium.



To calculate the area of a trapezium, divide it into a rectangle and two triangles as shown below.



Now, piece together the triangular ends so that the trapezium is divided into a triangle and rectangle. The base of the triangle is the difference between the lengths of two parallel sides. That is, *a* – *b*.



∴ Area of a trapezium = area of the rectangle + area of the triangle

= bh + ½ (a – b)h

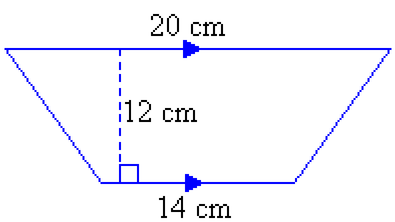
= h

= h[2b + a – b/2]

=h[a + b/2]

= (half the sum of parallel sides) X (perpendicular distance between the parallel sides)

Example



Find the area of the following trapezium

Solution

a = 20 cm, b = 14 cm, h = 12 cm

A = ½(a + b)h

= ½ (20 + 14) X 12

= ½ X 34 X 12

= 204

Area of Circles

There is only one formula for the area of a circle:

$A = \pi r^2$

We must therefore remember to use the **radius** each time.

ASSESSMENT

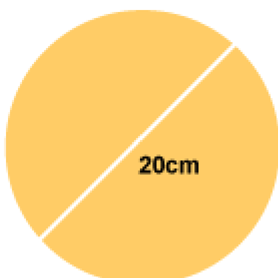
Find the area of a circle with a diameter of 20 cm. Use the π button on your calculator, and give the answer correct to 1 dp.

We have been told that the diameter is 20 cm. Therefore, the radius is 10 cm.

$A = \pi r^2$

$A = \pi \times 10 \times 10 = 314.159...\ cm^2$

$A = 314.2\ cm^2$ (1 dp)

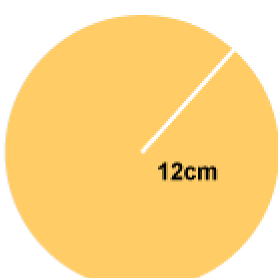


Note: Units of area are always written as squares. For example, cm^2, m^2 etc.

πr^2 means $\pi \times r \times r$ (only the r is squared).

Find the area of a circle with a radius of 12 cm.

Use the π button on your calculator, and give your answer correct to 3 significant figures (s.f.).



The answer is 452 cm^2.

You need to calculate $\pi \times 12 \times 12 = 452.3893...$ and then round your answer to 3 s.f.

<u>Week 5&6</u>

<u>Topic: Three Dimensional Shapes</u>

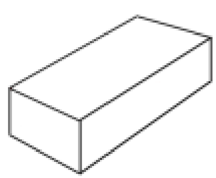
Three-dimensional shapes

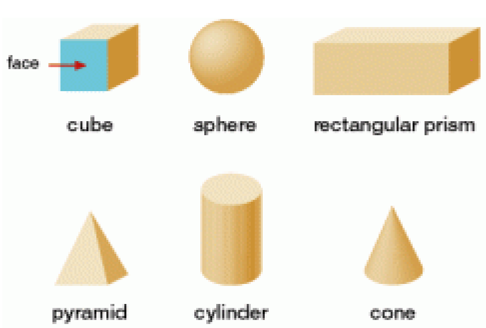
Nearly everything that we can see and touch takes up space. These things are either **gases, liquids or solids**. You will study some of the properties of liquids and gases in science.

Most solids, or **three-dimensional shapes,** such as stones and trees, have rough and **irregular** shape. This usually occurs in nature. However, some three-dimensional shapes, such as tin, can and houses, have **regular** shapes. These are usually made by people. We often call then **geometrical solids**.

Look at the photograph below, it show groundnut pyramids

The diagram below show that each pyramid is made up from a square-based pyramid and a cuboid.

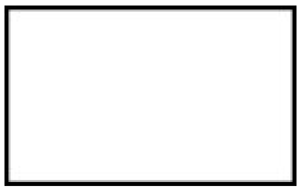




Cuboids and cubes

Look at a matchbox. The name of its shape is a **cuboid**

A face may be flat (**plane**) or curved. A cuboid has 6 plane faces. Each face is in the shape of a rectangle.



A rectangle

An edge is a line where two faces meet. It may be straight or curved. A cuboid has 12 straight edges.

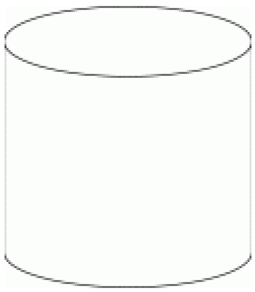
A **vertex** is a point or corner where three or more edges meet The plural of vertex is **vertices.** A cuboid has 8 vertices.

Drawing cuboids

There are many ways of drawing cuboids. In each case it is impossible to show the solid as it really is. A **skeleton view** is very useful since it shows all the edges. The drawings below show how to draw a skeleton view very quickly. Notice that some edges are hidden from view. We usually show these as broken lines.

Cubes

A cube is a cuboid in which all six faces are squares

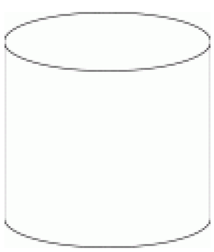


Exercise

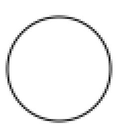
Write down five everyday objects which are cuboids

Cylinders and prisms

Cylinder



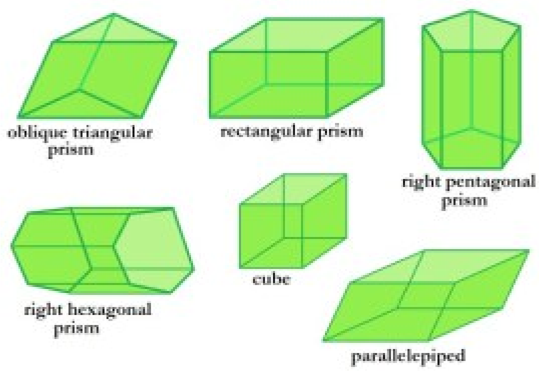
The cylinder has two plane faces and one curved face. It has no vertices and two curved edges> the two plane faces are both circles.



A skeleton view of a cylinder is drawn in much the same way as that of a cuboid

Prism

The base and top faces of a prism are always the same shape. The names of prisms come from the shape of their base and top faces. The side faces of right prisms are always rectangular.



The cuboid is a rectangular prism and the cylinder can be thought of as a special prism.

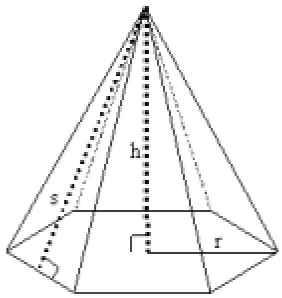
Cones and Pyramids

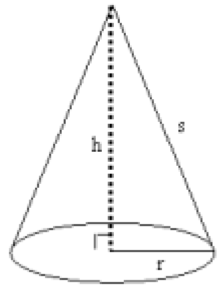
Pyramid: A 3-dimensional solid in which the base is a polygon and the sides are triangles which meet in one point called the vertex. We shall examine regular pyramids in which the base is a regular polygon and the sides are congruent triangles.

A Right Circular Cone**:** A 3-dimensional solid in which the base is a circle. The side of a cone is formed by straight lines which connect the circular base to a vertex. The height is the perpendicular distance from the vertex to the base and meets the base in the center of the circle.

The Lesson:

The diagrams below show a pyramid and a cone. Both have a height of **h** and radius of **r**. In the pyramid at left, **r** is the radius of the regular hexagon that is the base of the pyramid. In the cone at right, **r** is the radius of the circular base. The slant height is **s** in both diagrams.



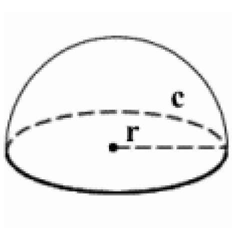


Sphere

Nearly every ball is sphere shaped

A tennis ball   Outline of a ball; sphere

Half of a sphere is called a hemisphere



Exercise

Write down five everyday objects each of which is either sphere-shaped or contains part of a sphere in its shape.

Which of the following solids will roll smoothly on a plane surface?

a. cube b. cone c. sphere d. cylinder e. cuboid

SOLIDS 2: VOLUME

The volume of a solid is a measure of the space it takes up. The cube is used as the shape for the basic unit of volume. A cube of edge 1 metre has a volume of **1 cube**

metre or **1 m^3.** A cube of edge 1 centimetre has a volume of **1 cubic centimeter** or **1 cm^3.**

It is different to measure volume directly. One way is to build a copy of the solid using basic units. For example, to measure the volume of the 6 cm by 3 cm by 4 cm cuboid in the figure below, a copy can be built from 1 cm^3 cubes.

Units of volume

The **cubic metre, m^3,** is the basic unit of volume.

1 m = 100 cm

1 m^3 = (100 X 100 X 100) cm^3 = 1 000 000 cm^3

Similarly

1 cm = 10 mm

1 cm^3 = (10 X 10 x 10) mm^3 = 1 000 mm^3

When calculating problems about volume, make sure that all dimension are in the same units.

Volume of Cuboids and Cubes

Cuboid

Notice that

a. the 6 cm by 3 cm by 4 cm cuboid in the above figure has a volume of 72 cm^3

b. 6 X 3 X 4 – 72.

We can the volume of any cuboid by finding the product of its length, breadth and height:

volume of cuboid = length X breadth X height

= area of base X height

= area of end face X length

= area of side face X breadth

Cube

Similarly, for a cube of side s,

Volume of cube = (side) X (side) X (side)

= s X s X s

= s^3

Example 1

Calculate the volume of a rectangular box which measures 30 cm X 15 cm X 10 cm.

Volume of box = (30 X 15 X 10)cm^3

= 4 500 cm^3

Example 2

A rectangular room 4 m long by 3 m wide contains 30 m^3 of air.

Calculate the height of the room.

Volume of room = 30 m^3

area of floor (base) = 4 m X 3 m = 12 m^2

height of room = 30/12 m = 2½ m

Capacity of containers

The **capacity of a container** is the measure of the space inside it. The basic unit of capacity is the **litre.** 1 litre of water will just fill a 10 cm by 10 cm by 10 cm cubic container.

Therefore in practice,

1 liter = (10 X 10 X 10) cm^3 = 1 000 cm^3

The table below shows the relation between units of capacity and units of volume.

	capacity	Volume

Kilolitre	1 kl = 1 000l	= 1 000 000 cm^3 = 1 m^3
Litre	1 l	= 1 000 cm^3
Millimeter	1 ml = 0.001 l	= 1 cm^3

Example

How many litres of water dos a 5 m X 4 m X 3 m tank hold?

volume of tank = (5 X 4 X 3) m^3 = 60 m^3

but, 1 m^3 = 1 000 litres

capacity of tank = 60 X 1 000 litres

= 60 000 litres

ASSESSMENT

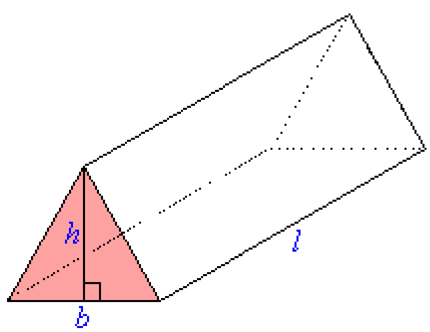
1. A rectangular tin measure 10 cm by 10 cm by 20 cm. What is its capacity in litres?

2. Calculate the capacity in litres of a tin 20 cm by 2 cm by 10 cm.

Volume of right-angled triangular prism

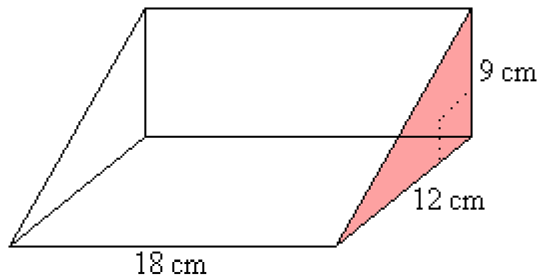
A triangular prism whose length is *l* units, and whose triangular cross-section has base *b* units and height *h* units, has a volume of *V* cubic units given by

v = Al = ½ bhl



3.

Find the volume of the triangular prism shown in the diagram.



V = Al

= ½ bhl

= ½ X 12 X 9 X 18

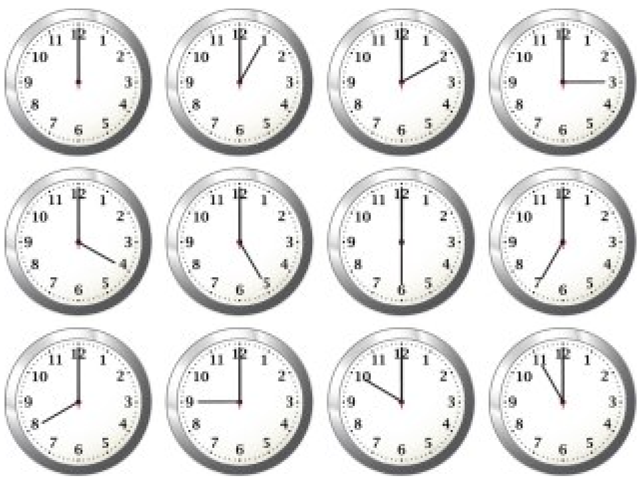
= 972

Week 7

Topic: Identification and Properties of Angles

Angles as rotation

We use the word **angle** for amount of turn. For example the figures below show the hands of a clock move between 12 'clock to 12 ' clock.



Both hands turn. In one hour the amount that each hand turns is different

The minute hand makes one complete **turn**, or one **revolution**

The angle turned = 1 revolution.

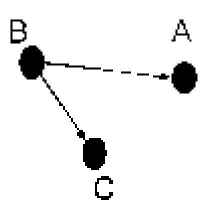
As we can measure length, we can measure angle. To avoid fractions, one revolution is divided into 360 equal parts. Each part is called a **degree**. We use the symbol ^{o} for degree.

1 revolution = 360 degrees or 360^{o}

1^{o} = 1/360 revolution

Angles between lines

The figure below shows two lines AB and CB. The lines meet at the point B.



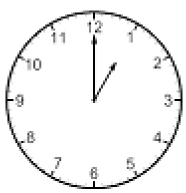
The angle between the two lines is the amount that one line must turn so that it points the same way as the other line.

Example

What is the angle between the hour hand and the minute hand of a clock at 1 o'clock?

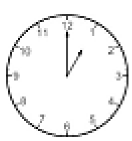
The angle between the hands is the amount that one hand must turn to reach the position of the other. That is 1/12 of a revolution.

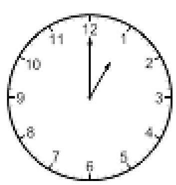
angle between hands = 1/12 of a revolution



= 1/12 of 360^{o} = 30^{o}

Notice, in the above example, that the size of the angle does not depend on the size of the hands. At 1 o'clock the angle between the hour hand and the minute hand is 30^{o} whether on a watch or on a clock.





Watch Clock

Again notice from the figures above that 330^{o} would also be a correct answer to the previous example above. The minute hand would turn through 330^{o} if it went the other way round.

$330^{o} + 30^{o} = 360^{o}$

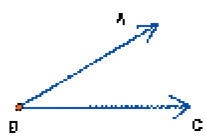
Copy and complete the table below

revs	**degrees**
1	360
2	
3	1080

10
1/12 180
1/3
90
1/10
1/8
1½

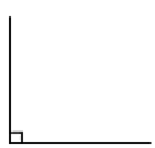
Naming angles

When lines BA and CB meet at the point B in the figure below, we say that angle ABC or angle CBA is the angle between them. Notice that B, the middle letter, is the vertex of the angle. The lines AB and OB are the arms of the angle.



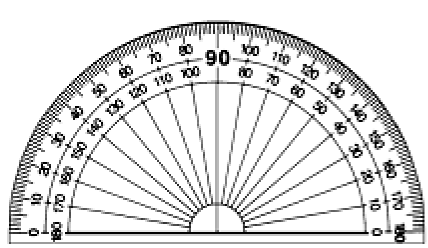
If BA is fixed and CB can turn, we get angles of different sizes. Some of these have special names.

The right angle is especially important. There are many examples where lines meet at right angles. A right angle is often shown on a diagram by drawing a small square at the vertex of the angle.

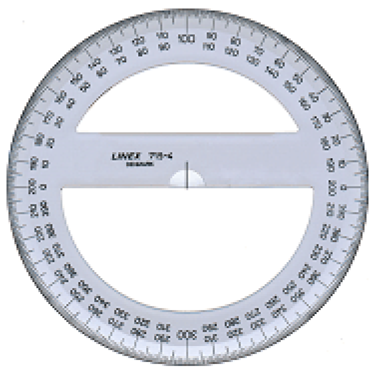


Measuring angles

We use a **protractor** to measure the number of degrees in angle. There are many kinds of protractor; two are shown in figures below;



Anticlockwise



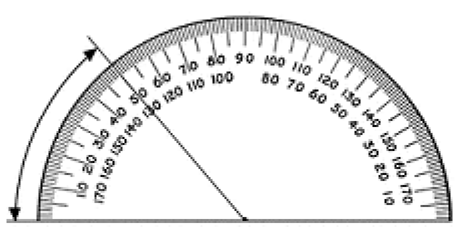
Clockwise

To measure an angel:

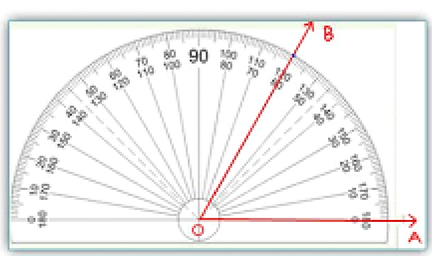
a. Place the protractor over the angle so that its centre, O, is exactly over the vertex of the of the angle and the base line is exactly along one arm of the angle.

b. Count the *degrees* from the base line to the other side of the angle. Most protractors have two rows of numbers. This is because you can measure the angle either **anticlockwise** or **clockwise** as in the two immediate figures above respectively.

This can be confusing because the reading seems to be either 50^{0} or 130^{0}. However, since the angle is acute, it must be 50^{0}.



Read the size of the angle between A and B in the figure below



Constructing angles

To draw an angle less than 180° with a protractor, proceed as follows:

Draw a straight line (i.e. an arm of the angle).

Place a dot at one end of the arm. This dot represents the vertex of the angle.

Place the centre of the protractor at the vertex dot and the baseline of the protractor along the arm of the angle.

Find the required angle on the scale and then mark a small dot at the edge of the protractor.

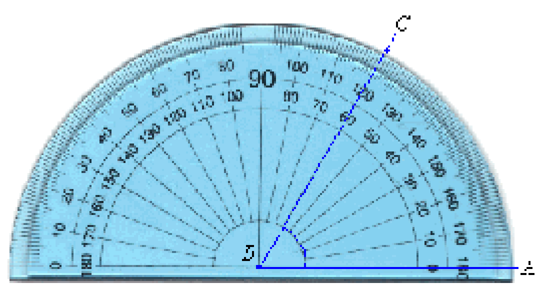
Join the small dot to the vertex with a ruler to form the second arm of the angle.

Label the angle with capital letters.

ASSESSMENT

Draw $\angle ABC = 60^{\circ}$ with a ruler and a protractor

Solution:



Draw a straight line *AB*.

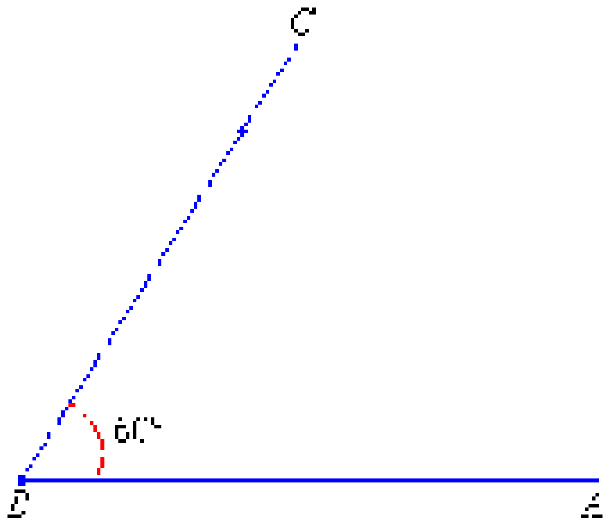
Place a dot at *B*. This dot represents the vertex of the angle.

Place the centre of the protractor at *B* and the baseline of the protractor along the arm *BA*.

Find 60° on the scale and mark a small dot at the edge of the protractor.

Join the vertex *B* to the small dot with a ruler to form the second arm, *BC*, of the angle.

Mark the angle with a small arc as shown below.

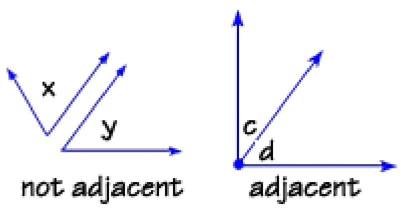

C
60°
D
A

Week 8

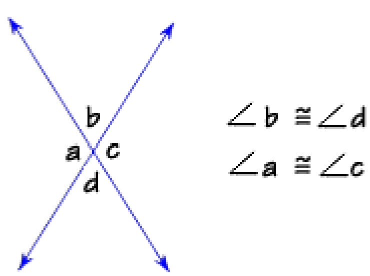
Topic: Theorems

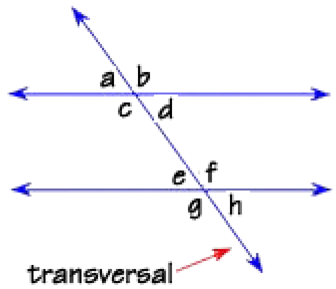
Angles between two lines

Adjacent means “next to.” But we use this word in a very specific way when we refer to adjacent angles. Study these two figures. Only the pair on the right is considered to be adjacent, angles *c* and *d*. Adjacent angles must share a common side and a common vertex, and they must not overlap each other.



Vertical angles are pairs of angles formed by two intersecting lines. Vertical angles are *not* adjacent angles — they are opposite each other. In this diagram, angles *a* and *c* are vertical angles, and angles *b* and *d* are vertical angles. Vertical angles are congruent.





These two lines are parallel, and are cut by a transversal, which is just a name given to a line that intersects two or more lines at different points. Eight angles appear, in four corresponding pairs that have the same measure, so therefore are congruent.

These four corresponding pairs are:

angles *a* and *e*
angles *c* and *g*
angles *b* and *f*
angles *d* and *h*

The angles that lie in the interior area, or the area between the two lines that are cut by the transversal, are called interior angles. Angles *c, d, e* and *f* are interior angles. Angles *a, b, g,* and *h* lie in the exterior area, and they are called “exterior angles.”

$\angle a \cong \angle h$

$\angle b \cong \angle g$

$\angle c \cong \angle f$

$\angle d \cong \angle e$

We call angles on opposite sides of the transversal alternate angles. Angles *c* and *f*, and *d* and *e*, are alternate interior angles. Angles *a* and *h*, and *b* and *g*, are alternate exterior angles. Note that these alternate pairs are also congruent.

When a transversal cuts two lines that are not parallel, as shown here, it still forms eight angles—four corresponding pairs. However, the corresponding pairs are not congruent as occurs with parallel lines.

Vertically opposite angles

What is vertically opposite angles?

When two straight lines intersect each other four angles are formed.

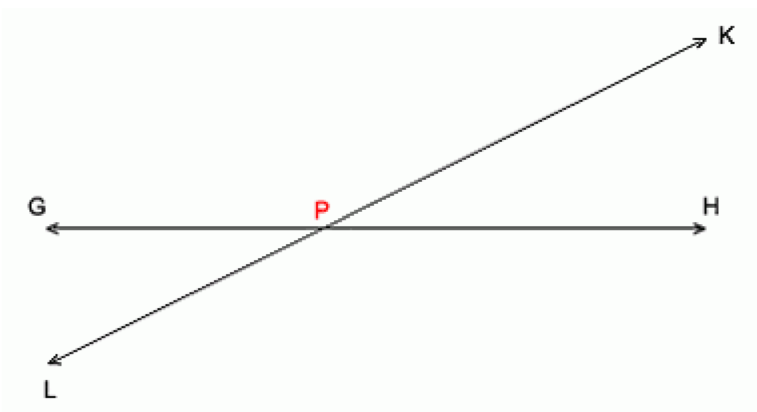
The pair of angles which lie on the opposite sides of the point of intersection are called vertically opposite angles.

In the given figure, two straight lines AB and CD intersect each other at point O. Angles AOD and BOC form one pair of vertically opposite angles; whereas angles AOC and BOD form another pair of vertically opposite angles.

Vertically opposite angles are always equal.
i.e., ∠AOD = ∠BOC
and ∠AOC = ∠BOD

Note:
In the given figure; rays OM and ON meet at O to form ∠MON (i.e. ∠a) and reflex ∠MON (i.e. ∠b). It must be noted that ∠MON means the smaller angle only unless it is mentioned to take otherwise.

For example,in the given figure, two lines GH and KL are intersecting at a point P.



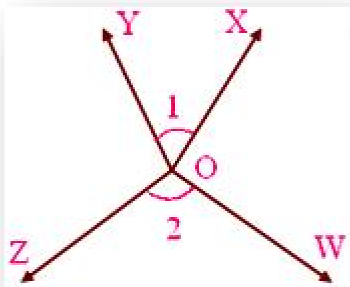
We observe that with the intersection of these lines, four angles have been formed. Angles ∠1 and ∠3 form a pair of vertically opposite angles; while angles ∠2 and ∠4 form another pair of vertically opposite angles.
Clearly, angles ∠1 and ∠2 form a linear pair.
Therefore, ∠1 + ∠2 = 180°
or, ∠1 = 180° − ∠2 ....(i)

again similarly, ∠2 and ∠3 form a linear pair.
Therefore, ∠2 + ∠3 = 180°
or, ∠3 = 180° − ∠2 ....(ii)

From (i) and (ii) we get;
∠1 = ∠3

Similarly, we can prove that ∠2 = ∠4
Thus, if two lines intersect then vertically opposite angles are always equal.

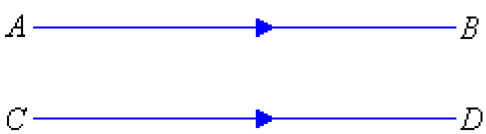
In the below figure, ∠1 and ∠2 are not vertically opposite angles, because their arms do not form two pairs of opposite rays.



Now, we will solve various examples on vertically opposite angles.

Parallel Lines

If two lines are in the same plane and do not intersect, then the lines are said to be **parallel**.

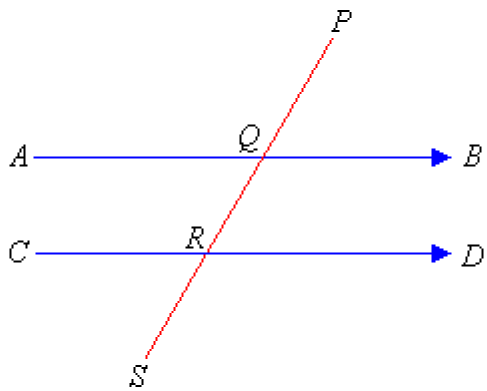


For example, AB is parallel to CD and we write it as AB || CD.

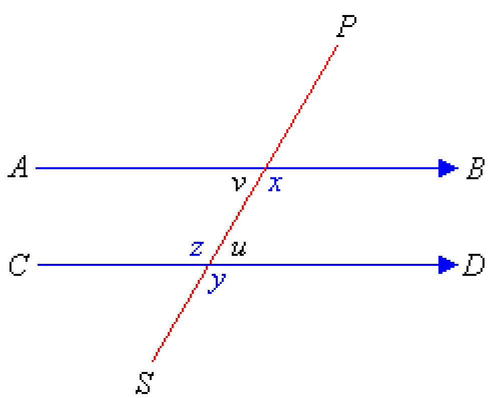
Note:

Arrows are placed on the lines *AB* and *CD* to indicate that they are parallel.

A line that meets two or more parallel lines is called a **transversal**. Line *PQRS* in the following diagram is a transversal.



If two parallel lines are cut by a transversal as shown in the next diagram, we refer as follows to the angles formed:



z and *x* (or *u* and *v*) are **alternate angles**

x and *y* are **corresponding angles**

u and *x* (or *z* and *v*) are **allied** (or **co-interior**) **angles**

y and *z* are **vertically opposite angles**

Remember that:

Alternate angles are always equal.

Corresponding angles are always equal.

Allied (or co-interior) angles are supplementary.

Vertically opposite angles are always equal.

Angles in a triangle

Introduction

There are many different shapes. There might seem to be hundreds of different shapes and you could try to learn them all but:

1. This is confusing
2. This is a waste of time
3. This is pointless. We need to see what they have in common and what is different about them.

It is much better to concentrate on the properties of shapes. This way you can remember a few facts and them use them to find out more about different shapes.

Properties of shapes

The 3 properties of shapes that we are going to look at are:

1. **The number of sides**
2. **The interior angles (the angles inside).**
3. **The length of the sides.**

These properties help use to remember which shapes are which and why they are so called (in some cases).A shape that has 3 sides: **TRIANGLES (tri- means 3).**

Triangles **ALWAYS** have 3 sides.
The interior angles of a triangle add up to 180 degrees.

ASSESSMENT

The 3 properties of shapes are?

Week 9

Topic: CONSTRUCTION: PARALLEL AND PERPENDICULAR LINES

In geometry, to **construct** a figure means to draw it accurately. Accurate construction depends on using measuring and drawing instruments properly.

There is a useful property to show that 2 given lines are parallel. This property states that if 2 given lines are both perpendicular to a third line, then the 2 lines are parallel. The figure below illustrates this property.

Example:

Construct a line parallel to *AB* and 2 cm above it.

Solution:

Step 1: Mark a point *C* anywhere on the line *AB.* Construct a line perpendicular to *AB* and passing through *C.*

Step 2 : Construct a line segment on the perpendicular line 2 cm above *C*. Label the point as *D.* Then, construct a line perpendicular to *CD* and passing through *D*. This line is parallel to *AB*.

Constructing Perpendiculars

Construct a perpendicular from a point on a line

Given: Point P is on a given line
Task: Construct a line through P perpendicular to the given line.

Directions

1. Place your compass point on P and sweep an arc of any size that crosses the line twice (below the line). You will be creating (at least) a semicircle. (Actually, you may draw this arc above OR below the line.)

2. STRETCH THE COMPASS LARGER!!
3. Place the compass point where the arc crossed the line on one side and make a small arc below the line. (The small arc could be above the line if you prefer.)

4. Without changing the span on the compass, place the compass point where the arc crossed the line on the OTHER side and make another arc. Your two small arcs should be crossing.
5. With your straightedge, connect the intersection of the two small arcs to point *P*.

This new line is perpendicular to the given line.

Explanation of construction: Remember the construction for bisect an angle? In this construction, you have bisected the straight angle P. Since a straight angle contains 180 degrees, you have just created two angles of 90 degrees each. Since two right angles have been formed, a perpendicular exists.

Construct a perpendicular to a line from a point on a line

Given: Point ***P*** is off a given line
Task: Construct a line through ***P*** perpendicular to the given line.

Directions

1. Place your compass point on P and sweep an arc of any size that crosses the line twice.
2. Place the compass point where the arc crossed the line on one side and make an arc ON THE OPPOSITE SIDE OF THE LINE.
3. Without changing the span on the compass, place the compass point where the arc crossed the line on the OTHER side and make another arc. Your two new arcs should be crossing on the opposite side of the line.

4. With your straightedge, connect the intersection of the two new arcs to point *P*.

This new line is perpendicular to the given line.

Explanation of construction: To understand the explanation, some additional labeling will be needed. Label the point where the arc crosses the line as points C and D. Label the intersection of the new arcs on the opposite side as

point E. Draw segments , , , and . By the construction, PC = PD and EC = ED. Now, remember a locus theorem: The locus ofpoints equidistant from two points (C and D), is the perpendicular bisector of the line segment

determined by the two points. Hence, is the perpendicular bisector of .
The fact that we created a bisector, as well as a perpendicular, is actually MORE than we needed – we only needed to create a perpendicular.

ASSESSMENT

Construct a line parallel to AB and 4 cm above it.

Week 10

Topic: STATISTICS 1 – DEFINITION

PURPOSE AND DATA COLLECTION

The need for statistics – statistical data

Suppose a stranger asks you for information about yourself. You could say lot of things. For example: your name, the town you live in; the school you go to; what you ate last night; the things you like; the things you don't like; etc.

You might also use numbers, for example: I am 12 years old; I have 4 brothers and 2 sisters; I am 171 cm tall and my mass is 48kg; I wear size 6 shoes; my village is 15km from the school; etc.

We use the word data for basic information like this. When we use numbers, the information is called **statistical data,** or just **statistics**. The table is showing statistical data about two teams.

	games played	**won**	**lost**
Eagles	18	10	5
Falcons	15	2	8

drawn goals for goals against Eagles 36021 **Falcons** 51937

The statistics in the table give a lot of information about the two teams. Eagles seem to be more successful than Falcons. A good player, looking at the statistics, might prefer to play for Falcons. Thus, statistics can help when taking decisions.

Purposes of statistics

There are many more serious reasons for gathering statistics than selecting which team to play for. For instance, statistics about population trend can inform the government whether they need to encourage people o have smaller families; statistics about the available of potable water (drinkable water) can inform state and district planners whether or not to budget for pumps and pipelines. The table below shows the leading causes of death by age group in 2002 in a country in East Africa.

	age group (years)			
Causes of death	**0-4**	**5-14**	**15-59**	**60+**
Malaria/Fever	40%	61%	16%	26%
HIV/AIDS/TB	4%	61%	56%	17%
Heart Disease	n/s	n/s	6%	23%
Injury/Accident	32%	17%	5%	n/s

n/s means not significant

Data like this in the table above might tell a Health Minister that more needs to be done about malaria for young people aged 0 to 14 and that HI/AIDS and TB need to be reduced in the 15 – 59 range.

ASSESSMENT

It would be impossible to give statistics unless data were collected beforehand. To be able to collect data, you need to be able to count. You also need to be able to write down, or **record**, the data clearly. The table below shows same data collected in two different ways by two students.

(a) bus, car, car, car, lorry, bicycle, bicycle, car, car, lorry, bicycle, car.

(b)

Vehicles	Tally	Total
Car	IIIIII	6
Bus	I	1
Lorry	II	2
Taxi		
Bicycle	III	3

Motorbike

The first student (a) tried to write down every vehicle as it came by. When two bicycles came by she did not have time to write down properly. It is easy to make mistakes when counting this student's totals.

The second student (b) spent some time before beginning to record. When a vehicle came by he made a tally. iT is easy to count his total. He wrote down all the vehicles he could think of in a column. When a vehicle came by he made a tally. It is easy to count his totals.

Week 11

Topic: STATISTICS (Continued) – GRAPHICAL PRESENTATION OF DATA

Types of presentation

Good **presentation** can make statistical data easy to read, understand and interpret. Therefore it is important to present data clearly.

i. There are two main ways of presenting data: presentation of numbers or values in **lists** and **tables;**

ii. Presentation using **graphs,** i.e. picture. We use the following examples to show the various kinds of presentation.

An English teacher gave an
essay to 15 students.

She graded the essays from A (very good), through B, C.D, E to f (very poor). The grades of the students were:

B, C, A, B, A, D, F, E, C, C, A, B, B, E, B

Lists and tables

Rank and order list

Rank order means in order from highest to lowest. The 15 grades are given in rank order below:

A, A, A, B, B, B, B, C, C, C, E, E, F

Notice that all the grades are put in the list even though most of them appear more than once. The ordered list makes it easier to find the following: the highest and lowest grades; the number of students who got each grades; the most common grade; the number of students above and below each grade; and so on.

Frequency table

Frequency means the number of times something happens. For example, three students got grade A.

The frequency of grade A is three. A **frequency table,** gives the frequency of each grade.

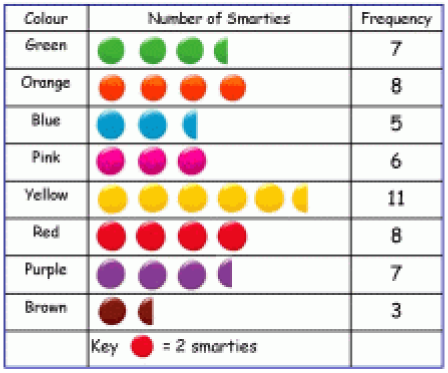
Grade	**A**	**B**	**C**	**D**	**E**	**F**
frequency	3	5	3	**1**	2	1

Graphical presentation

In most cases, a picture will show the meaning of statistical data more clearly than a list of or table or numbers. The following methods of presentation give the data of the example in picture, or **graph,** form.

Pictogram

A **pictogram** uses pictures or drawings to give a quick and easy meaning to statistical data.



Colour	Number of Smarties	Frequency
Green		7
Orange		8
Blue		5
Pink		6
Yellow		11
Red		8
Purple		7
Brown		3
	Key = 2 smarties	

Bar chart

A bar chart represents the data as horizontal or vertical bars. The length of each bar is proportional tothe amount that it represents.

There are 3 main types of bar charts.

Horizontal bar charts, vertical bar chart and double bar charts.

When constructing a bar chart it is important to choose a suitable scale to represent the frequency.

The following table shows the number of visitors to a park for the months January to March.

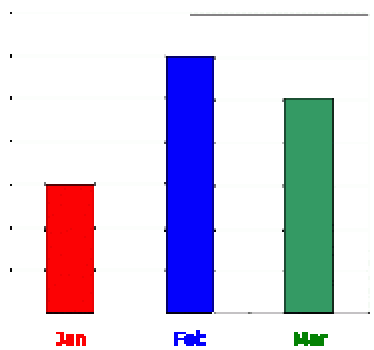
Month	January	February	March
Number of visitors	150	300	250

a) Construct a vertical and a horizontal bar chart for the table.

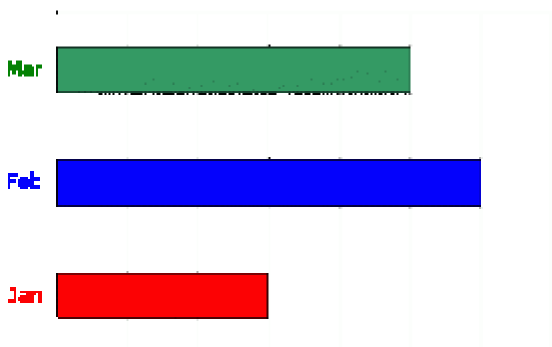
Solution

a) If we choose a scale of 1:50 for the frequency then the vertical bar chart and horizontal bar chart will be as shown.

Vertical bar chart



Horizontal bar chart



Pie Chart

Pie charts are useful to compare different parts of a whole amount. They are often used to present financial information. E.g. A company's expenditure can be shown to be the sum of its parts including different expense categories such as salaries, borrowing interest, taxation and general running costs (i.e. rent, electricity, heating etc).

A pie chart is a circular chart in which the circle is divided into sectors. Each sector visually represents an item in a data set to match the amount of the item as a percentage or fraction of the total data set.

Example

A family's weekly expenditure on its house mortgage, food and fuel is as follows:

Expenses N

Mortgage	300
Food	225
Fuel	75

Draw a pie chart to display the information.

Solution

The total weekly expenditure = N300 + N225 + N75 = N600

We can find what percentage of the total expenditure each item equals.

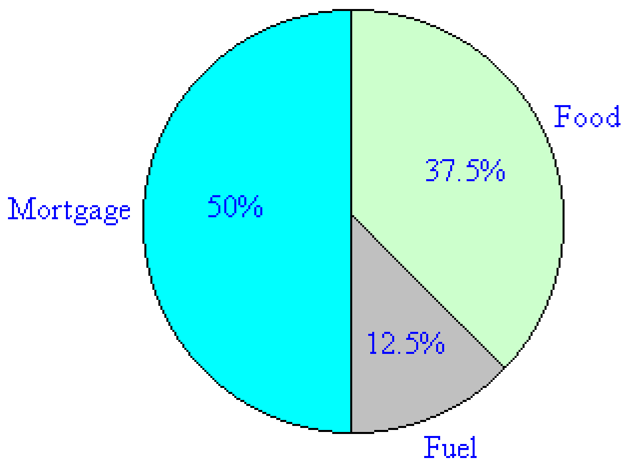
Percentage of weekly expenditure on:

Mortgage = 300/600 X 100% = 50%

Food = 225/600 X 100% = 37.5%

Fuel = 75/600 X 100% = 12.5%

To draw a pie chart, divide the circle into 100 percentage parts. Then allocate the number of percentage parts required for each item.



Note

It is simple to read a pie chart. Just look at the required sector representing an item (or category) and read off the value. For example, the weekly expenditure of the family on food is 37.5% of the total expenditure measured.

A pie chart is used to compare the different parts that make up a whole amount.

ASSESSMENT

The following is a rank order list of an exam result: 87, 82, 78, 76, 75, 70, 66, 64, 59, 59, 59, 51, 49, 48, 41.

a. How many students took the exam?

b. What was the highest rank?

c. What was the lowest rank?

d. What is the mark of the student who came 6th?

e. What is the position of the student who got 76 marks?

f. Three students got 59 marks. What is their position?

g. How many students got less than 75 marks?

Week 12

Topic: STATISTICS – AVERAGES

Averages

The **average** of a set of numbers is a very important statistics. The average is typical of the set of numbers and gives information them. For example:

a. If a football team's **average score** is 5.2 goals, we know that the team is good at scoring goals.

b. If two classes have **average ages** of 8.7 years and 16.9 years, we expect that the first is a Primary School class and the second is a Secondary School class.

c. If the **average life** of a battery is 20 hours, we expect a new battery to last about 20 hours, may be a little more or a little less.

There are many kinds of averages, here we will find out about three:
the **mean,** the **median and the mode.**

The Mean

Example

Four tests results: 15, 18, 22, 20
The sum is: 75
Divide 75 by 4: 18.75

The 'Mean' (Average) is 18.75

(Often rounded to 19)

The Median

The Median is the 'middle value' in your list. When the totals of the list are odd, the median is the middle entry in the list after sorting the list into increasing order. When the totals of the list are even, the median is equal to the sum of the two middle (after sorting the list into increasing order) numbers divided by two. Thus, remember to line up your values, the middle number is the median! Be sure to remember the odd and even rule.

Examples

Find the Median of: 9, 3, 44, 17, 15 (Odd amount of numbers)
Line up your numbers: 3, 9, 15, 17, 44 (smallest to largest)
The Median is: 15 (The number in the middle)

Find the Median of: 8, 3, 44, 17, 12, 6 (Even amount of numbers)
Line up your numbers: 3, 6, 8, 12, 17, 44
Add the 2 middles numbers and divide by 2: 8 12 = 20 ÷ 2 = 10
The Median is 10.

The Mode

The mode in a list of numbers refers to the list of numbers that occur most frequently. A trick to remember this one is to remember that mode starts with the same first two letters that most does. Most frequently – Mode. You'll never forget that one!

Examples

Find the mode of:
9, 3, 3, 44, 17 , 17, 44, 15, 15, 15, 27, 40, 8,
Put the numbers is order for ease:
3, 3, 8, 9, 15, 15, 15, 17, 17, 27, 40, 44, 44,
The Mode is 15 (15 occurs the most at 3 times)

It is important to note that there can be more than one mode and if no number occurs more than once in the set, then there is no mode for that set of numbers.

Occasionally in Statistics you'll be asked for the 'range' in a set of numbers. The range is simply the smallest number subtracted from the largest number in your set. Thus, if your set is 9, 3, 44, 15, 6 – The range would be 44-3=41. Your range is 41.

A natural progression once the 3 terms in statistics are understood is the concept of probability. Probability is the chance of an event happening and is usually expressed as a fraction. But that's another topic!

ASSESSMENT

1. Calculate the mean of the sets of numbers

a. 9, 11, 13

b. 1, 9, 4, 6

c. 7, 8, 124cm, 7cm, 1cm, 6cm

2. Find the median of

a. 8.3, 11.3, 9.4, 13.8, 12.9, 10.5

b. 1, 2, 4, 5, 7

c. 1, 3, 5, 6, 6, 7, 8

c. 12, 14, 15, 16, 17, 18, 19, 19, 20