MATHEMATICS

For

Senior Secondary School

3

EDUBASE

SS 3
FIRST TERM NOTES ON
MATHEMATICES

TABLE OF CONTENT

FIRST TERM

<u>WEEK 1</u>

SSS 3 FIRST TERM MATHEMATICS

Topic: THEORY OF LOGARITHM

Laws of Logarithms

1. **Product law**

$Log_a(MN) = log_a M + log_a N$

e.g. $log_2(2 x 4) = log_2 + log_2 4$

2. **Quotient law**

$Log_a (M/N) = log_a M – log_a N$

e.g. $log_{10}(1000/10) = log_{10}1000 – log_{10}10$

3. **Power law**

$Log_a M^p = P log_a M$

e.g. $log_2 2^3 = 3 log_2 2$

4a. $log_a 1 = 0$ where a ^{1} 1

4b. $log_a a = 1$ e.g. $log_{10}10 = 1$, $log_2 2 = 1$

5. **Fractional Power**

$Log_a Mx/y = x log_a M/y$ e.g. $log_{10}100^{3/2}$

6. **Root Law**

In a special case of 5 when x = 1

$Log_a M1/x = log_a {}^yÖM = log_a M/y$ e.g. $log_2 16^{1/4} = log_2 {}^4Ö16 = log_2 16/4$

7. **Change of base**

$log_a M = 1/log_m{}^a$ e.g. $log_2 8 = log_8 2$

Calculations based on the application of the basic rule

Simplify without using table

1. $Log_2 {}^3Ö8/log_2 4$

$Log_2 8^{1/3}/log_2 2^2$

$1/3 \log_2 2^3/2\log_2 2$

$Log_2 2/2\log_2 2$ = ½

2. $3 \log_{10} 10^2 + \log_{10} 10^{-3}$

$(2)3 \log_{10} 10 + -3 \log_{10} 10$

$6 \log_{10} 10 + (-3) \log_{10} 10$

6 + (-3) = 3

3. ½ $\log_5 25 - \log_5 0.2$

$= \log_5 25^{1/2} - \log_5 1/5$

= $\log_5$Ö25 – $\log_5 1/5$

$= \log_5 5 - \log_5 1/5$

$= \log_5 (5/1/5)$

$= \log_5 25 = \log_5 5^2 = 2\log_5 5 = 2$

Application of Logarithm in Solving Problem Involving Calculations

APPLICATIONS OF EXPONENTIAL AN LOGARITHMIC FUNCTIONS

To solve an exponential or logarithmic word problem, convert the narrative to an equation and solve the equation.

In this section, we will review population problems. We will also discuss why the base of e is used so often with population problems.

Example 1: Suppose that you are observing the behavior of cell duplication in a lab. In one experiment, you started with one cell and the cells doubled every minute. Write an equation with base 2 to determine the number (population) of cells after one hour.

Solution and Explanations:

First record your observations by making a table with two columns: one column for the time and one column for the number of cells. The number of cells (size of population) depends on the time. If you were to graph your findings, the points would be formed by (specific time, number of cells at the specific time). For example, at t = 0, there is 1 cell, and the corresponding point is (0, 1). At t = 1, there are 2 cells, and the corresponding point is (1, 2). At t = 2, there are 4 cells, and the

corresponding point is (2, 4). At t = 3, there are 8 cells, and the corresponding point is (3, 8).
It appears that the relationship between the two parts of the point is exponential. At time 0, the number of cells is 1 or $2^0 = 1$. After 1 minute, when t = 1, there are two cells or $2^1 = 2$. After 2 minutes, when t = 2, there are 4 cells or $2^2 = 4$. Therefore, one formula to estimate the number of cells (size of population) after t minutes is the equation (model)
$f(t) = 2^t$.

Determine the number of cells after one hour:
Convert one hour to minutes. · = 60 min
Substitute 60 for t in the equation. $f(t) = 2^t$:
$f(60) = 2^{60} = 1.15\times10^{18}$

Example 2: Determine how long it would take the population (number of cells) to reach 100,000 cells.

Solution and explanation:
In this example, you know the number of cells at the beginning of the experiment (1) and at the end of the experiment (100,000), but you do not know the time.
Substitute 100,000 for f(t) in the equation $f(t) = 2^t$:
$100,000 = 2^t$
Take the natural logarithm of both sides:
$\ln(100,000) = \ln(2^t)$
Simplify the right side of the equation using the third rule of logarithms:
$\ln(100,000) = t\ln(2)$
Divide both sides by ln(2):
$t = = 16.60964$ min
It would take 16.6 minutes, rounded, for the population (number of cells) to reach 100,000.

Example 3: Write an equation with base 5 that is equivalent to the equation $f(t) = 2^t$.

Solution and Explanation:

Let's start with a generic exponential equation with base 5:

$f(t) = a \cdot 5^{bt}$.

The f(t) represents the size of the population at time t, the t represents the time, and the a and b represent adjusters when we change the base. The value of a is the number of cells (size of population) at the beginning of the study, and the value of b is the relative growth rate based on a base of 5. We need to find the values of a and b.

We know that the population is 1 at time 0, so insert these numbers in the equation

$f(t) = a \cdot 5^{bt}$.

We have

$1 = a \cdot 5^{b \cdot 0} = a \cdot 5^0 = a \cdot 1 = a$.

We now know that the value of a in the adjusted equation is 1.

Rewrite the equation

$f(t) = a \cdot 5^{bt}$

with $a = 1$.

$f(t) = 1 \cdot 5^{bt}$

which in turn can be rewritten as

$f(t) = 5^{b \cdot t}$

We know that the population after 1 minute is 2 cells, so insert these numbers in the equation

$f(t) = 5^{b \cdot t}$

to obtain

$2 = 5^{b \cdot 1}$.

Solve for b by taking the natural logarithm of both sides of the equation $2 = 5^b$.
$\ln(2) = \ln(5^b)$.

Simplify the right side of the equation using the third rule of logarithms:
$\ln(2) = b \ln(5)$.

Divide both sides of the equation by ln(5) and simplify:
$b = \ln(2)/\ln(5) = 0.43067658075$,
rounded to 0.4307.

Insert this value of b in the equation
$f(t) = 5^{b \cdot t}$,
and the equation is simplified to
$f(t) = 5^{0.4307t}$

We know that the population is 8 after 3 seconds, so use these values to check the validity of the above equation. Substitute 3 for t in the right side of the above equation. If the answer is 8, or close to 8 because we rounded, then the model (equation) is correct. $5^{0.4307(3)} = 8.00906$, rounded to 8.
We know from the original equation that after 4 seconds, the population is 16. Let's do a second check. $5^{0.4307(4)} = 16.02415$, rounded to 16 cells. The check would be closer had we rounded b to more decimals.

ASSESSMENT

- Suppose that you are observing the behavior of cell duplication in a lab. In one experiment, you started with one cell and the cells tripled every minute. Write an equation with base 2 to determine the number (population) of cells after one hour.

WEEK 2

SSS 3 FIRST TERM MATHEMATICS

Introduction to Rational and Irrational Numbers

A rational number is part of a whole expressed as a fraction, decimal or a percentage. In the same vein, a number is also rational if it can written as a fraction where the top number of the fraction and bottom number are both whole numbers.

The term rational is derived from the word 'ratio' because the rational numbers are figures which can be written in the ratio form.

Every whole number, including negative numbers and zero, is a rational number. This is because every whole number 'n' can be written in the form n/1

For example, 3 = 3/1 and therefore 3 is a rational number.

Numbers such as 3/8 and -4/9 are also rational because their numerators and denominators are both whole numbers.

Recurring decimals such as 0.26262626…, all integers and all finite decimals, such as 0.241, are also rational numbers.

Alternatively, an **irrational number** is any number that is not rational. It is a number that cannot be written as a ratio of two integers (or cannot be expressed as a fraction).

For example, the square root of 2 is an irrational number because it cannot be written as a ratio of two integers.

The square root of 2 is not a number of arithmetic: no whole number, fraction, or decimal has a square of 2. Irrational numbers are square roots of non-perfect squares. Only the square roots of square numbers are rational.

Similarly Pi (π) is an irrational number because it cannot be expressed as a fraction of two whole numbers and it has no accurate decimal equivalent.

Pi is an unending, never repeating decimal, or an irrational number. The value of Pi is actually 3.14159265358979323… There is no pattern to the decimals and you cannot write down a simple fraction that equals Pi.

Euler's Number (e) is another famous irrational number. Like Pi, Euler's Number has been calculated to many decimal places without any pattern showing. The value of e is 2.7182818284590452353… and keeps going much like the value of Pi. The golden ratio (whose symbol is the Greek letter "phi") is also an irrational number. It is a special number approximately equal to 1.618 but again its value is never ending: 1.61803398874989484820…

Moving on then, the root of a positive real quantity is called a surd when its value cannot be exactly determined.

For example, each of the quantities $\sqrt{3}$, $\sqrt[3]{7}$, $\sqrt[4]{19}$, (16)^2/5 etc. is a surd.

From the definition it is evident that a surd is an incommensurable quantity, although its value can be determined to any degree of accuracy. It should be noted that quantities $\sqrt{9}$, $\sqrt[3]{64}$, $\sqrt[4]{(256/625)}$ etc. expressed in the form of surds are commensurable quantities and are not surds (since $\sqrt{9} = 3$, $\sqrt[3]{64} = 4$, $\sqrt[4]{(256/625)} = 4/5$ etc.). In fact, any root of an algebraic expression is regarded as a surd.

Thus, each of $\sqrt{m}$, $\sqrt[3]{n}$, x2−−√5x25 etc. may be regarded as a surd when the value of m (or n or x) is not given. Note that $\sqrt{m} = 8$ when $m = 64$; hence, in this case $\sqrt{m}$ does not represent a surd. Thus, $\sqrt{m}$ does not represent surd for all values of m.

Worked Examples

1 – Recognising Surds

A surd is a square root which cannot be reduced to a whole number.

For example,

4–√=24=2

is not a surd, because the answer is a whole number.

Alternatively

5–√5

is a surd because the answer is not a whole number.

You could use a calculator to find that

5–√=2.236067977…5=2.236067977…

but instead of this we often leave our answers in the square root form, as a surd.

2 – Simplifying Surds

During your exam, you will be asked to simplify expressions which include surds.

In order to correctly simplify surds, you must adhere to the following principles:

ab−−√=a−−√∗b√ab=a∗b

a−−√∗a−−√=aa∗a=a

Example

(a) – Simplify

27−−√27

Solution

(a) – The surd √27 can be written as:

27−−√=9–√∗3–√27=9∗3

9–√=39=3

Therefore,

27−−√=33–√27=33

Example

(b) – Simplify

12−−√3–√123

Solution

(b) –

12−−√3–√=12−−√∗3–√=(12∗3)−−−−−−√=36−−√123=12∗3=(12∗3)=36

36−−√=636=6

Therefore,

12−−√3–√=6123=6

Example

(c) – Simplify

45−−√5–√455

Solution

(c) –

45−−√5–√=45/5−−−−√=9–√=3455=45/5=9=3

Therefore,

45−−√5–√=3455=3

3 – Adding and Subtracting Surds

In order to add and subtract surds, the numbers which are being square rooted (or cube rooted) must be the same.

Example

(a) – Simplify

12−−√+27−−√12+27

Solution

(b) – The numbers which are being square rooted must be the same, so it is necessary to find a common multiple of 12 and 27

12−−√=(3∗4)−−−−−√=3–√∗4–√=2∗3–√12=(3∗4)=3∗4=2∗3

Similarly,

27−−√=(9∗3)−−−−−√=9–√∗3–√=3∗3–√27=(9∗3)=9∗3=3∗3

Therefore,

12−−√+27−−√=23–√+33–√=53–√12+27=23+33=53

By making the numbers which are being square rooted the same, you can easily add and subtract surds.

Example

(a) – Simplify

90−−√−45−−√90−45

Solution

(a) –

90−−√=(16∗5)−−−−−−√=16−−√∗5–√=45–√90=(16∗5)=16∗5=45

45−−√=(9∗5)−−−−−√=9–√∗5–√=35–√45=(9∗5)=9∗5=35

Therefore,

90−−√−45−−√=45–√−35–√=5–√90−45=45−35=5

4 – Rationalising Surds

The term 'rationalising an expression' simply means removing any surds from the denominators of fractions. This process of simplifying fractions with surds in the denominator often involves rationalising the expression.

Example

(a) – Simplify

8–√6–√86

Solution

(a) –

8–√∗6–√6–√∗6–√8∗66∗6

(48−−√)6(48)6

16∗3−−−−−√616∗36

43–√6436

23–√3233

Exam Tips

1. Memorise the general principles of surds as mentioned in the guide above
2. Remember that a rational number is part of a whole expressed as a fraction, decimal or a percentage. A number is rational if we can write it as a fraction where the top number of the fraction and bottom number are both whole numbers
3. Remember that an irrational number is any number which is not rational, such as Pi or e
4. Write down every stage of your working out in order to score maximum method marks

Topic Summary

When solving problems related to surds, rational and irrational numbers, it is extremely important that you clearly demonstrate your working out and double check your method. Otherwise, a small mistake could cost you vital marks.

However, with care and attention, you can be experienced at recognising rational and irrational number in order to solve related mathematical problems.

ASSESSMENT

1. What is difference between a rational number and an irrational number?
2. Simplify the expression; 60−−√−45−−√30−25

WEEK 3

SSS 3 FIRST TERM MATHEMATICS

Introduction

Trigonometry involves three ratios – **sine**, **cosine** and **tangent** which are

abbreviated to , and .

The three ratios are calculated by calculating the ratio of two sides of a right-angled triangle.

-
-
-

A useful way to remember these is:

Accurate trigonometric ratios for 0°, 30°, 45°, 60° and 90°

The trigonometric ratios for the angles 30°, 45° and 60° can be calculated using two special triangles.

An equilateral triangle with side lengths of 2 cm can be used to calculate accurate values for the trigonometric ratios of 30° and 60°.

The equilateral triangle can be split into two right-angled triangles.

The length of the third side of the triangle can be calculated using Pythagoras' theorem.

Use the trigonometric ratios to calculate accurate values for the angles 30° and 60°.
A square with side lengths of 1 cm can be used to calculate accurate values for the trigonometric ratios of 45°.
Split the square into two right-angled triangles.

Calculate the length of the third side of the triangle using Pythagoras' theorem.

Use the trigonometric ratios to calculate accurate values for the angle 45°.
The accurate trigonometric ratios for 0°, 30°, 45°, 60° and 90° are:

is undefined because and division by zero is undefined.

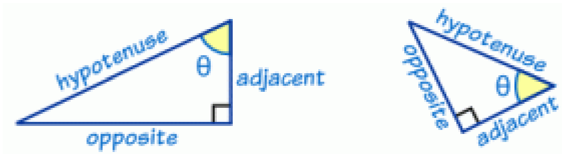
More on Sine and Cosine of Angle

The **sine** (abbreviated "**sin**") and cosine ("**cos**") are the two most prominent trigonometric functions. All other trig functions can be expressed in terms of them. In fact, the sine and cosine functions are closely related and can be expressed in terms of each other.
Definition 1 is the simplest and most intuitive definition of the sine and cosine function. The sine definition basically says that, on a right triangle, the following measurements are related:

- the measurement of one of the non-right angles (q)
- the length of the side opposite to that angle
- the length of the triangle's hypotenuse

Alternately, the cosine definition basically says that, on a right triangle, the following measurements are related:

- the measurement of one of the non-right angles (q)
- the length of the side adjacent to that angle
- the length of the triangle's hypotenuse

- 

Furthermore, Definition I gives exact equations that describe each of these relations:

sin (q) = opposite / hypotenuse

cos(q) = adjacent / hypotenuse

This first equation says that if we evaluate the sine of that angle q, we will get the exact same value as if we divided the length of the side **opposite** to that angle by the length of the triangle's hypotenuse. This second equation says that if we evaluate the cosine of that angle q, we will get the exact same value as if we divided the length of the side **adjacent** to that angle by the length of the triangle's hypotenuse. These relations holds for any right triangle, regardless of size.

The main result is this: If we **know** the values of any two of the above quantities, we can use the above relation to mathematically **derive** the third quantity. For example, the sine function allows us to answer any of the following three questions:

"Given a right triangle, where the measurement of one of the non-right **angles** (q) is known and the length of the side **opposite** to that angle q is known, find the length of the triangle's **hypotenuse**."

"Given a right triangle, where the measurement of one of the non-right **angles** (q) is known and the length of the triangle's **hypotenuse** is known, find the length of the side**opposite** to that angle q."

"Given a right triangle, where the length of the triangle's **hypotenuse** and the length of one of the triangle's other sides is known, find the measurement of the **angle** (q) **opposite**to that other side."

The cosine is similar, except that the adjacent side is used instead of the opposite side.

The functions takes the forms **y = sin(q)** and **x = cos(q)**. Usually, q is an angle measurement and x and y denotes lengths.

The sine and cosine functions, like all trig functions, evaluate differently depending on the units on q, such as **degrees, radians, or grads**. For example, sin(90°) = 1, while sin(90)=0.89399….explaination

Both functions are trigonometric **cofunctions** of each other, in that function of the complementary angle, which is the "cofunction," is equal to the other function:
sin(x) = cos(90° – x) and
cos(x) = sin(90° – x).
Furthermore, sine and cosine are mutually **orthogonal**.

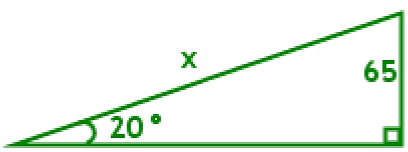
Use of sine and cosine

Sines and cosines of angles are used to find the lengths of unknown sides in triangles. The table below shows the sines and cosines of some chosen angles.

angle A	sin A	cos A
30^0	0.5000	0.8660
35^0	0.5736	0.8192
40^0	0.6428	0.7660
45^0	0.7071	0.7071
50^0	0.7660	0.6428
55^0	0.8192	0.5736
60^0	0.8660	0.5000

Example

In the triangle shown below, find the value of *x*, accurate to three decimal places.



They've given me an angle measure and the length of the side "opposite" this angle, and have asked me for the length of the hypotenuse. The sine ratio is "opposite over hypotenuse", so I can turn what they've given me into an equation:

sin(20°) = 65/*x*

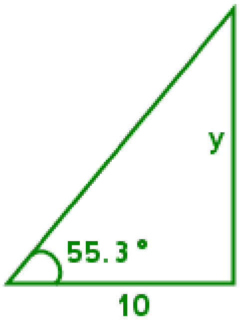
x = 65/*sin*(20°)

I have to plug this into my calculator to get the value of *x*: *x* = 190.047286...

x = 190.047

Note: If your calculator displayed a value of 71.19813587..., then check the "mode": your calculator is set to "radians" rather than to "degrees". You'll learn about radians later.

For the triangle shown, find the value of *y*, accurate to four decimal places.



They've given me an angle, a value for "adjacent", and a variable for "opposite", so I can form an equation:

tan(55.3°) = *y*/10

10*tan*(55.3°) = *y*

Plugging this into my calculator, I get *y* = 14.44183406….

y = 14.4418

Using sine and cosine tables

1. In the sine table, as angles increase from 0^0 to 90^0, their sines increase from 0 to 1.

2. In the cosine table, as angles increase from 0^0 to 90^0, their cosine decrease from 0 to 1.

Table of sin (angle)

Angle	sin (a)	Angle	sin (a)	Angle	sin (a)	Angle	sin (a)
0.0	0.0	25.0	.4226	46.0	.7193	71.0	.9455
1.0	.0174	26.0	.4384	47.0	.7314	72.0	.9511
2.0	.0349	27.0	.4540	48.0	.7431	73.0	.9563
3.0	.0523	28.0	.4695	49.0	.7547	74.0	.9613
4.0	.0698	29.0	.4848	50.0	.7660	75.0	.9659
5.0	.0872	30.0	.5000	51.0	.7772	76.0	.9703
6.0	.1045	31.0	.5150	52.0	.7880	77.0	.9744
7.0	.1219	32.0	.5299	53.0	.7986	78.0	.9781
8.0	.1392	33.0	.5446	54.0	.8090	79.0	.9816
9.0	.1564	34.0	.5592	55.0	.8191	80.0	.9848
10.0	.1736	35.0	.5736	56.0	.8290	81.0	.9877
11.0	.1908	36.0	.5878	57.0	.8387	82.0	.9903
12.0	.2079	37.0	.6018	58.0	.8480	83.0	.9926
13.0	.2249	38.0	.6157	59.0	.8571	84.0	.9945
14.0	.2419	39.0	.6293	60.0	.8660	85.0	.9962
15.0	.2588	40.0	.6428	61.0	.8746	86.0	.9976
16.0	.2756	41.0	.6561	62.0	.8829	87.0	.9986
17.0	.2924	42.0	.6691	63.0	.8910	88.0	.9994
18.0	.3090	43.0	.6820	64.0	.8988	89.0	.9998
19.0	.3256	44.0	.6947	65.0	.9063	90.0	1.00
20.0	.3420	45.0	.7071	66.0	.9135		
21.0	.3584			67.0	.9205		
22.0	.3746			68.0	.9272		
23.0	.3907			69.0	.9336		
24.0	.4067			70.0	.9397		

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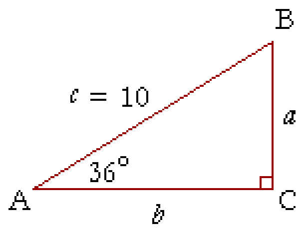
Solving right-angled triangles

To SOLVE A TRIANGLE means to know all three sides and all three angles. When we know the ratios of the sides, we use the method of similar figures. That is the method to use when solving an isosceles right triangle or a 30°-60°-90° triangle. When we do not know the ratio numbers, then we must use the Table of ratios. The following example illustrates how.

The general method

Example 1. Given an acute angle and one side**.** Solve the right triangle ABC if angle A is 36°, and side *c* is 10 cm.



Solution

Since angle A is 36°, then angle B is 90° − 36° = 54°.

To find an unknown side, say *a*, proceed as follows:

1. Make the unknown side the numerator of a fraction, and make the known side the denominator.

Unknown/known = a/10

2. Name that function of the angle.

Unknown/known = a/10 = 36^0

3. Use the trigonometric Table to evaluate that function.

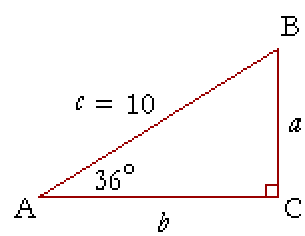
Unknown/known = a/10 = 36^0 = 0.588

4. Solve for the unknown side.

a = 10 × .588 cm = 5.88 cm

Example

Solve the triangle for side *b*.



Unknown/known = b/a = cos 36^0 0.809

B = 10 X 8.09cm

ASSESSMENT

- Proof the trigonometric ratios formula

WEEK 4

SSS 3 FIRST TERM MATHEMATICS

What is a Matrix?

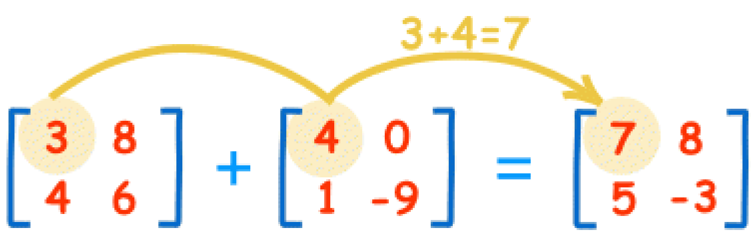
Simply put, a Matrix is an array of numbers:

(This one has 2 Rows and 3 Columns)

We talk about one **matrix**, or several **matrices**. And in Mathematics, there are many things we can do with them as discussed below-

Adding

To add two matrices: add the numbers in the matching positions:



These are the calculations:

3+4=7	8+0=8
4+1=5	6−9=−3

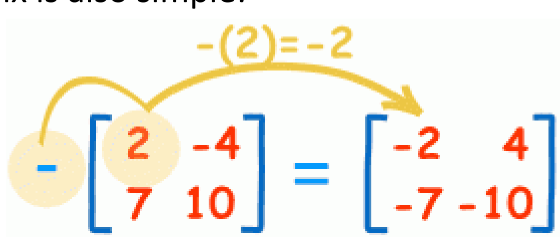
The two matrices must be the same size, i.e. the rows must match in size, and the columns must match in size.

Example: a matrix with **3 rows** and **5 columns** can be added to another matrix of **3 rows** and **5 columns**.

But it could not be added to a matrix with **3 rows** and **4 columns** (the columns don't match in size)

Negative

The negative of a matrix is also simple:



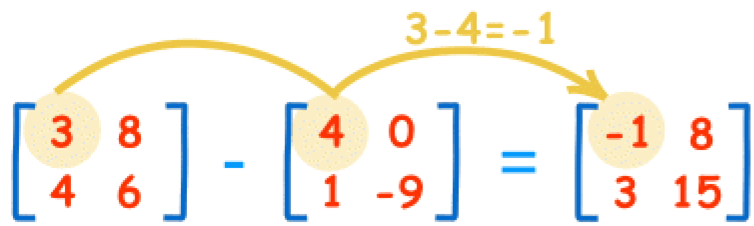
These are the calculations:

−(2)=−2 −(−4)=+4

−(7)=−7 −(10)=−10

Subtracting

To subtract two matrices: subtract the numbers in the matching positions:



These are the calculations:

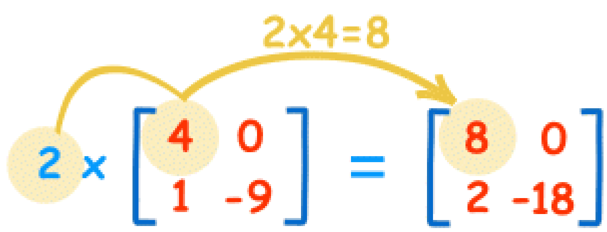
3−4=−1 8−0=8

4−1=3 6−(−9)=15

*Note: subtracting is actually defined as the **addition** of a negative matrix: A + (−B)*

Multiply by a Constant

We can multiply a matrix by a **constant** *(the value 2 in this case)*:



These are the calculations:

2×4=8 2×0=0

2×1=2 2×−9=−18

We call the constant a **scalar**, so officially this is called “scalar multiplication”.

Multiplying by another Matrix

To **multiply two matrices together** is a bit more difficult … read Multiplying Matrices to learn how.

Dividing

And what about division? Well we **don’t** actually divide matrices, we do it this way:

$A/B = A \times (1/B) = A \times B^{-1}$

where $\mathbf{B^{-1}}$ means the "inverse" of B.

So we don't divide, instead we **multiply by an inverse**.

And there are special ways to find the Inverse, learn more at Inverse of a Matrix.

Transposing

To "transpose" a matrix, swap the rows and columns.

We put a "T" in the top right-hand corner to mean transpose:

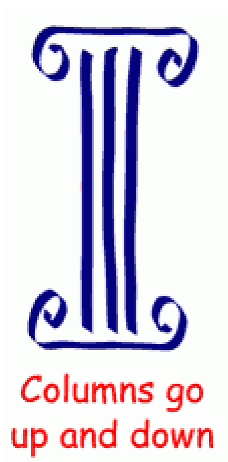
$$\begin{bmatrix} 6 & 4 & 24 \\ 1 & -9 & 8 \end{bmatrix}^T = \begin{bmatrix} 6 & 1 \\ 4 & -9 \\ 24 & 8 \end{bmatrix}$$

Notation

A matrix is usually shown by a **capital letter** (such as A, or B)

Each entry (or "element") is shown by a **lower case letter** with a "subscript" of **row,column**:

$$A = \begin{bmatrix} a_{1,1} & a_{1,2} & a_{1,3} \\ a_{2,1} & a_{2,2} & a_{2,3} \end{bmatrix}$$



Rows and Columns

So which is the row and which is the column?

- Rows go **left-right**
- Columns go **up-down**

To remember that rows come before columns use the word **"arc"**:

$$a_{r,c}$$

Example:

B =

Here are some sample entries:

$b_{1,1}$ = 6 *(the entry at row 1, column 1 is 6)*

$b_{1,3}$ = 24 *(the entry at row 1, column 3 is 24)*

$b_{2,3}$ = 8 *(the entry at row 2, column 3 is 8)*

Determinant of a Matrix

The determinant of a matrix is a **special number** that can be calculated from a square matrix.

A Matrix is an array of numbers:

$$\begin{bmatrix} 3 & 8 \\ 4 & 6 \end{bmatrix}$$

A Matrix

(This one has 2 Rows and 2 Columns)

The determinant of that matrix is (calculations are explained later):

$3 \times 6 - 8 \times 4 = 18 - 32 = \mathbf{-14}$

What is it for?

The determinant tells us things about the matrix that are useful in systems of linear equations, helps us find the inverse of a matrix, is useful in calculus and more.

Symbol

The **symbol** for determinant is two vertical lines either side.

Example:

|A| means the determinant of the matrix **A**

(Exactly the same symbol as absolute value.)

Calculating the Determinant

First of all the matrix must be **square** (i.e. have the same number of rows as columns). Then it is just basic arithmetic. Here is how:

For a 2×2 Matrix

For a 2×2 matrix (2 rows and 2 columns):

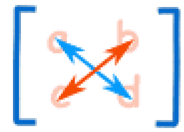
$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

The determinant is:

$$|A| = ad - bc$$

"The determinant of A equals a times d minus b times c"

It is easy to remember when you think of a cross:



- Blue is positive (+ad),
- Red is negative (−bc)

Example:

$$B = \begin{bmatrix} 4 & 6 \\ 3 & 8 \end{bmatrix}$$

$$|B| = 4\times 8 - 6\times 3$$
$$= 32-18$$
$$= 14$$

For a 3×3 Matrix

For a 3×3 matrix (3 rows and 3 columns):

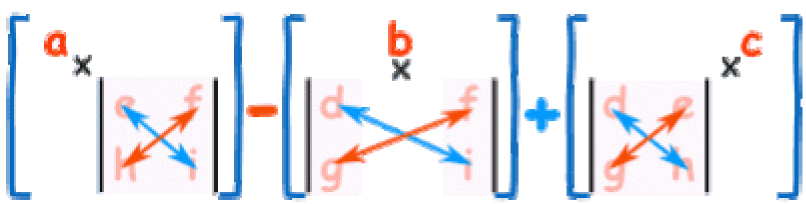
$$A = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$$

The determinant is:

$$|A| = a(ei - fh) - b(di - fg) + c(dh - eg)$$

"The determinant of A equals ... etc"

It may look complicated, but **there is a pattern**:



To work out the determinant of a **3×3** matrix:

- Multiply **a** by the **determinant of the 2×2 matrix** that is **not in a**'s row or column.
- Likewise for **b**, and for **c**
- Add them up, but remember that **b** has a negative sign!

As a formula *(remember the vertical bars || mean "determinant of")*:

$$|A| = a \cdot \begin{vmatrix} e & f \\ h & i \end{vmatrix} - b \cdot \begin{vmatrix} d & f \\ g & i \end{vmatrix} + c \cdot \begin{vmatrix} d & e \\ g & h \end{vmatrix}$$

"The determinant of A equals a times the determinant of … etc"

Example:

$$C = \begin{bmatrix} 6 & 1 & 1 \\ 4 & -2 & 5 \\ 2 & 8 & 7 \end{bmatrix}$$

|C| = 6×(−2×7 − 5×8) − 1×(4×7 − 5×2) + 1×(4×8 − (−2×2))

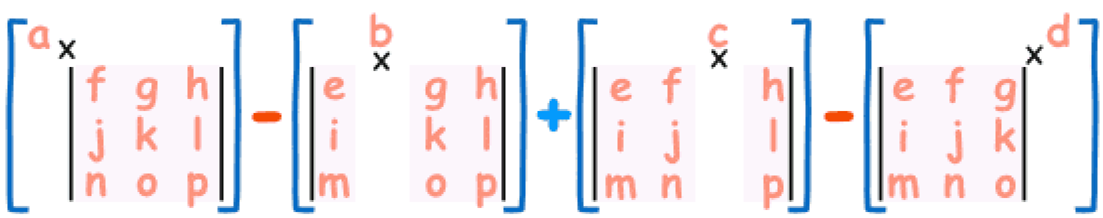
= 6×(−54) − 1×(18) + 1×(36)

= **−306**

For 4×4 Matrices and Higher

The pattern continues for 4×4 matrices:

- **plus a** times the determinant of the matrix that is **not** in **a**'s row or column,
- **minus b** times the determinant of the matrix that is **not** in **b**'s row or column,
- **plus c** times the determinant of the matrix that is **not** in **c**'s row or column,
- **minus d** times the determinant of the matrix that is **not** in **d**'s row or column,



As a formula:

$$|A| = a \cdot \begin{vmatrix} f & g & h \\ j & k & l \\ n & o & p \end{vmatrix} - b \cdot \begin{vmatrix} e & g & h \\ i & k & l \\ m & o & p \end{vmatrix} + c \cdot \begin{vmatrix} e & f & h \\ i & j & l \\ m & n & p \end{vmatrix} - d \cdot \begin{vmatrix} e & f & g \\ i & j & k \\ m & n & o \end{vmatrix}$$

Notice the **+−+−** pattern (+a… −b… +c… −d…). This is important to remember.

The pattern continues for 5×5 matrices and higher. Usually best to use a Matrix Calculator for those!

Not The Only Way

This method of calculation is called the “Laplace expansion” and I like it because the pattern is easy to remember. But there are other methods (just so you know).

Summary

- For a 2×2 matrix the determinant is **ad – bc**
- For a 3×3 matrix multiply **a** by the **determinant of the 2×2 matrix** that is **not** in **a**‘s row or column, likewise for **b** and **c**, but remember that **b** has a negative sign!
- The pattern continues for larger matrices: multiply **a** by the **determinant of the matrix** that is **not** in **a**‘s row or column, continue like this across the whole row, but remember the + − + − pattern.

ASSESSMENT

1. Go through the explanation thoroughly and look for further questions to solve.

WEEK 5

SSS 3 FIRST TERM MATHEMATICS

What is a Linear Equation?

Linear Equations

A linear equation looks like any other equation. It is made up of two expressions set equal to each other. A linear equation is special because:

1. It has one or two variables.
2. No variable in a linear equation is raised to a power greater than 1 or used as the denominator of a fraction.
3. When you find pairs of values that make the linear equation true and plot those pairs on a coordinate grid, all of the points for any one equation lie on the same line. Linear equations graph as straight lines.

A linear equation in two variables describes a relationship in which the value of one of the variables depends on the value of the other variable. In a linear equation in x and y, x is called x is the independent variable and y depends on it. We call y the dependent variable. If the variables have other names, yet do have a dependent relationship, the independent variable is plotted along the horizontal axis. Most linear equations are functions (that is, for every value of x, there is only one corresponding value of y). When you assign a value to the independent variable, x, you can compute the value of the dependent variable, y. You can then plot the points named by each (x,y) pair on a coordinate grid. The real importance of emphasizing graphing linear equations with your students, is that they should already know that any two points determine a line, so finding many pairs of values that satisfy a linear equation is easy: Find two pairs of values and draw a line through the points they describe. All other points on the line will provide values for x and y that satisfy the equation.

Slope

- Introducing the Concept
- Developing the Concept

Two-step Linear Equations with Rational Numbers.

- Introducing the Concept
- Developing the Concept

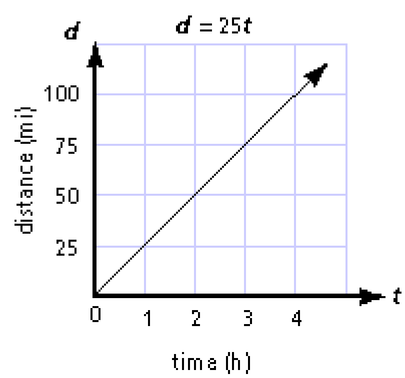
Describing Linear Relationships

The graphs of linear equations are always lines. One important thing to remember about those lines is: Not every point on the line that the equation describes will necessarily be a solution to the problem that the equation describes.

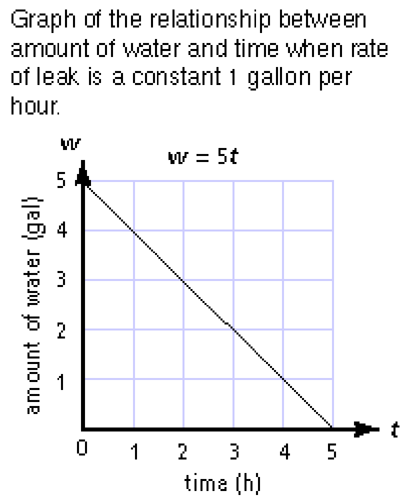
Examples of Linear Relationships

- **distance = rate × time**In this equation, for any given steady rate, the relationship between distance and time will be linear. However, distance is usually expressed as a positive number, so most graphs of this relationship will only show points in the first quadrant. Notice that the direction of the line in the graph below is from bottom left to top right. Lines that tend in this direction have positive slope. A positive slope indicates that the values on both axes are increasing from left to right.

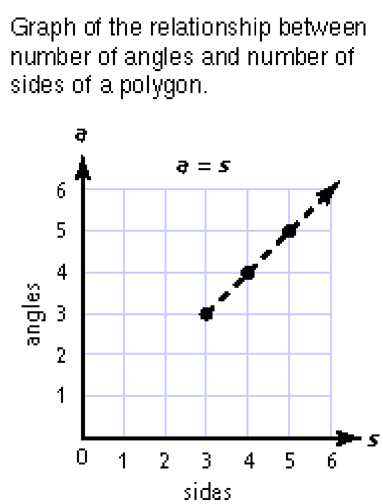
Graph of the relationship between distance and time when rate is a constant 25 miles per hour.



- **amount of water in a leaky bucket = rate of leak × time**In this equation, since you won't ever have a negative amount of water in the bucket, the graph will also show points only in the first quadrant. Notice that the direction of the line in this graph is top left to bottom right. Lines that tend in this direction have negative slope. A negative slope indicates that the values on the *y* axis are decreasing as the values on the *x* axis are increasing.

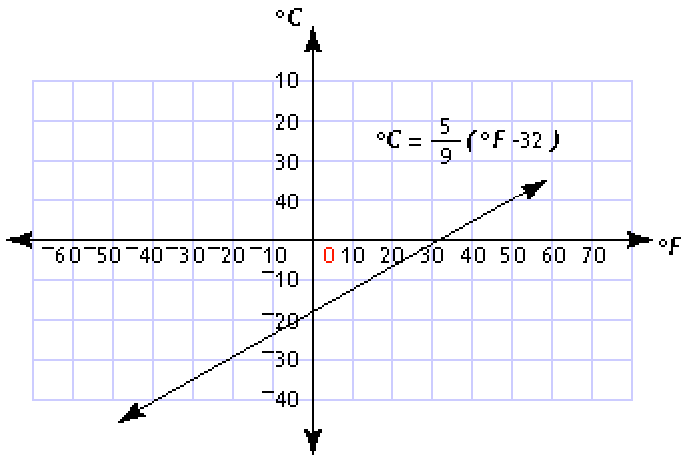


- **number of angles of a polygon = number of sides of that polygon**Again in this graph, we are relating values that only make sense if they are positive, so we show points only in the first quadrant. In this case, since no polygon has fewer than 3 sides or angles, and since the number of sides or angles of a polygon must be a whole number, we show the graph starting at (3,3) and indicate with a dashed line that points between those plotted are not relevant to the problem.



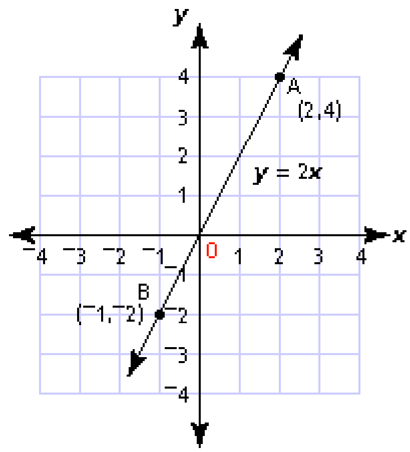
- **degrees Celsius = 5/9 (degrees Fahrenheit – 32)**Since it's perfectly reasonable to have both positive and negative temperatures, we plot the points on this graph on the full coordinate grid.

Graph of the relationship between degrees Celsius and degrees Fahrenheit.



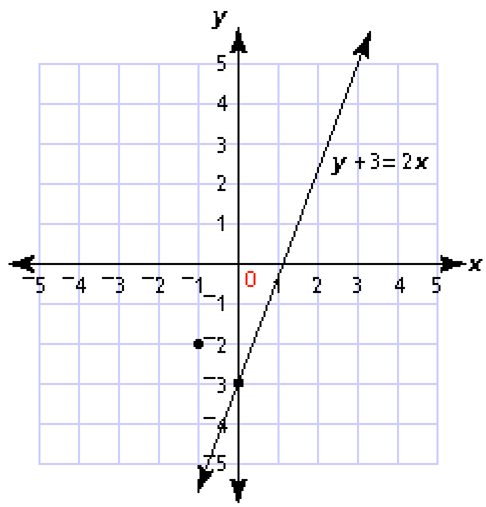
Slope

- **Steepness and Direction**The slope of a line tells two things: how steep the line is with respect to the *y* axis and whether the line slopes up or down when you look at it from left to right. When you're plotting data, slope tells you the rate at which the dependent variable is changing with respect to the change in the independent variable. This gives you a valuable clue about how to find slope: Pick any two points on the line. To find how fast *y* is changing, subtract the *y* value of the second point from the *y* value of the first point ($y_2 - y_1$). To find how fast *x* is changing, subtract the *x*value of the second point from the *x* value of the first point ($x_2 - x_1$). To find the rate at which *y* is changing with respect to the change in *x*, write your results as a ratio: $(y_2 - y_1)/(x_2 - x_1)$.



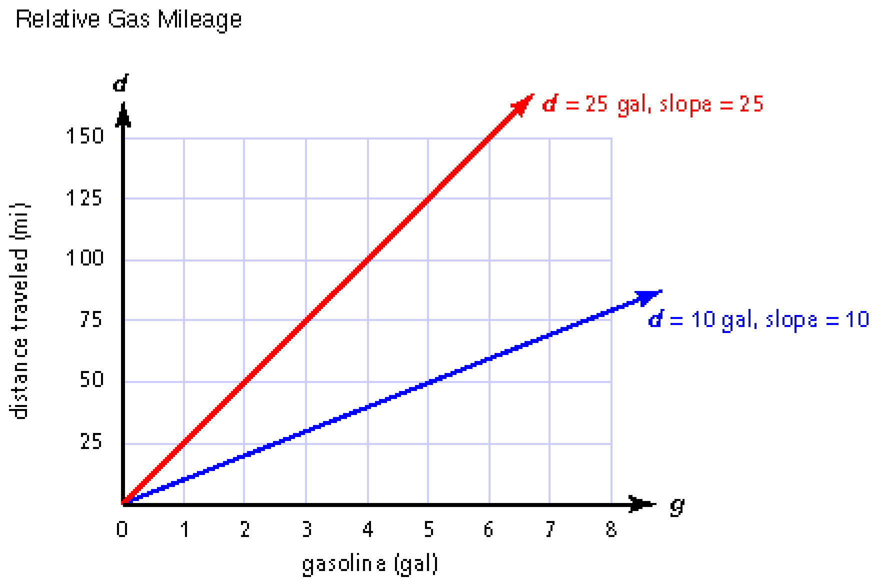
If we designate Point *A* as the first point and Point *B* as the second point, the slope of the line is (-2 – 4)/(-1 – 2) = -6/-3. or 2. It does not matter which point you designate as point 1, just as long as you use the same point as the first point when calculating change in y and change in *x*. If we designate Point *B* as the first point and Point *A* as the second point, the value of the slope is the same: (4 – -2)/(2 – -1) = 6/3, or 2. It is also the same value you will get if you choose any other pair of points on the line to compute slope.

- **Slope-Intercept Form**The equation of a line can be written in a form that gives away the slope and allows you to draw the line without any computation. If students are comfortable with solving a simple two-step linear equation, they can write linear equations in slope-intercept form. The slope-intercept form of a linear equation is $y = mx + b$. In the equation, *x* and *y* are the variables. The numbers *m* and *b* give the slope of the line (*m*) and the value of *y* when *x* is 0 (b). The value of *y* when *x* is 0 is called the *y*-intercept because (0,y) is the point at which the line crosses the *y* axis. You can draw the line for an equation in this form by plotting (0,*b*), then using *m* to find another point. For example, if *m* is 1/2, count +2 on the *x* axis, then +1 on the *y* axis to get to another point (1, b + 2)

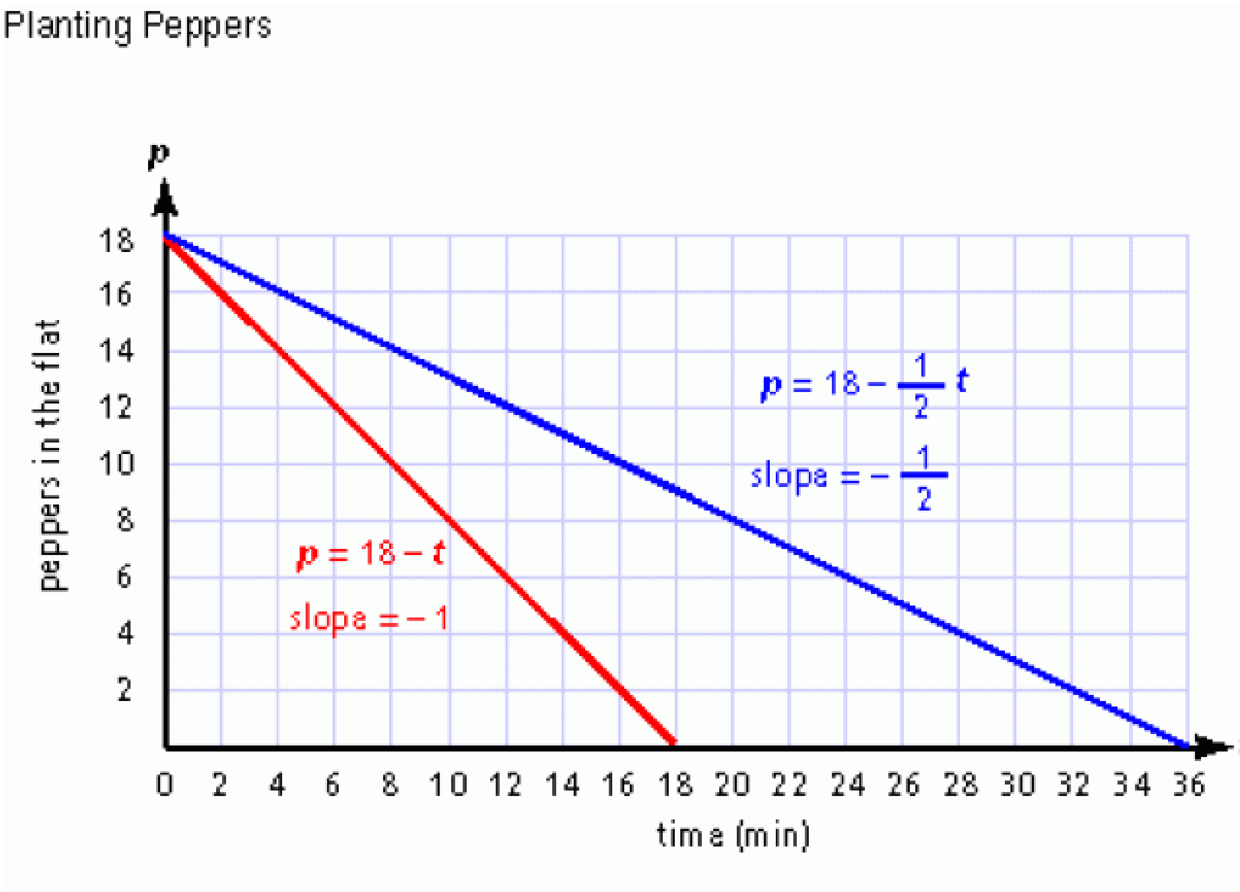


The equation for this line is $y + 3 = 2x$. In slope-intercept form, the equation is $y = 2x – 3$. You can see that the slope $m = 2$ and the slope really is 2 since for every +2 change in y, there is a +1 change in x. Now look at b in the equation: −3 should be the y value where $x = 0$ and it is.

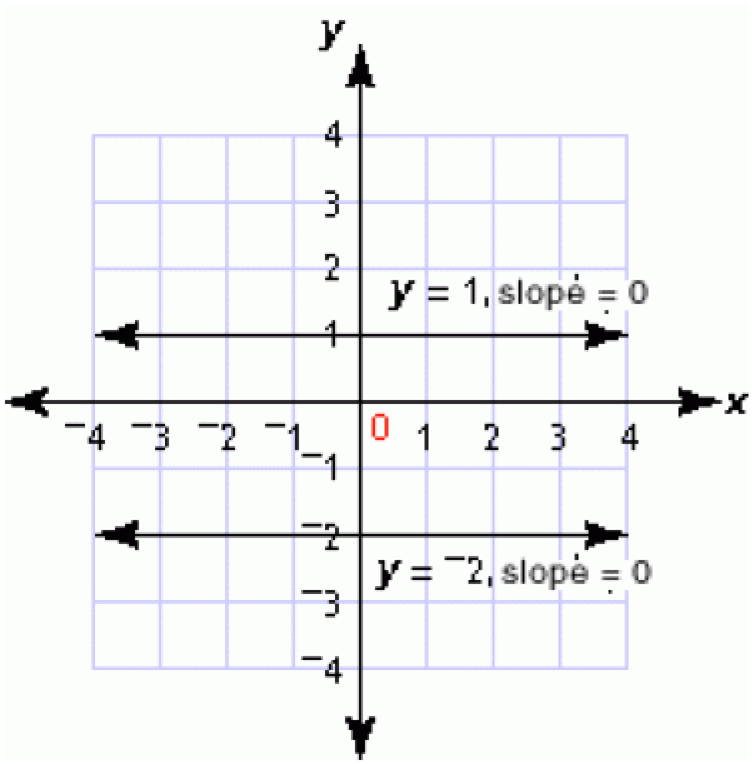
- **Positive Slope**When a line slopes up from left to right, it has a positive slope. This means that a positive change in *y* is associated with a positive change in *x*. The steeper the slope, the greater the rate of change in *y* in relation to the change in *x*. When you are dealing with data points plotted on a coordinate plane, a positive slope indicates a positive correlation and the steeper the slope, the stronger the positive correlation.Consider gas mileage. If you drive a big, heavy, old car, you get poor gas mileage. The rate of change in miles traveled is low in relation to the change in gas consumed, so the value *m* is a low number and the slope of the line is fairly gradual. If you drive a light, efficient car, you get better gas mileage. The rate of change in the number of miles you travel is higher in relation to the change in gas consumed, so the value of *m* is a greater number and the line is steeper. Both rates are positive, because you still travel a positive number of miles for every gallon of gas you consume.



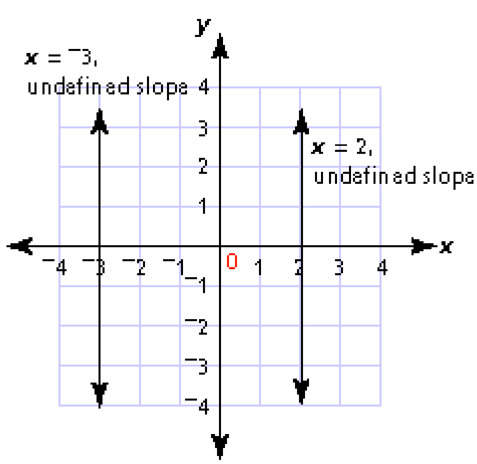
- **Negative Slope**When a line slopes down from left to right, it has a negative slope. This means that a negative change in *y* is associated with a positive change in *x*. When you are dealing with data points plotted on a coordinate plane, a negative slope indicates a negative correlation and the steeper the slope, the stronger the negative correlation.Consider working in your vegetable garden. If you have a flat of 18 pepper plants and you can plant 1 pepper plant per minute, the rate at which the flat empties out is fairly high, so the absolute value of *m* is a greater number and the line is steeper. If you can only plant 1 pepper plant every 2 minutes, you still empty out the flat, but the rate at which you do so is lower, the absolute value of *m* is low, and the line is not as steep.



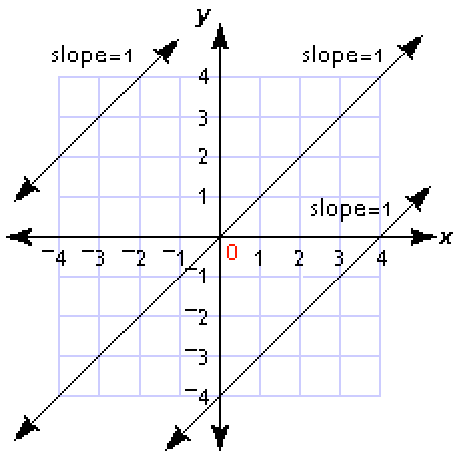
Zero Slope When there is no change in *y* as *x* changes, the graph of the line is horizontal. A horizontal line has a slope of zero.



- **Undefined Slope**When there is no change in *x* as *y* changes, the graph of the line is vertical. You could not compute the slope of this line, because you would need to divide by 0. These lines have undefined slope.



- **Lines with the Same Slope**Lines with the same slope are either the same line, or parallel lines.



In all three of these lines, every 1-unit change in *y* is associated with a 1-unit change in *x*. The rate of change is 1/1. All three have a slope of 1.**Solving Two-Step Linear Equations with Rational Numbers**When a linear equation has two variables, as it usually does, it has an infinite number of solutions. Each solution is a pair of numbers (*x*,*y*) that make the equation true. Solving a linear equation usually means finding the value of *y* for a given value of *x*.

- **When the Equation is Already in Slope-Intercept Form**If the equation is already in the form $y = mx + b$, with *x* and *y* variables and *m* and *b* rational numbers, then the equation can be solved in algebraic terms. To find ordered pairs of solutions for such an equation, choose a value for *x*, and compute to find the corresponding value for *y*. You'll notice that the easiest value to choose for *x* is often 0, because in that case, $y = b$. Students may be asked to make tables of values for linear equations. These are simply T-tables with lists of values for *x* with the corresponding computed values for *y*.Two-step equations involve finding values for expressions that have more than one term. The terms in an expression are separated by addition or subtraction symbols. $2x$ is an expression with one term. $2x + 6$ has two terms. To find a value for *y* given a value for *x*, substitute the value for *x* into the expression and compute. First, find the value of the term that contains x, then find the value of the entire expression.Consider the equation $y = 2x + 6$. Find the value for y when $x = 5$.

First, substitute the value for x into the equation .; $y = 2(5) + 6$

Then, find the value of the term that contains x. $y = 10 + 6$

Last, find the value of the entire expression. $y = 16$

- **When the Equation is Not in Slope-Intercept Form**When a linear equation is not in slope-intercept form ($y = mx + b$), students can still make a table of values to find solutions for the equation, but it may be simpler to put the equation in slope-intercept form first. This requires mirroring operations (balancing) on each side of the equation until y is by itself on the one side of the equation, set equal to an expression involving x. You can manipulate the equation in this way because of the equality properties:

If $a = b$, then $a + c = b + c$
If $a = b$, then $a – c = b – c$
If $a = b$, then $ac = bc$
If $a = b$, then $a \div c = b \div c$

Consider $2x + y – 6 = 0$. This equation is not in slope-intercept form. There are two ways to put it in slope-intercept form.

1. Show the original equation. $2x + y – 6 = 0$

Subtract y from each side. $2x + y – y – 6 = 0 – y$

$2x – 6 = 0 – y$

Multiply each side by -1. $-1(2x – 6) = -1(y)$

$-2x + 6 = y$

2. Show the original equation. $2x + y – 6 = 0$

Add 6 to each side. $2x + y – 6 + 6 = 0 + 6$

$2x + y = 6$

Subtract $2x$ from each side. $2x – 2x + y = 6 – 2x$

$y = 6 – 2x$

The two equations, $-2x + 6 = y$ and $y = 6 - 2x$ are equivalent because you can turn one into the other by using the symmetric property of equality, which states that if $a = b$, then $b = a$ and the commutative property, which states that $a + b = b + a$.

commutative property $-2x + 6 = y$ → $6 - 2x = y$

symmetric property $6 - 2x = y$ → $y = 6 - 2x$

Quadratic Equation

Quadratic equation refers to polynomial equation that have a general form of $ax^2+bx+c=0$, where a, b and c are co-efficient. $a \neq 0$ otherwise it would be a linear equation and c is constant. The quadratic formula is defined as $x= (-b \pm \sqrt{b^2-4ac})/2a$, where "a" is the co-efficient of the x^2, b is the linear co-efficient of the x and c is the constant term. Therefore, the quadratic formula involves substituting the co-efficient from a given quadratic equation into the formula.

Derivation of the quadratic formula.

For all quadratic equations, we have the general form:

$ax^2 + bx + c = 0$

1. Moving the "non x" to the right,

we get: $ax^2 + bx = - c$

2. Dividing by 'a' (the coefficient of x^2),

we get: $x^2 + bx/a = - c/a$

3. We take the coefficient of x, divide it by 2, square the result and then add that to both sides of the equation. The coefficient of x is b/a, one half of that is (b/2a) and squaring that,

we get $b^2/4a^2$.

Adding that both sides of the equation,

we have $x^2 + bx/a + b^2/4a^2 = - c/a + b^2/4a^2$

4. Taking the square roots of both sides,

we get: $x + b/2a = \sqrt{-c/a + b^2/4a^2}$

Moving b/2a to the right ,

$x =-b/2a \pm \sqrt{-4ac/4a^2 + b^2/4a^2}$

Simplifying $x = -b/2a \pm \sqrt{b^2-4ac}/2a$

Finally, $x = -b \pm \sqrt{b^2-4ac}/2a$

Questions:

Solve for the roots for the general form of quadratic equation

1. $4x^2 - 5x + 1 = 0$

A. x = 1, ¼ B. x = -1, ¼ C. 1, 1 D. 1/4, 1

2. $2x^2 - 14x - 13 = 0$

A. $5+5\sqrt{3}/2$, $7+5\sqrt{3}/2$ B. $7+5\sqrt{3}/2$, $5+5\sqrt{3}/2$ C. $7+5\sqrt{3}/2$, $7-5\sqrt{3}/2$ D. $7+5\sqrt{6}/2$, $7+\sqrt{3}/2$

3. $3x^2 + x - 2 = 0$

A. x = 2/3, -1 B. x = -2/3, 1 C. x = -2/3, -1 D. x = -2/3, 1

4. 3×2+4x+1=0

A. x =-1, -1/3 B. x = 1, -1/3 C. x=-1, 1/3 D. x=1, 1/3

5. $2x^2 - 3x - 6$

A. ¾, 4/9 B. ¾, -9/4 C. 9/4, ¾ D. 9/5, ¾

Answers

1. A 2. C 3. C 4. A 5. B

WEEK 6

SSS 3 FIRST TERM MATHEMATICS

Word Problem Involving Quadratic Equations

Write a "let" statement and equation for each example. Explain the difference.

1. Find two consecutive integers whose sum is 13

Let x = first integer

x + 1 = second integer

x + x + 1 = 13

Find two consecutive two consecutive integers whose products is 42

Let x = first integer

X + 1 = second integer

x(x + 1) = 42

Write "let" statement (if consecutive integers, use x; x + 1; x + 2 if consecutive even or odd, use x; x+2;x+4)

Write equation using keywords from statement: sum (add), product (multiply, more than (add), less than (subtract), etc.

Solve equation:

Check in original problem statement

1. Let x = first integer, x+2= second integer

x(x + 2) = 143

$x^2 + 2x = 143$

$x^2 + 2x - 143 = 0$

(x + 13)(x – 11) = 0

Integers: -13 and -11 i.e. 11 and 13

Find two numbers whose sum is 9 and whose product is 20.

Solution: Let one of the numbers be x

The other number is 9-x

Their product is 20 : x(9-x) = 20

$x(9 – x) = 20$
i.e. $9x – x^2 – 20 = 0$
Þ $x^2 + 9x + 20 = 0$ (the resulting quadratic equation)
Factorizing: $(x – 4)(x – 5) = 0$
\x = 4 or 5 √
\ The numbers are 4 and 5
If check sum of numbers: 4 + 5 = 9
The product of numbers:4 x 5 = 20

Example
The difference of two numbers is 2 and their product is 224. Find the numbers.
Let x and y be the numbers. Their difference is 2, so I can write
$x – y = 2$
Their product is 224, so
$xy = 224$
From $x – y = 2$, I get $x = y + 2$. Plug this into $xy = 224$ and solve for y:
$(y + 2)y =224$
$y^2 + 2y =224$
$y^2 + 2y – 224 = 0$
$(y + 16)(y – 14) =224$
If y = -16, then x = – 16 + 2 = – 14.
If y = 14, then x = 14 + 2 = 16.
So two pairs work: -14 and -16, and 14 and 16

Problem: Motorboat moving upstream and downstream on a river. A motorboat makes a round trip on a river 56 miles upstream and 56 miles downstream, maintaining the constant speed 15 miles per hour relative to the water.
The entire trip up and back takes 7.5 hours.
What is the speed of the current?

Solution: Denote the unknown current speed of the river as x miles/hour.
When motorboat moves upstream, its speed relative to the bank of the river is 15 – x miles/hour, and the time spent moving upstream is 56/15 – x hours.

When motorboat moves downstream, its speed relative to the bank of the river is 15 + x miles/hour, and the time spent moving downstream is 56/15 + x hours.
So, the total time up and back is 56/15 – x + 56/15 + x, and it is equal to 7.5 hours, according to the problem input.
This gives an equation 56/15 – x + 56/15 + x = 7.5.
To simplify the equation, multiply both sides by (15 – x)(15 + x) and collect common terms. Step by step, you get 56(15 + x) + 56(15 – x) = 7.5(15 – x)(15 + x),
,$1680 = 7.5(15^2 - x^2)$
$1680/7.5 = 225 - x^2$
$224 = 225 - x^2$
$x^2 = 1$.

Problem: Andrew and Bill, working together, can cover the roof of a house in 6 days. Andrew, working alone, can complete this job in 5 days less than Bill. How long will it take Bill to make this job?

Solution: Denote x number of days for Bill to cover the roof, working himself, if Andrew works alone, he can complete this job in x – 5 days.
Thus, in one single day Andrew covers 1/x -5 part of the roof area, while Bill covers 1/x part of the roof area.
Working together, Andrew and Bill make 1/x – 5 + 1/x of the whole work in each single day.
Since they can cover the entire roof in 6 days working together, the equation for the unknown value x is as follows: 6/x – 5 + 6/x = 1.
To simplify this equation, multiply both sides by (x – 5)x , then transfer all terms from the right side to the left with the opposite signs, then collect common terms and adjust the signs. In this way you get 6x + 6(x – 5) = x(x-5),
$6x - 6x - 30 = x^2 - 5x$,
$-x^2 + 6x + 6x + 5x - 30 = 0$,
$- x^2 - 17x - 30 = 0$,
You get the quadratic equation. Apply the quadratic formula to solve this equation. You get
x = 17 ± Ö17^2 – 4.30/2 = 17 ± Ö289 – 120/2 = 17 ± Ö169/2

Note: square root represent Ö

The equation has two roots: x_1 = 17 + 13/2 and x_2 = 17 -13/2 = 2 and. The second root x_2 = 2 does not fit the given conditions (if Bill covers the roof in two days, then Andrew has 2 – 5= -3 days, what has no sense).

So, the potentially correct solution is x_1 = 15: Bill covers the roof in 15 days.

Let us check it. If Bill gets the job done in 15 days, then Andrew makes it in 10 days, working separately.

Since 6/10 + 6/15 = 1, this solution is correct.

Answer: Bill covers the roof in 15 days

EXERCISES

Lets see how much you've learnt, attach the following answers to the comment below

1. One leg of a right triangle exceeds the other leg by four inches. The hypotenuse is 20 inches. Find the length of the shorter leg of the right triangle. Hint: Pythagorean Theorem. A. 12 B. 14 16 D. 182.
2. The product of two consecutive integers is 56. Find the integers. Hint: Pythagorean Theorem A. -8, 7 -8, -7 C. 8, -7 D. -8, -8
3. The area of a rectangle is 80cm^2. If the length is 2cm more than the width, find the width. A. 9cm B 10cm C. 6cm D. 8cm
4. Solve this quadratic equation $x^2 - 7x + 12.25 = 0$, using Quadratic formula.
5. Solve these equations by factoring x: $y = x^2 - 5x + 7$ A. 1 and 7 B. 2 and 6 C. 1 and 6 D. 2 and 7

Surface Area and Volume of Sphere and Hemispherical Shape

A sphere is the locus of points in space equidistant from one point called the centre of the sphere.

Note: the difference between the sphere and the circle. The sphere is the locus of a point ***in space***, while the circle is the locus of a point in ***a plane***.

In general, surface area is the sum of all the shapes that cover the surface of an object. To calculate the **surface area of a sphere** we multiply 4 by pi by the radius of the sphere squared. Given this formula, we can find the surface area of a sphere when given the radius. Similarly, we can find the radius of a sphere is we are given the surface area.

The image below shows a shape of a sphere

The area of a sphere is mathematically given as $4\pi r^2$

The half of a sphere is called the **hemisphere**.

When we're talking about the surface area of the sphere, you can think of it as how much paint would you need to cover a tennis ball or if you'd looked at a baseball and you took all the stitching apart, how much leather would you need to make that ball?

Well, to find the surface area of a sphere, you're going to use the formula that surface area equals 4 times pi times the radius squared. Now, notice the dimensional here. We have r to the second power which agrees with what we know about surface area which is it's a two dimensional property. So the only thing that you need to know in order to calculate the surface area of a sphere is this formula $4\pi r^2$. Let's look at a very basic example of this application.

If the radius of a sphere is 3 centimetres, what is the surface area? Well we'll start off by writing our surface area formula. Surface area equals 4 pi r squared and then we'll say our radius is 3 centimetres. So then we just need to substitute in and we'll know our surface area.

$\mathbf{4\pi r^2}$

$4 \times 22/7 \times 3^2$

$4 \times 3.142 \times 9$

113cm; thus gives the surface area of the sphere.

So when you have a surface area problem and they tell you the radius, all you need to do is to substitute into your formula and simplify.

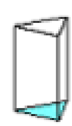
Volume of a Sphere

In determining the volume of a sphere the following activities are carried out.

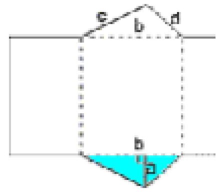
1. Cut a hollow ball vertically and horizontally into four equal parts with two open surfaces.

2. Take one part and join any of the two surfaces to form a solid that almost looks like a cone. Notice that the radius of then ball R is equal to the height h and then base r of the cone formed (i.e. R = h = r). But the volume of a cone = 1/3 $\pi r^2 h$, where h is the height and r is the base radius of the cone.
So, the volume of the cone formed = 1/3 πr^3 (since R = h =r)
Since there four equal parts of the ball there are four such cones.
∴ Volume of the entire ball = 4 x1/3 πR^3 = 4/3 πR^3 cubic units
∴ Since a ball is spherical in nature it is likely that the volume of a sphere is derived thus.
∴ Volume of sphere = 4/3 πR^3 cubic units where R is the radius of the sphere.
Examples
Given a sphere with the radius 7cm. Find the volume of this sphere. Take **π** as 3.14.
Since the values for *r*, and **π** are given, we can substitute *r* with 5 and **π** with 3.14. After substituting these values, we can calculate **V** as shown below:
$V = 4/3\ \pi r^3$
$V = 4/3(3.14)(7)^3$
$V = 4/3(3.14)(343)$
= 4/3(1077.02)
= 4(1077.03)/3
= 4308.08/3
= $1436.03cm^3$
Surface area of Hemisphere is $S = ½(4\pi r^2)$

Surface area and Volume of composite shape

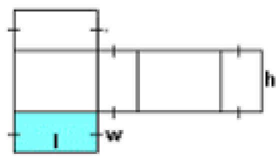
Geometric Shape	**Surface Area** **B** = area of the **b**ase **P** = **p**erimeter of the base	Volume **B** = area of the **b**ase **P** = **p**erimeter of the base
prism (general)	SA = 2B + Ph	V = Bh
Triangular Prism	SA = 2B + PhSA = 2(1/2ab) + (b + c + d)hSA	V = BhV = 1/2 abh 

= ab + (b + c + d)h

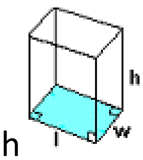


Rectangular prism

SA = 2B + PhSA = 2(lw) + (2l + 2w)h

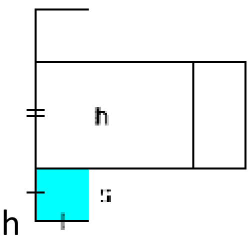


V = lwh



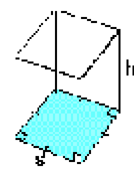
Regular square prism

SA = 2B + PhSA = 2(s^2) +



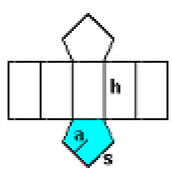
(4s)h

V = BhV = s^2h



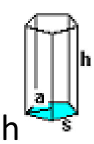
Regular pentagonal prism

SA = 2B + PhSA = 2(1/2ans) + nshSA = 2(1/2a)(5)s + 5shSA = 5as



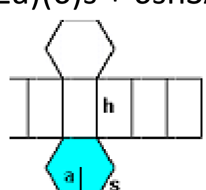
+ 5sh

V = BhV = 1/2anshV = 1/2a(5)shV = 5/2ash



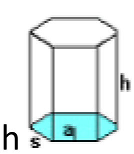
Regular hexagonal prism

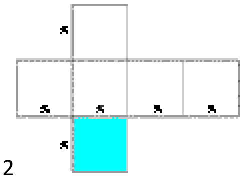
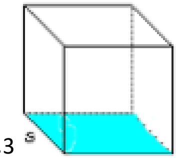
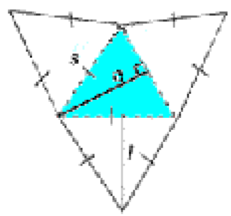
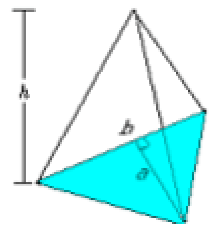
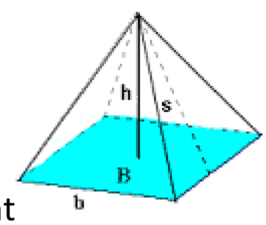
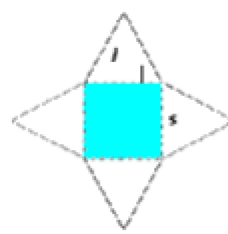
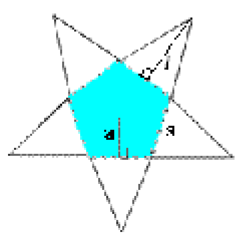
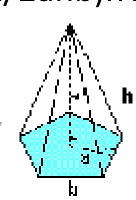
SA = 2B + PhSA = 2(1/2ans) + nshSA = 2(1/2a)(6)s + 6shSA =6as

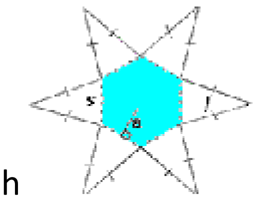
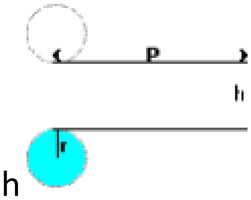
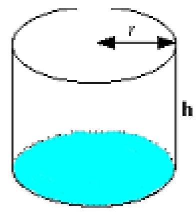


+ 6sh

V = BhV = 1/2anshV = 1/2a(6)shV = 3ash



Cube	SA = 2B + PhSA = 2(s^2) +  (4s)s = 6s^2	V = BhV = s^3 
Regular pyramid (general)	SA = B + n(1/2s*l*) *l* = slant height	V = 1/3Bh
Regular triangular pyramid	SA = B + n(1/2s*l*)SA = 1/2as + (3)(1/2s*l*)SA = 1/2as + 3/2s*l* *l* = slant height 	V = 1/3BhV = 1/3(1/2 ab)hV = 1/6 abh 
Regular square pyramid	SA = B + n(1/2s*l*)SA = s^2 + (4)(1/2s*l*)SA = s^2+ 2s*l* *l* = slant  height	V = 1/3BhV = 1/3(b^2)h=  1/3b^2h
Regular pentagonal pyramid	SA = B + n(1/2s*l*)SA = 1/2a(5)s + (5)(1/2s*l*)SA = 5/2as + 5/2s*l* *l* = slant height 	V = 1/3BhV = 1/3(1/2anb)hV = 1/6 anbh 

Regular hexagonal pyramid	SA = B + n(1/2s*l*)SA = 1/2a(6)s + (6)(1/2s*l*)SA = 3as + 3s*l* *l* = slant  height	V = 1/3BhV = 1/3(1/2anb)hV = 1/6  anbh
Cylinder	SA = 2B + phSA = 2(π r^2)  + (2πr)h	V = BhV = π r^2h 

Questions

1. Calculate the volume of the sphere with radius 3.2. Take π =3.14.

A. 207.04 cm^3 B. 137.2 cm^3 C. 189. 5 cm^3 D. 200 cm^3

Find the volume of the following spheres with

2. Diameter 8.6 cm

A. 432 cm^3 B. 333 cm^3 C. 453 cm^3 D. 234 cm^3

Calculate the surface area of the spheres with

3. Radius 3.2 cm

A. 125 cm B. 128 cm C. 121 cm D. 230 cm

4. Calculate the surface area of the sphere, given diameter 5.7 cm

A. 122 cm B. 102.01 cm C. 201.10 cm D. 98.4 cm

5. Which of these is correct for the Volume of a Regular Square Pyramid

A. v = ½ b^2h B. v = 1/3 b^2h C. b v = ½ b^3h D. v = 1/3 b^3h

Answer

1. B 2. B 3. C 4. B 5. B

WEEK 7

SSS 3 FIRST TERM MATHEMATICS

Latitude

Latitude is the angular distance of a point on the earth's surface, measured in degree from centre of the earth. Latitude (00), called equator, divides the earth into two equals- the Northern and Southern hemisphere. All other latitudes are parallel to the equator and to each other; hence, the latitudes are also called parallels of latitude.

Importance of latitude lines include:

(i) The equator 0o

(ii) Tropic of Cancer (lat 231/2oN), tropic of Capricorn (lat 231/2oN), arctic circle (661/2oN), Antarctic Circle (661/2oS), North pole (90oN) and South pole (90oS)

Uses of Lines of Latitude

(i) The lines of latitude in conjunction with that of longitude are used to determine the exact location of places on the atlas map.

North pole 90oN

Arctic circle 661/2oN

Tropic of cancer 231/2oN

Equator 0o

Tropic of Capricorn 231/2oS

Antarctic Circle 661/2oS

South pole 90oS

(ii) Lines are also used to calculate the distance between two places on the earth surface

Calculation of distances using lines of latitude

The distance between one line of the latitude and another is 111km

Two points lying North or South of equator are subtracted to get the latitude difference.

Example: Calculate the appropriate distance in a straight line between Tama (lat 60N) and London (lat 520 N)
Solution: Latitude difference = 520 – 60 = 460
Average distance = 111km
460 = 46 x 111km = 5, 106km

Calculation of Distances using Lines of Latitude

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Two points lying North or South of equator are subtracted to get the latitude difference.
Example: Calculate the appropriate distance in a straight line between Tama (lat 60N) and London (lat 520 N)
Solution: Latitude difference = 520 – 60 = 460
Average distance = 111km
460 = 46 x 111km = 5, 106km

Calculation of Local time and Longitude

The earth rotates at an angle of 10 in 4minutes; 15^0 in 1 hour and 360^0 in 24 hours.
Example: if the time in Cameroon 30^0E is 3:00pm. What time will it be in London?
Solution: 30^0 = 30 x 1 x 4minutes = 120 minutes = 2 hours
Time in London = 3:00pm – 2 hours
15 hours – 2 hours
Time in London = 13hours
Time in London = 1:00 pm

International Dateline

Time and Time Zones;

Time zones are based on the fact that Earth moves through 15 degrees of longitude (imaginary vertical lines drawn on the globe) each hour. Therefore, there are 24 standard time zones (24 hours x 15º=360º), which is the sum of angles in a circle.

Time zones are counted from the **Prime Meridian** (0º longitude), which runs through Greenwich, England. Each time zone is based on a central meridian, counted at 15º intervals from the Prime Meridian, and extends 7½º to either side of the central meridian.

For example, Nigeria lies between the longitudes 3° and 14° E of the Greenwich Meridian, much of which is in the zone of the 15ºE central meridian (GMT+1), and the time zone for this meridian includes all locations between 7½ºE and 22½ºE.

Lagos, though lies at longitude 3° 27′ E, and falls under the time zone for the Greenwich Meridian (7½°W – 7½°E), but for the sake of simplicity, and so that all parts of the country will operate at one time, it has been grafted into the GMT+1 time zone.

The need for standard time zones emerged with the spread of high speed transportation systems – first, trains and later, airplanes.The system of time zones that is in use till date was agreed upon by delegates from 27 countries who met in Washington DC in 1884.

Countries like the United States operate at 4 time zones – while it is 9am (GMT -5), Wednesday in New York; it is 6am (GMT -8), Wednesday in Los Angeles – because the country spans about 3,000 miles (over 4,800 km) from East to West. But China, though spanning over 3,000 miles (over 5,000 km) from East to West operates at just one time zone (GMT+8).

Countries like New Zealand start the day first, followed by Japan, Australia, Russia, China and India, followed by countries in Europe and Africa, followed by countries in North and South America, and ending on islands out in the Pacific ocean, like Hawaii and finally Samoa.

ASSESSMENT

The distance between one line of the latitude and another is 140km. Calculate the appropriate distance in a straight line between Tama (lat 120N) and London (lat 470 N)

SSS 3
SECOND TERM NOTES ON MATHEMATICE

TABLE OF CONTENT

SECOND TERM

WEEK 1

SSS 3 SECOND TERM MATHEMATICS

Introduction to the Coordinate Geometry of Straight Line

Straight lines in coordinate geometry are the same idea as in regular geometry, except that they are drawn on a coordinate plane and we can do more with them. Consider the line in Fig 1. How would I define that particular line? What information could I give you over the phone so that you could draw the exact same line at your end?

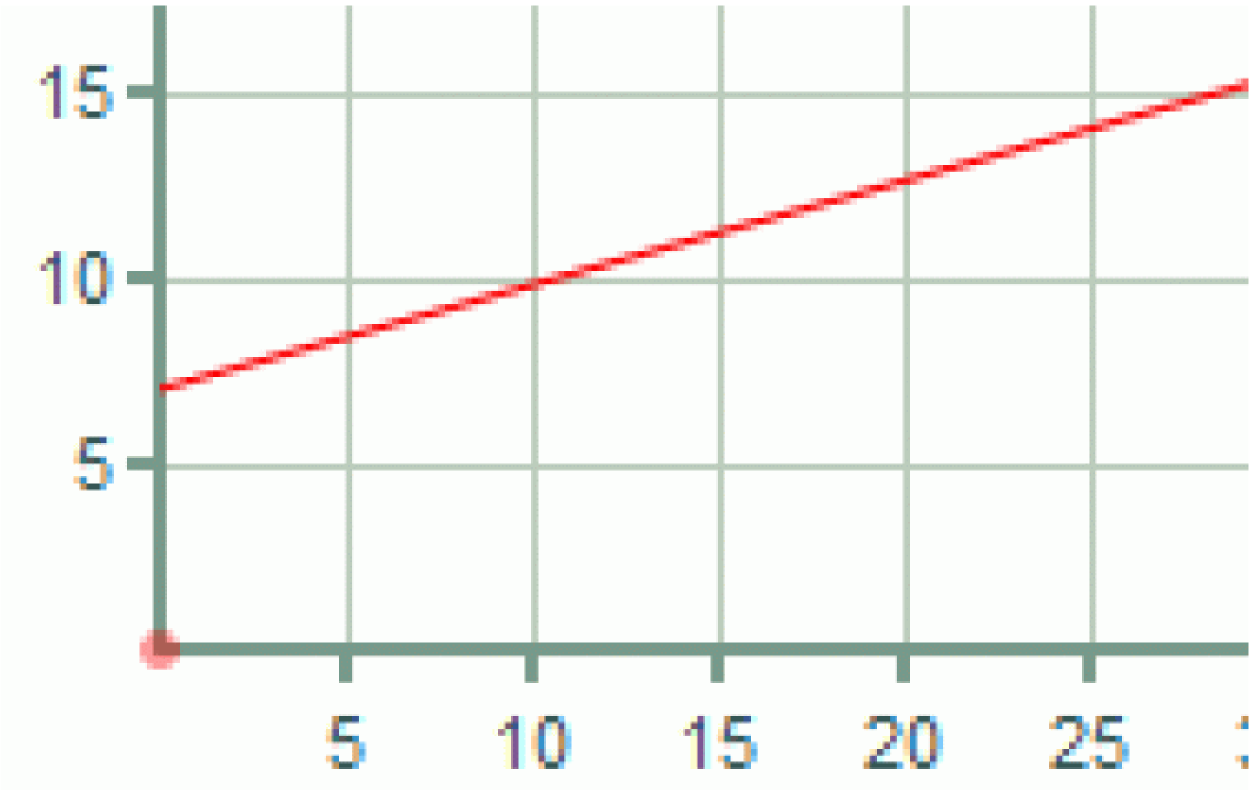


Fig 1. How to define this line?

There are three ways commonly used in coordinate geometry:

1. Give the coordinates of any two points on the line
2. Give the coordinates of one point on the line, and the slope of the line
3. Give an equation that defines the line.

It does not matter whether we are talking about a line, ray or line segment. In all cases any of the above three methods will provide enough information to define the line exactly.

Moving on then, the Coordinate Geometry of Straight Line can also be seen as any straight line has an equation of the form-

$y=mx+c$,

where m, the gradient, is the height through which the line rises in one unit step in the horizontal direction, and c, the intercept, is the y-coordinate of the point of intersection between the line and the y-axis. This is shown in Figure 1.

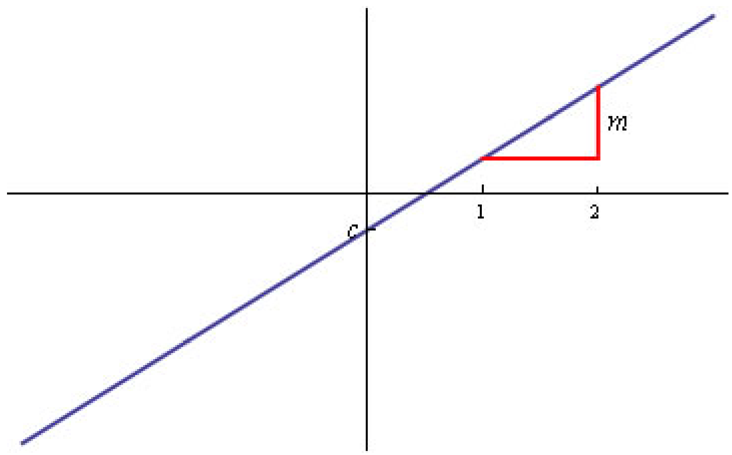


Figure 1: The straight line, $y=mx+c$

If we know the gradient m of a straight line with unknown intercept c, and the coordinates (x1,y1) of a point through which it passes, then we know that y1=mx1+c and therefore c=y1−mx1.

If we substitute into y=mx+c, we obtain y=mx−mx1+y1 which we can rearrange to give y−y1=m(x−x1).

So, for example, the straight line through the point (3,1) with gradient 2 is given by y−1=2(x−3), which gives y=2x−5.

If we know two points (x1,y1) and (x2,y2) through which passes a line with unknown gradient m and intercept c, then y1 y2 = = mx1+c, mx2+c.

Subtracting the first equation from the second gives y2−y1=m(x2−x1) and therefore m=x2−x1y2−y1.

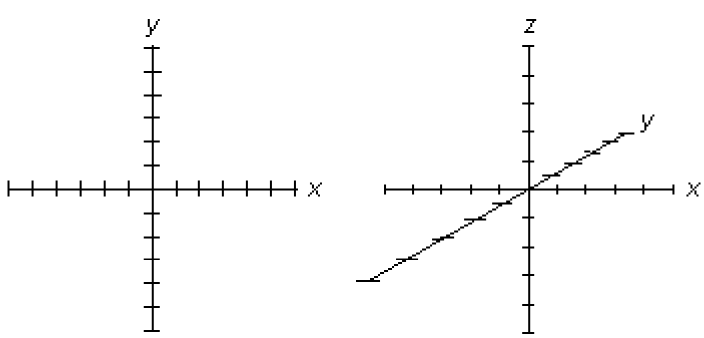
The equation of the line is therefore y−y1=x2−x1y2−y1(x−x1).

So, for example, the straight line through (−1,−2) and (2,7) has equation y+2=2+17+2(x+1), which gives y=3x+1.

Cartesian Rectangular Coordinate

Cartesian coordinates, also called rectangular coordinates, provide a method of rendering graphs and indicating the positions of points on a two-dimensional (2D) surface or in three-dimensional (3D) space. The scheme gets its name from one of the first people known to have used it, the French mathematician and philosopher René Descartes (1596-1650). Cartesian coordinates are used to define positions on computer displays, in 3D models and virtual reality (VR) renderings. The coordinate system is also employed in mathematics, physics, engineering, navigation, robotics, economics and other sciences.

The Cartesian plane consists of two perpendicular axes that cross at a central point called the origin. Positions or coordinates are determined according to the *east/west* and *north/south* displacements from the origin. The east/west axis is often called the *x* axis, and the north/south axis is called the *y* axis. For this reason, the Cartesian plane is also known as the *xy* -plane. The *x* and *y* axes are linear number lines, meaning that each division on a given axis always represents the same increment. However, the increments on different axes can differ. For example, in the illustration at left below, each increment on the *x* axis might represent 2 units, while each increment on the *y* axis represents 5 units. Points or coordinates are indicated by writing an opening parenthesis, the *x* value, a comma, the *y* value, and a closing parenthesis in that order. An example is (*x,y*) = (2,-5). The origin is usually, but not always, assigned the value (0,0).



Cartesian three-space, also called *xyz* -space, has a third axis, oriented at right angles to the *xy* plane. This axis, usually called the *z* axis, passes through the origin of the *xy* -plane. Positions or coordinates are determined according to the east/west (x), north/south (y), and *up/down* (z) displacements from the origin. As is the case with the x and y axes, the z axis is a linear number line. For example, in the illustration at right above, each increment on the x axis might represent 5 units, each increment on the y axis 10 units, and each increment on the z axis 2 units. Points or coordinates are indicated by writing an opening parenthesis, the x value, a comma, the y value, another comma, the z value, and a closing parenthesis in that order. An example is (x,y,z) = (5,-10,-4). The origin is usually, but not always, assigned the value (0,0,0).

Plotting the Linear Graph

If the rule for a relation between two variables is given, then the graph of the relation can be drawn by constructing a table of values.

To plot a **straight line graph** we need to find the coordinates of *at least two points* that fit the rule.

Example 6

Plot the graph of $y = 3x + 2$.

Solution:

Construct a table and choose simple x values.

x	-2	-1	0	1	2
y					

In order to find the y values for the table, substitute each x value into the rule $y = 3x + 2$.

When $x = -2$, $y = 3(-2) + 2$

$= -6 + 2$

$= -4$

When $x = -1$, $y = 3(-1) + 2$

$= -3 + 2$

$= -1$

When $x = 0$, $y = 3 \times 0 + 2$

$= 0 + 2$

$= 2$

When $x = 1$, $y = 3\times1+2$

$= 3+2$

$= 5$

When $x = 2$, $y = 3\times 2+2$

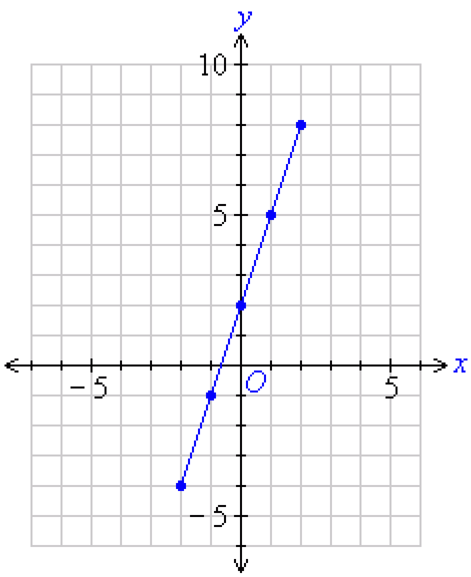
$= 6+2$

$= 8$

The table of values obtained after entering the values of *y* is as follows:

x	-2	-1	0	1	2
y	-4	-1	2	5	8

Draw a Cartesian plane and plot the points. Then join the points with a ruler to obtain a straight line graph.



Setting out:

Often, we set out the solution as follows.

$\boldsymbol{y = 3x+2}$

When $x = -2$, $y = 3(-2)+2$

$= -6+2$

$= -4$

When $x = -1$, $y = 3(-1)+2$

$= -3+2$

$= -1$

When $x = 0$, $y = 3\times0+2$

$= 0+2$

$= 2$

When $x = 1$, $y = 3 \times 1 + 2$

$= 3 + 2$

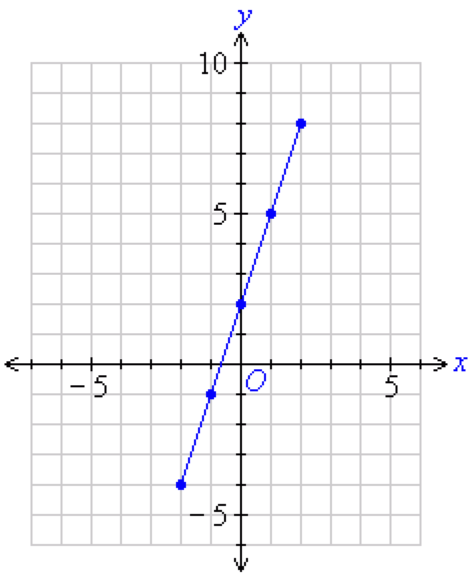
$= 5$

When $x = 2$, $y = 3 \times 2 + 2$

$= 6 + 2$

$= 8$

x	-2	-1	0	1	2
y	-4	-1	2	5	8



Example 7

Plot the graph of *y* = –2*x* + 4.

Solution:

$y = -2x + 4$

When $x = -2$, $y = -2(-2) + 4$

$= 4 + 4$

$= 8$

When $x = -1$, $y = -2(-1) + 4$

$= 2 + 4$

$= 6$

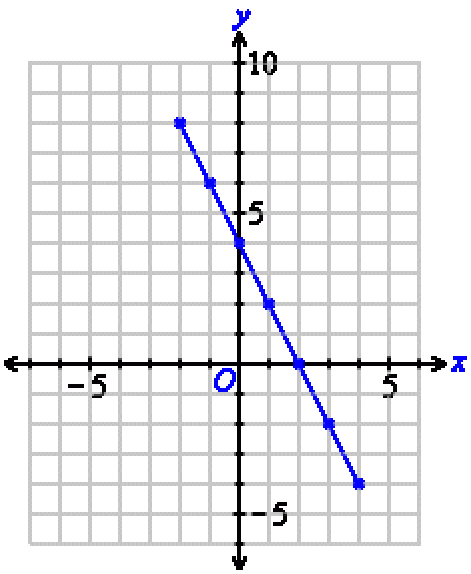
When $x = 0$, $y = -2 \times 0 + 4$

$= 0 + 4$

$= 4$

$$\text{When } x = 1, y = -2(1) + 4$$
$$= -2 + 4$$
$$= 2$$
$$\text{When } x = 2, y = -2(2) + 4$$
$$= -4 + 4$$
$$= 0$$
$$\text{When } x = 3, y = -2(3) + 4$$
$$= -6 + 4$$
$$= -2$$
$$\text{When } x = 4, y = -2(4) + 4$$
$$= -8 + 4$$
$$= -4$$

x	-2	-1	0	1	2	3	4
y	8	6	4	2	0	-2	-4



ASSESSMENT

1. If a line passes through point A(0, c) and has gradient 'm' then equation will be
   (a) y = mx + c
   (b) c = xy + m
   (c) m = xy + c
   (d) cx = y + m
2. According to Pythagoras theorem, distance between points (-3, 8) and (8, -5) is
   (a) 19.03 units

(b) 11.03 units

(c) 15.03 units

(d) 17.03 units

3. If coordinates of A and B are (2, 2) and (9, 11) respectively then length of line segment AB is

   (a) 11.4

   (b) 13.4

   (c) 15.4

   (d) 17.4

4. If points of straight line are M(7, 1) and N(7, 2) then line MN is

   (a) horizontal line with equation with x = 1

   (b) vertical line with equation with x = 2

   (c) vertical line with equation with x = 7

   (d) vertical line with equation with x = 7

5. Straight line equation y = 5x – 2 has gradient of

   (a) x + y

   (b) xy

   (c) 2

   (d) 5

ANSWERS

1. a
2. d
3. a
4. c
5. d

WEEK 2

SSS 3 SECOND TERM MATHEMATICS

Determine the Distance between Two Coordinate Points

To determine the distance between two coordinate points, a formula used. said formula is an algebraic expression which can be seen thus- (x_1, y_1) and (x_2, y_2).

D=(x2−x1)2+(y2−y1)2 √D=(x2−x1)2+(y2−y1)2

Example

Find the distance between (-1, 1) and (3, 4).

This problem is solved simply by plugging our x- and y-values into the distance formula:

D=(3−(−1))2+(4−1)2 √=D=(3−(−1))2+(4−1)2=

=16+9 √=25−−√=5=16+9=25=5

Sometimes you need to find the point that is exactly between two other points. This middle point is called the "midpoint". By definition, a midpoint of a line segment is the point on that line segment that divides the segment in two congruent segments.

If the end points of a line segment is (x_1, y_1) and (x_2, y_2) then the midpoint of the line segment has the coordinates:

(x1+x22,y1+y22)(x1+x22,y1+y22)

Finding the Mid-Point of the Line Joining Two Points

If we want to find the distance between two points on a number line we use the distance formula:

AB=|b−a|or|a−b|AB=|b−a|or|a−b|

Example

Point A is on the coordinate 4 and point B is on the coordinate -1.

AB=|4−(−1)|=|4+1|=|5|=5AB=|4−(−1)|=|4+1|=|5|=5

If we want to find the distance between two points in a coordinate plane we use a different formula that is based on the Pythagorean Theorem were (x_1,y_1) and (x_2,y_2) are the coordinates and d marks the distance:

d=(x2−x1)2+(y2−y1)2 √d=(x2−x1)2+(y2−y1)2

The point that is exactly in the middle between two points is called the midpoint and is found by using one of the two following equations.

Method 1: For a number line with the coordinates a and b as endpoints:

midpoint=a+b2midpoint=a+b2

Method 2: If we are working in a coordinate plane where the endpoints has the coordinates (x_1,y_1) and (x_2,y_2) then the midpoint coordinates is found by using the following formula:

midpoint=(x1+x22,y1+y22)

Practical Application of Coordinate Geometry

The coordinate geometry is an important branch of mathematics. It mainly helps us to locate the points in a plane. Its uses are spread in all fields like trigonometry, calculus ,dimensional geometry etc. And the subject have obvious applications in statistics ,physics also. Here we will see some applications through examples. First we will see the coordinate plane, which is made up of x and y axes. The horizontal line is the x axis and the vertical line is the y axis.

We can see in the figure two intersecting lines, that is the x and y axis,and they meet at the origin (0,0). We are marking the coordinates using x and y ordinates respectively ,with a separating comma. We can see the numbers marked in the plane ,at a unit difference. This will help us to locate the points easily.

For example: We can mark the point (1,1) directly up to the point 1 in x axis and directly straight to 1 in y axis to the right side. And for drawing graphs we have to plot the points like this and have to join them.

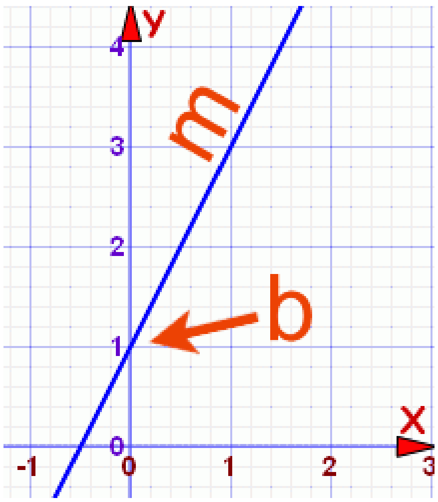
Gradient and Intercept of a Straight Line

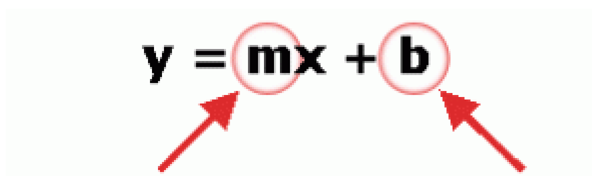
The equation of a straight line is usually written this way:

y = mx + b

(or "y = mx + c" in the UK see below)

What does it stand for?





Slope (or Gradient) Y Intercept

y = how far up

x = how far along

m = Slope or Gradient (how steep the line is)

b = the Y Intercept (where the line crosses the Y axis)

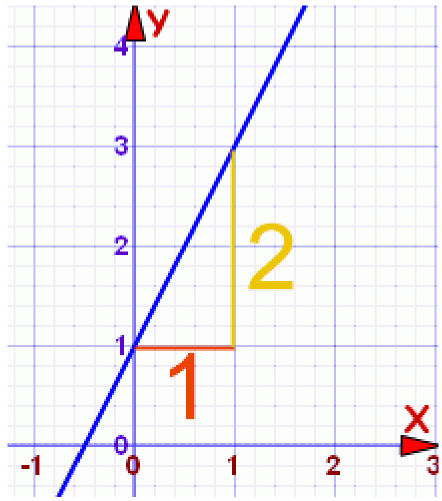
How do you find "m" and "b"?

- b is easy: just see where the line crosses the Y axis.
- m (the Slope) needs some calculation:

m = *Change in Y***Change in X**

Knowing this we can work out the equation of a straight line:

Example 1



m = *2***1** = 2

b = 1 *(where the line crosses the Y-Axis)*

So: y = 2x + 1

With that equation you can now …

… choose any value for x and find the matching value for y

For example, when x is 1:

y = 2×**1** + 1 = 3

Check for yourself that x=1 and y=3 is actually on the line.

Or we could choose another value for x, such as 7:

y = 2×**7** + 1 = 15

And so when x=7 you will have y=15

Example 2

m = −*3***1** = −3

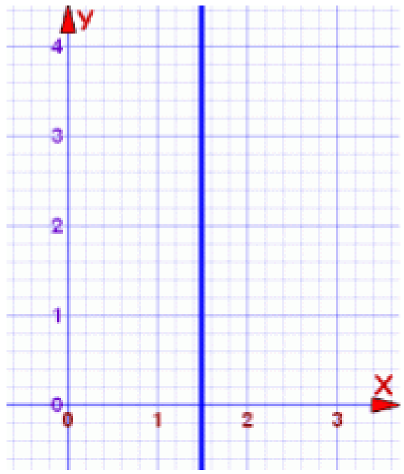
b = 0

This gives us **y = −3x + 0**

We do not need the zero!

So: y = −3x

Example 3: Vertical Line



What is the equation for a vertical line?

The slope is **undefined** … and where does it cross the Y-Axis?

In fact, this is a **special case**, and you use a different equation, not "**y**=…", but instead you use "**x**=…".

Like this:

x = 1.5

Every point on the line has **x** coordinate **1.5**,

that is why its equation is **x = 1.5**

Rise and Run

Sometimes the words "rise" and "run" are used.

- Rise is how far up
- Run is how far along

And so the slope "m" is:

m = *rise***run**

You might find that easier to remember

ASSESSMENT

1. If a line passes through point A(0, c) and has gradient 'm' then equation will be
   (a) y = mx + c
   (b) c = xy + m
   (c) m = xy + c
   (d) cx = y + m
2. According to Pythagoras theorem, distance between points (-3, 8) and (8, -5) is
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   (c) 15.03 units
   (d) 17.03 units
3. If coordinates of A and B are (2, 2) and (9, 11) respectively then length of line segment AB is
   (a) 11.4
   (b) 13.4
   (c) 15.4
   (d) 17.4
4. If points of straight line are M(7, 1) and N(7, 2) then line MN is
   (a) horizontal line with equation with x = 1
   (b) vertical line with equation with x = 2
   (c) vertical line with equation with x = 7
   (d) vertical line with equation with x = 7
5. Straight line equation y = 5x – 2 has gradient of
   (a) x + y
   (b) xy
   (c) 2
   (d) 5

ANSWERS

1. a
2. d
3. a
4. c
5. d

WEEK 3

SSS 3 SECOND TERM MATHEMATICS

Introduction

An algebraic function is a function that can be written using a finite number of the basic operations of arithmetic (i.e., addition, multiplication, and exponentiation). In order to take the derivative of these functions, we will need the power rule.

The Power Rule: The power rule states that if , then .

Exponential and Logarithmic Functions

Exponential Functions: An exponential function with base is its own derivative. That is to say, if , then as well.

Logarithmic Function: Logarithmic functions with base have derivatives of the following form: if , then .

Rules of Differentiation for Algebraic Function

1. Derivative of a constant function. The derivative of f(x) = c where c is a constant. f ‘(x) = 0 Example : f(x) = 5 , then f ‘(x) = 0
2. Derivative of a power function. The derivative of f(x) = x^n where n is a constant real number. f ‘(x) = n x ^(n- 1) Example : f(x) = x^7 then, f ‘(x) = 7 x^(7-1) = 7x^6
3. Derivative of the sum of functions The derivative of f(x) = g(x) + h(x) is given by f ‘(x) = g ‘(x) + h ‘(x) Example: f(x) = 3x^4 + 2x let g(x) = 3x^4 and h(x) = 2x then, f ‘(x) = g ‘(x) + h ‘(x) = 12x^3 + 2
4. Derivative of the difference of functions. The derivative of f(x) = g(x) – h(x) is given by f ‘(x) = g ‘(x) – h ‘(x) Example: f(x) = 5x – x^-2 let g(x) = 5x and h(x) = x^-2 then, f ‘(x) = g ‘(x) – h ‘(x) = 5 (-2x^-3) = 5 + 2^-3
5. Derivatives of a composite functions Example : f(x) = (2x^3 + 5)^4 let a = 2, k = 4 and n = 3 thus, f'(x) = kanx^(n-1)(ax^n + b)^(k-1) = 4(2)(3)x^2(2x^3 + 5)^3 = 24x^2 (2x^3 + 5)^3
6. Derivative of the product of two functions The derivative of f(x) = g(x) h(x) is given by f ‘(x) = g(x) h ‘(x) + h(x) g ‘(x) Example:

More Tips: Differentiate each function with respect to its independent variable-

ASSESSMENT

1. f(x) = 7x – 3

---

2. y = 2 – 4x + $6x^5$

3. $g(t) = (4t^3 + 6t - 54)^{63}$

4. $u = (3t - 2)(4t + 6)^3$

5.

$$y = \frac{2x}{3x-1}$$

6.

7.

$$y = \frac{2 - x}{x^2}$$

8.

9.

$$y = \frac{1}{x^2} - \frac{2x}{1 - x}$$

10.

11.

$$f(x) = \frac{mx + b}{rx + c}$$

12.

13. $y = \text{sqrt}(x) + 3x^{-2}$

14.

$$g(t) = \frac{1}{t + \text{sqrt}(3t)}$$

15.

16. $y = x^{2.7} + x^2 - 4$

17. $y = x^e - x^{-2/3}$

18. Find all points on the graph of the following functions at which the tangent line is horizontal (slope = 0).

$y = 4 + x + x^2$ $y = x^3 - 27x + 1$ $y = (x-1)^4 + 2(x-1)^2$

$y = \frac{3x}{x^2 + 4}$ $y = \frac{1}{x^2 + x + 1}$ $y = x^{1/3} - x^{4/3}$

19. How many tangent lines to the graph of $f(x) = 1/x$ pass through the point (1,-1)? At which points do these tangent lines touch the graph of f?

20. Find all points on the graph of $y = 2x^3 - 3x^2 - 9x - 1$ at which the tangent line is parallel to the line $y = 3x - 2$. (Parallel lines have the same slope.)

21. Find the x-coordinates of all points on the graph of $y = 2x^3 - 3x^2 - 9x - 1$ at which the tangent line is perpendicular to the line $y = 3x - 2$. (The product of the slopes of two nonvertical perpendicular lines is -1.)

WEEK 4

SSS 3 SECOND TERM MATHEMATICS

What is Algebraic Function?

In mathematics, an **algebraic function** is a function that can be defined as the root of a polynomial equation. Quite often algebraic functions can be expressed using a finite number of terms, involving only the algebraic operations addition, subtraction, multiplication, division, and raising to a fractional power.

Examples of such functions are:

-
-
-

Some algebraic functions, however, cannot be expressed by such finite expressions (this is Abel–Ruffini theorem). This is the case, for example, of the Bring radical, which is the function implicitly defined by

.

In more precise terms, an algebraic function of degree *n* in one variable *x* is a function y= hat satisfies a polynomial equation

where the coefficients $a_i(x)$ are polynomial functions of *x*, with coefficients belonging to a set *S*. Quite often, , and one then talks about "function algebraic over ", and the evaluation at a given rational value of such an algebraic function gives an algebraic number.

A function which is not algebraic is called a transcendental function, as it is for example the case of . A composition of transcendental functions can give an algebraic function- .

As an equation of degree *n* has *n* roots, a polynomial equation does not implicitly define a single function, but *n* functions, sometimes also called branches.

Consider for example the equation of the unit circle: This determines *y*,

except only up to an overall sign; accordingly, it has two branches:
An **algebraic function in *m* variables** is similarly defined as a function *y* which solves a polynomial equation in *m* + 1 variables:

It is normally assumed that *p* should be an irreducible polynomial. The existence of an algebraic function is then guaranteed by the implicit function theorem. Formally, an algebraic function in *m* variables over the field *K* is an element of the algebraic closure of the field of rational functions $K(x_1,\ldots,x_m)$.

ASSESSMENT

1. In function y = ƒ(x), 'f' is classified as
   (a) name of function
   (b) value of function
   (c) upper limit of function
   (d) lower limit of function
2. In function y = ƒ(x), 'y' is classified as
   (a) dependent variable
   (b) lower limit variable
   (c) upper limit variable
   (d) independent variable
3. Value h(x) is $6x^3$-3x+9 and g(x) = 3x then rational function is written as(a) 3x-$6x^3$-3x+9
   (b) 3x+$6x^3$-3x+9
   (c) 3x⁄$6x^3$-3x+9
   (d) $6x^3$-3x+9⁄3x
4. By putting value for x as 3 in function y = 10x, answer will be
   (a) 3.334

(b) 7
(c) 30
(d) 13

5. Consider function y = 10 – 4x, if value of x = -1 then value of ‘y’ is
(a) 6
(b) −2
(c) 14
(d) 2

ANSWERS

1. a
2. a
3. c
4. c
5. c

WEEK 5

SSS 3 SECOND TERM MATHEMATICS

What is Algebraic Function?

In mathematics, an **algebraic function** is a function that can be defined as the root of a polynomial equation. Quite often algebraic functions can be expressed using a finite number of terms, involving only the algebraic operations addition, subtraction, multiplication, division, and raising to a fractional power.

Examples of such functions are:

- f(x)=1/x
- f(x)={\sqrt {x}}
- {\displaystyle f(x)={\frac {\sqrt {1+x^{3}}}{x^{3/7}-{\sqrt {7}}x^{1/3}}}}

Some algebraic functions, however, cannot be expressed by such finite expressions (this is Abel–Ruffini theorem). This is the case, for example, of the Bring radical, which is the function implicitly defined by

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In more precise terms, an algebraic function of degree *n* in one variable *x* is a

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algebraic over “, and the evaluation at a given rational value of such an algebraic function gives an algebraic number.

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   (d) independent variable
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   (b) 3x+$6x^3$-3x+9

(c) $3x/6x^3-3x+9$

(d) $6x^3-3x+9/3x$

4. By putting value for x as 3 in function y = 10x, answer will be
   (a) 3.334
   (b) 7
   (c) 30
   (d) 13
5. Consider function y = 10 – 4x, if value of x = -1 then value of 'y' is
   (a) 6
   (b) −2
   (c) 14
   (d) 2

ANSWERS

1. a
2. a
3. c
4. c
5. c