MATHEMATICS

For

Senior Secondary School

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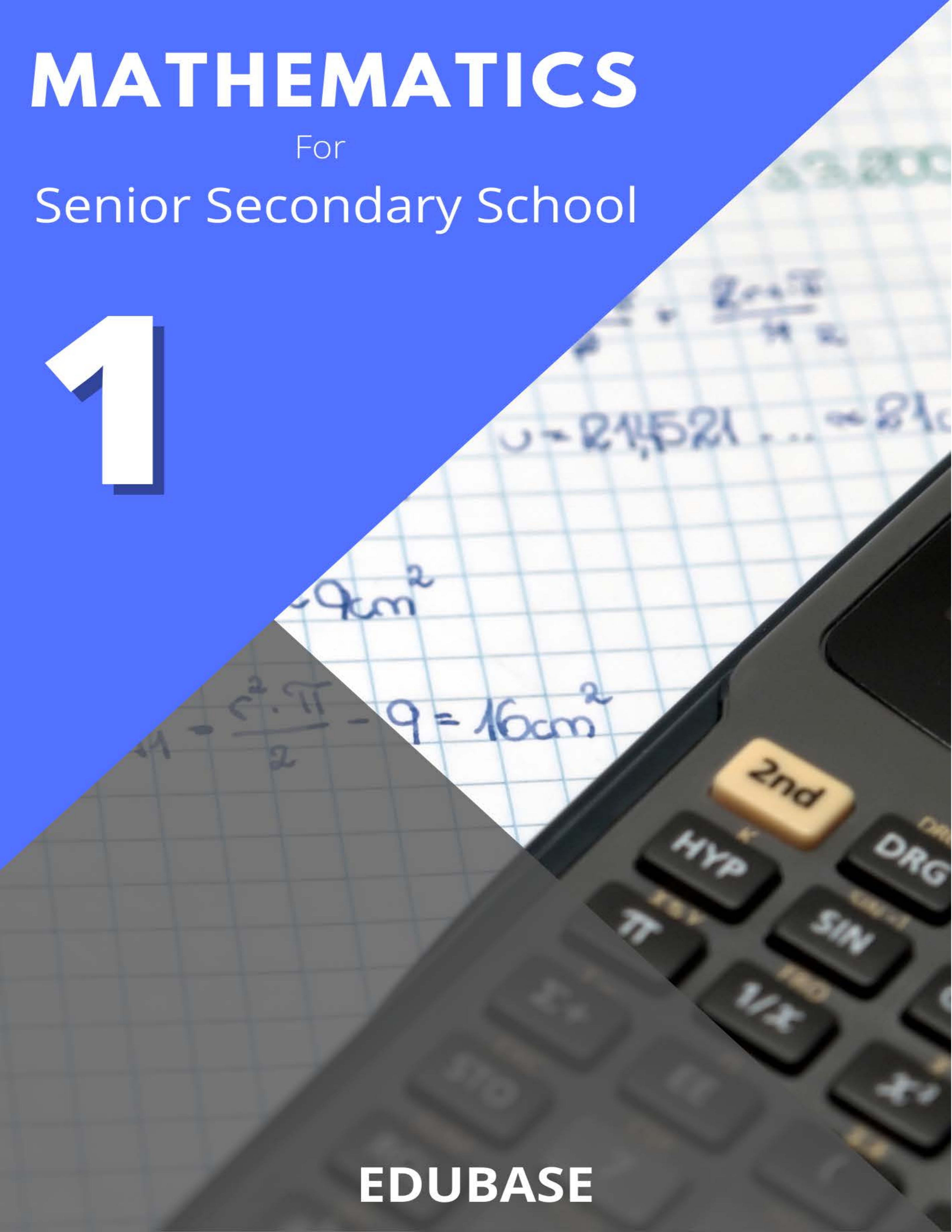


EDUBASE

SS1

MATHEMATICS

FIRST TERM

TABLE OF CONTENT

Week 1

What is Mensuration?

Mensuration is a branch of mathematics that deals with the measurement of areas and volumes of various geometrical figures. Figures such as cubes, cuboids, cylinders, cones and spheres have volume and area. Mensuration deals with the development of formulas to measure their areas and volumes.

CUBES

A cube is a solid of uniform cross-section. It is formed by squares and has 8 vertices. An example is processed cubed sugar.

The length of a side of a cube is ‘e’ which is the length of all sides since a cube is formed with squares.

TOTAL SURFACE AREA OF A CUBE

A cube has 6 faces. The surface area of each side = e^2 as each side is a square.

Therefore, Total surface area of a cube (all 6 sides) = $6e^2$

The surface area of a cube is gotten by the formula

Surface area= $6e^2$ sq. units

VOLUME OF A CUBE

In a cube all sides are equal. Length=e, height=e and width=e

Therefore, Volume of a cube= length× width× height

Volume of a cube= e^3 cubic units

CYLINDERS

A Cylinder is a uniform circular cross-section. Examples of cylinders are unsharpened pencils like HB or 2B pencils, garden rollers, tins of milk or tomato et cetera

TOTAL SURFACE AREA OF A CYLINDER

There are two types of cylinders;

(1) A closed cylinder and

(2) An open cylinder

TOTAL SURFACE AREA OF A CLOSED CYLINDER

The total surface area of a closed cylinder consists of a sum of the areas of (i) the curve surface and (ii) The two circular end faces.

The curved surface when opened out is a rectangle. This rectangle has length equal to the length of the

Cylinder and the width are equal to the circumference of the circular end face.

Area of curved surface of a cylinder = area of rectangle of dimensions length (L) and width

(Circumference of base)

$= 2\pi rl$

Area of the two circular end faces= twice the area of one circular face

$= \pi r^2$

Hence the total surface area of the closed cylinder

$= 2\pi rl + 2\pi r2$ sq. units

TOTAL SURFACE AREA OF AN OPEN CYLINDER

The total surface area of an open cylinder is the area of the curved surface which is the area of the rectangle the cylinder forms when spread.

Sometimes we are given a thick hollow cylinder. The total surface area is the sum of;

(i) the area of the external curved surface

(ii) the area of the internal curved surface and

(iii) the area of the end annular faces which will be shaded.

VOLUME OF A CYLINDER

A right circular cylinder is a solid of uniform cross-section. If a paper is wrapped round a cylinder, on opening it, a rectangle will be found.

Thus if the height of a cylinder

= h units

And the base radius

= r units

Then the volume of a cylinder

= Area of base by height

= $\pi r^2 h$ cubic units

TRIANGULAR PRISMS

A prism is a solid with uniform cross-section of a shape of a triangle or a trapezium or any other polygon.

TOTAL SURFACE AREA OF A TRIANGULAR PRISM

In the case of a triangular shaped prism, the total surface area is the sum of the surface areas of the five faces that make up the prism.

VOLUME OF A TRIANGULAR PRISM

The volume of a prism is the area of its cross-section multiplied by the distance between the end faces. Examples are funnel, Chinese hat, cut periwinkle shell et cetera.

CONES

A cone is a figure with circular base and sides slanting to a common point or vertex. There are two types of cones (i) a right circular cone, where the line joining the vertex is symmetrical and perpendicular to the base of the cone and (ii) the non-right circular cone but this isn't in the syllabus.

SURFACE AREA OF A CONE

Since a cone is formed from a sector of circle, then the surface area of a cone is equal to the area of the sector that formed it. Let L be the radius of the sector, then L becomes the slant height of the cone. If r is the radius of the base of the cone, then the length of arc of the sector is equal to 2pi.r which equals the circumference of the base of the cone.

If the sector subtends an angle- which it always does- then the area of the sector will be equal to

$= \theta /360 \times \pi l^2$ = curved surface of a cone

But $2\pi r$ = length of arc of a sector.

Therefore, $2\pi r = \theta /360 \times 2\pi l$

Finally, surface area of a cone = πrl

TOTAL SURFACE AREA OF A CONE

The total surface area of cone is the sum of (i) the curved surface area πrl sq. units and (ii) the area of the base of the cone πr^2 sq. units.

Therefore, total surface area of a cone = $\pi r^2 + \pi rl$

ASSESSMENT

1. What is mensuration?
2. What are the two types of cylinder?
3. What is the formula for the total surface area of a cone?
4. What is the formula for the volume of a cylinder?

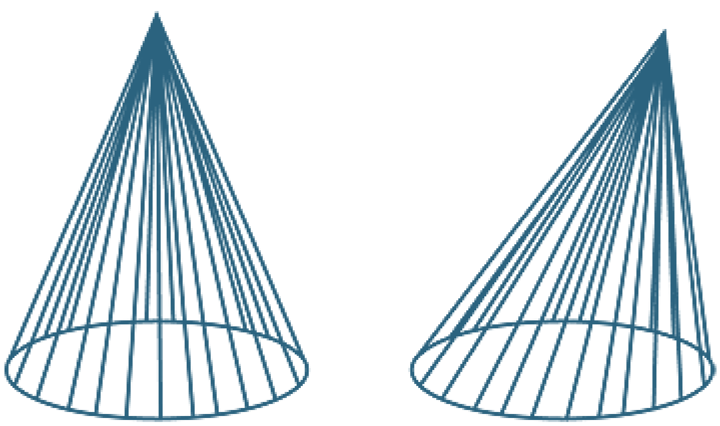
Volumes of Frustums of Cone, Rectangular-based Pyramid and other Pyramids

Week 2

SOLIDS 1 : Volume, Clinder and Surface Area

CONES

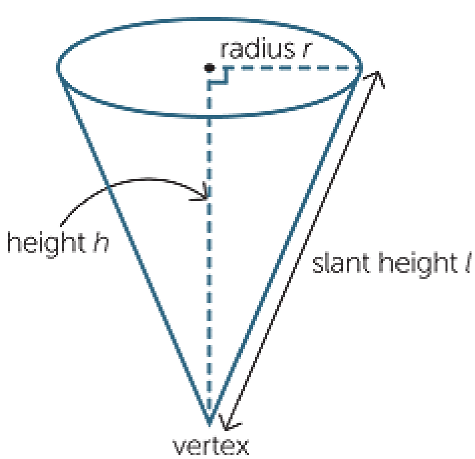
To create a cone we take a circle and a point, called the vertex, which lies above or below the circle. We then join the vertex to each point on the circle to form a solid.



If the vertex is directly above or below the centre of the circular base, we call the cone a right cone. In this section only right cones are considered.

If we drop a perpendicular from the vertex of the cone to the circular base, then the length of this perpendicular is called the height h of the cone.

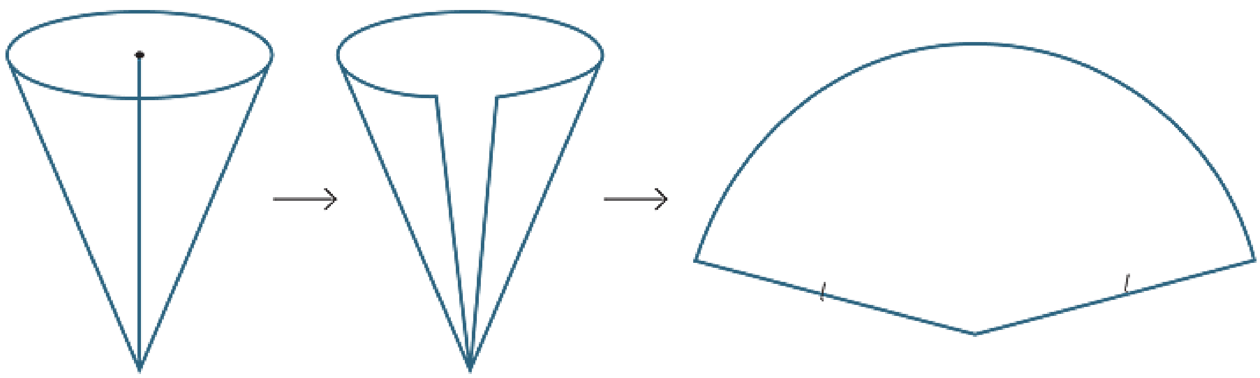
The length of any of the straight lines joining the vertex to the circle is called the slant height of the cone. Clearly l2 = r2 + h2 , where r is the radius of the base.



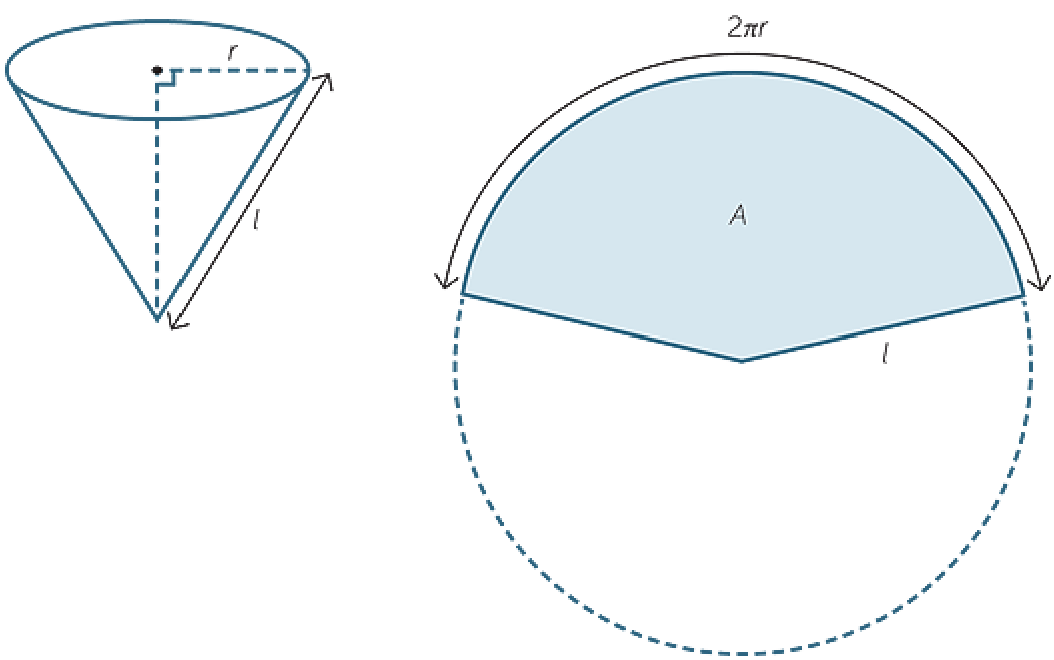
Surface area of a cone

Suppose the cone has radius r, and slant height l, then the circumference of the base of the cone is 2πr.

To find the area of the curved surface of a cone, we cut and open up the curved surface to form a sector with radius l, as shown below.



In the figure to the right below the ratio of the area of the shaded sector to the area of the circle is the same as the ratio of the length of the arc of the sector to the circumference of the circle.



Thus the fraction of the area of the whole circle taken up by the sector is

$$\frac{\text{circumference of the sector}}{\text{circumference of the circle}} = \frac{2\pi r}{2\pi l}.$$

Hence, the area of the sector is $\frac{2\pi r}{2\pi l} \times \pi l2 = \pi rl$.

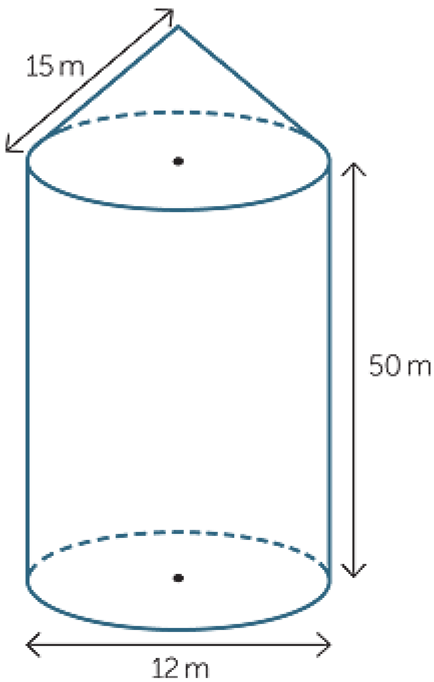
In conclusion, the area of the curved surface of the cone is πrl

Adding this to the base, we have

Surface area of a cone = $\pi rl + \pi r2$.

EXERCISE

Find the surface area of the solid with dimensions shown.



__Volume of a cone__

When developing the formula for the volume of a cylinder in the module Area Volume and Surface Area, we approximated the cylinder using inscribed polygonal prisms. By taking more and more sides in the polygon, we obtained closer and closer approximations to the volume of the cylinder. From this, we deduced that the volume of the cylinder was equal to the area of the base multiplied by the height.

We can use a similar approach to develop the formula for the volume of a cone.

Given a cone with base radius r and height h, we construct a polygon inside the circular base of the cone and join the vertex of the cone to each of the vertices of the polygon, producing a polygonal pyramid. By increasing the number of sides of the polygon, we obtain closer and closer approximations to the cone. Hence,

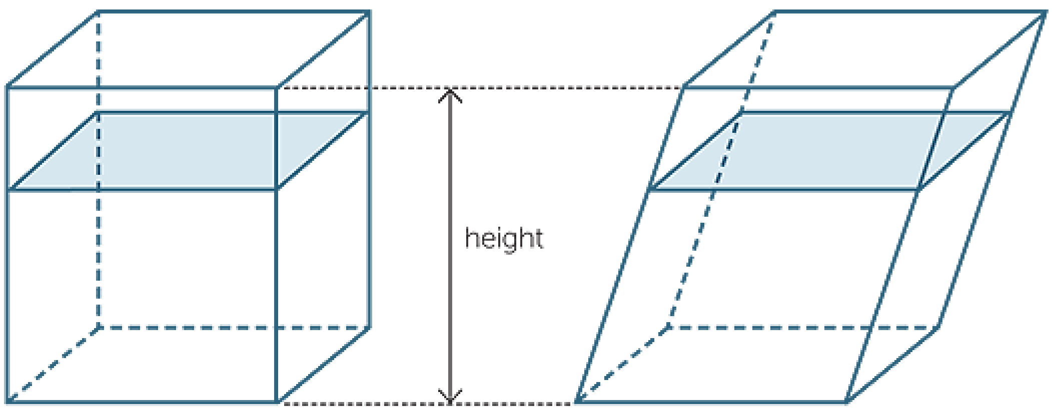
Volume of a cone = $\frac{1}{3}$ × area of the base × height

= $\frac{1}{3}$πr2h

OBLIQUE PRISMS, CYLINDERS AND CONES

We have seen that the volume of a right rectangular prism is area of the base multiplied by the height. What happens if the base of the prism is not directly below the top?

Cavalieri's first principle states that if the cross-sections of two solids, taken at the same distance above the base, have the same area, then the solids have the same volume.

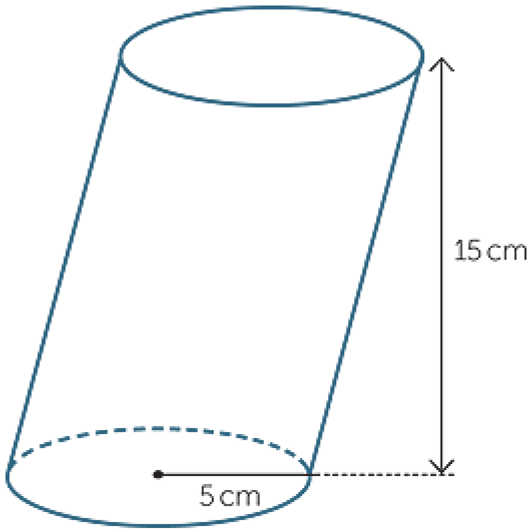


We will not give a proof of Cavalieri's principle here. To present a rigorous proof requires integration and slicing ideas.

It allows us to say that the volume of any rectangular prism, right or oblique, is given by the area of the base multiplied by the height.

The same applies to oblique cylinders and cones.

EXAMPLE



Find the volume of the cylinder shown in the diagram.

SOLUTION

The volume of the cylinder is V = π × 52 × 15 = 375π cm2.

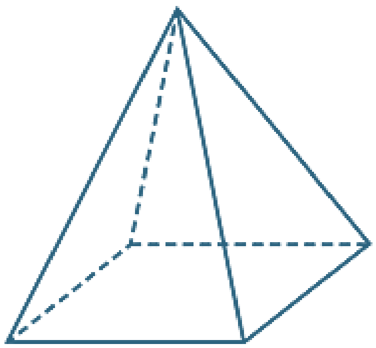
Cavalieri's second principle states that if the cross-sections of two solids, taken at the same distance above the base, have areas in the ratio a:b, then the solids have the volumes in the ratio a:b.

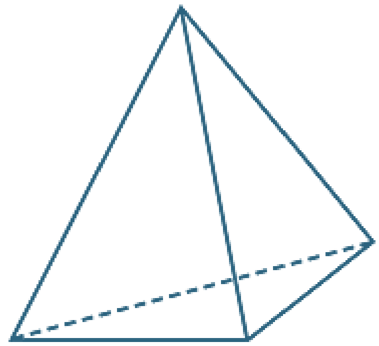
EXERCISE 6

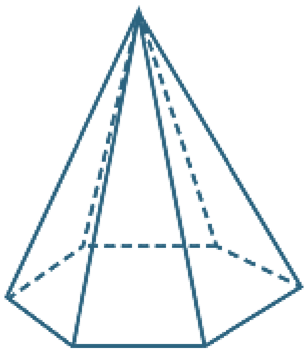
We showed earlier that the volume of a square based pyramid with base length 2x and height x has volume $\frac{1}{3}$ × 2x × 2x × x. Use Cavalieri's second principle to show that the volume of a pyramid whose base is a rectangle with side lengths c and d and height h is $\frac{1}{3}$ × cd × x.

PYRAMIDS

A pyramid is a polyhedron with a polygonal base and triangular faces that meet at a point called the vertex. The pyramid is named according to the shape of the base.





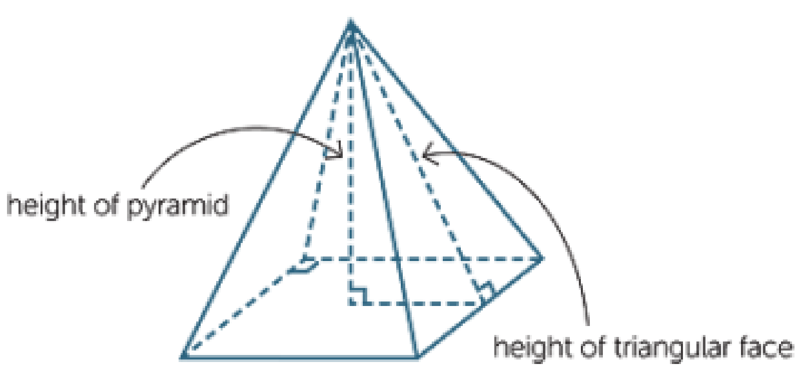


square-based pyramid triangular-based pyramid hexagonal-based pyramid

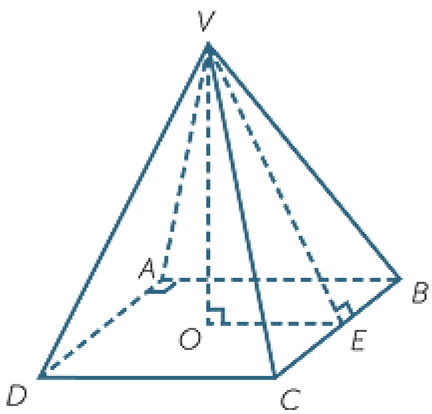
If we drop a perpendicular from the vertex of the pyramid to the base, then the length of the perpendicular is called the height of the pyramid.

Surface area of a right pyramid

The faces bounding a right pyramid consist of a number of triangles together with the base. To find the surface area, we find the area of each face and add them together. Depending on the information given, it may be necessary to use Pythagoras' Theorem to calculate the height of each triangular face. If the base of the pyramid is a regular polygon, then the triangular faces will be congruent to each other.

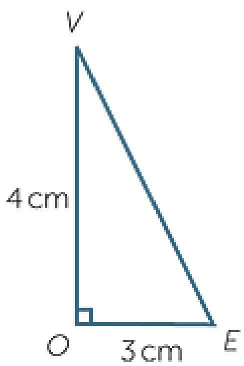


EXAMPLE



VABCD is a square-based pyramid with vertex V and base ABCD, with V vertically above the centre of the square base. The height of the pyramid is 4 cm and the side length of the base is 6 cm, find the surface area of the pyramid.

SOLUTION



We need to find the height VE of triangle VBC, using Pythagoras' Theorem.

VE2 = VO2 + OE2
= 42 + 32
= 25

Hence VE = 5 cm.

Area of VCB = $\frac{1}{2}$ × CB × VE
= $\frac{1}{2}$ × 6 × 5
= 15 cm2

Area of base = 6 × 6
= 36 cm2

Surface area = 4 × 15 + 36
= 96 cm2

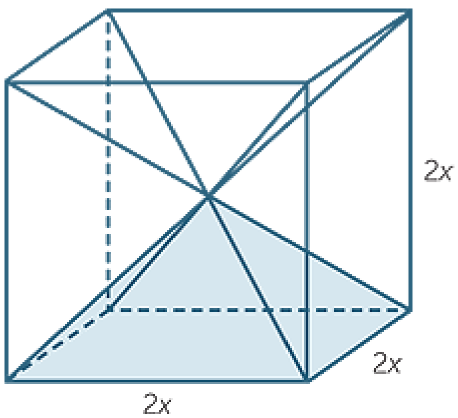
The surface area of the pyramid is 96 cm2.

EXERCISE 1

When it was built, the Great Pyramid of Cheops in Egypt had a height of 145.$\frac{3}{4}$ m and its base was a square of side length 229 m. Find its surface area in square metres, correct to three significant figures.

Volume of a pyramid

Here is a method for determining the formula for the volume of a square-based pyramid.



Consider a cube of side length 2x. If we draw the four long diagonals as shown, then we obtain six square-based pyramids, one of which is shaded in the diagram.

Each of these pyramids has base area 2x × 2x and height x. Now the volume of the cube is 8x3. So the volume of each pyramid is $\frac{1}{6}$ × 8x3 = $\frac{4}{3}$x3. Since the base area of each pyramid is 4x2 it makes sense to write the volume as

Volume = $\frac{1}{3}$ × 4x2 × x = $\frac{1}{3}$ × area of the base × height.

We can extend this result to any pyramid by using a geometric argument, giving the following important result.

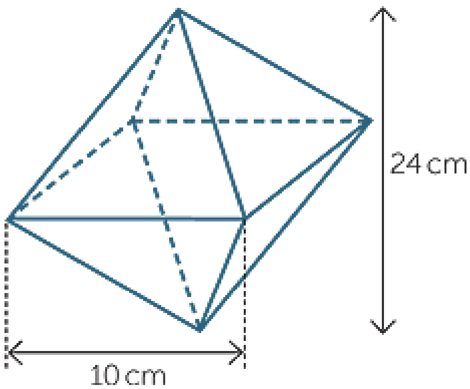
Volume of a pyramid = $\frac{1}{3}$ × area of the base × height.

See the appendix on the pyramid for details.

EXERCISE 2

Find the volume of the Great Pyramid of Cheops whose height is 145.75 m and whose base is a square of side length 229 m. Give answer in cubic metres correct to two significant figures.

EXERCISE 3



Find the volume of the 'diamond', with height 24 cm and side length 10 cm as shown.

Lesson Two

How to Prove that the sum of the angles in a triangle is 180 degrees

Theorem
If ABC is a triangle then <)ABC + <)BCA + <)CAB = 180 degrees.

Proof
Draw line a through points A and B. Draw line b through point C and parallel to line a.

Since lines a and b are parallel, <)BAC = <)B'CA and <)ABC = <)BCA'.
It is obvious that <)B'CA + <)ACB + <)BCA' = 180 degrees.
Thus <)ABC + <)BCA + <)CAB = 180 degrees.

Lemma
If ABCD is a quadrilateral and <)CAB = <)DCA then AB and DC are parallel.

Proof
Assume to the contrary that AB and DC are not parallel.
Draw a line trough A and B and draw a line trough D and C.
These lines are not parallel so they cross at one point. Call this point E.

Notice that <)AEC is greater than 0.
Since <)CAB = <)DCA, <)CAE + <)ACE = 180 degrees.
Hence <)AEC + <)CAE + <)ACE is greater than 180 degrees.
Contradiction. This completes the proof.

Definition
Two Triangles ABC and A'B'C' are congruent if and only if
|AB| = |A'B'|, |AC| = |A'C'|, |BC| = |B'C'| and,
<)ABC = <)A'B'C', <)BCA = <)B'C'A', <)CAB = <)C'A'B'.

Week 3

GEOMETRICAL CONSTRUCTION

When making a geometrical constructions, it is important to remember to do the following-

1. Use a hard pencil with a sharp point. This gives thin lines which are more accurate.

2. Check that your ruler has good straight edge. A damaged ruler is useless for construction work.

3. Check that your compasses are not too loose. Tighten loose compasses with a small screw driver.

4. All construction lines must be seen. Do not rub out anything which leads to the final result.

5. Always take great care, especially when drawing a line through a point.

6. Where possible, arrange that the angles of intersection between lines and arcs are about 90^{o}.

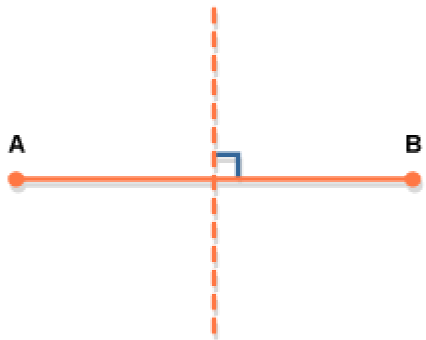
Perpendicular bisector of a line segment

The locus of a point which moves so that it is an equal distance from two points, A and B, is the perpendicular bisector of the line joining A and B.

Perpendicular means **at right angles to**.

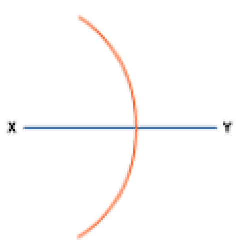
Bisector means **cuts in half**.

To construct this locus, you do the following (try this yourself on a piece of paper):

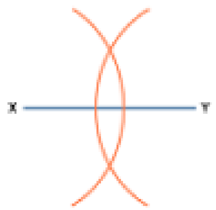


Draw the line segment XY.

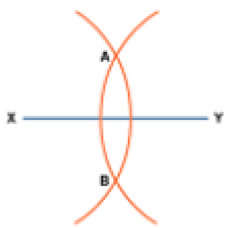
Put your compass on X and set it to be over half way along the line. Draw an arc.



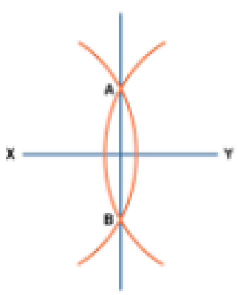
Without adjusting your compass put it on Y and draw another arc.



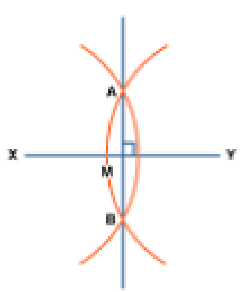
Label these points A and B.



Draw a straight line through A and B.

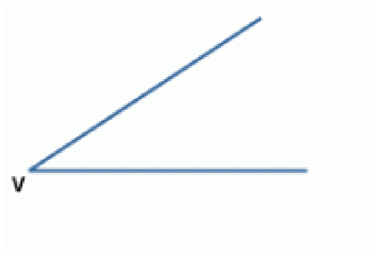


The point M where the lines cross is the midpoint of XY. And AB is perpendicular to XY.

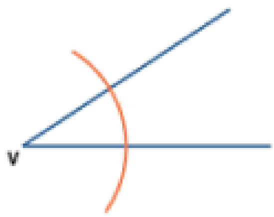


Bisecting an angle

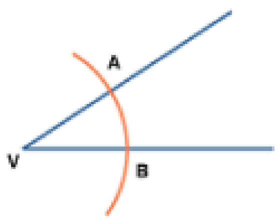
V is the vertex of the angle we want to bisect.



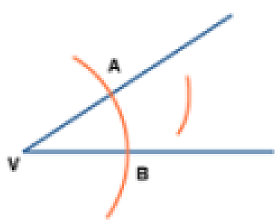
Place your compass on V and draw an arc that crosses both sides of the angle.



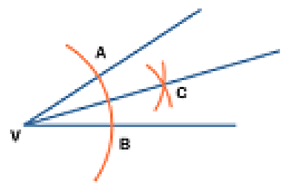
Label the crossing points A and B.



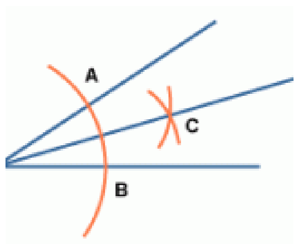
Place your compass on A and draw an arc between the two sides of the angle.



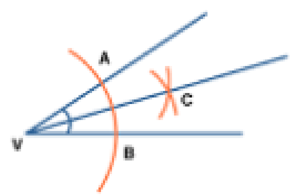
Without adjusting your compass place it on B and draw another arc that cuts the one you just drew. Label the point where they cross C.



Draw a straight line through V and C.



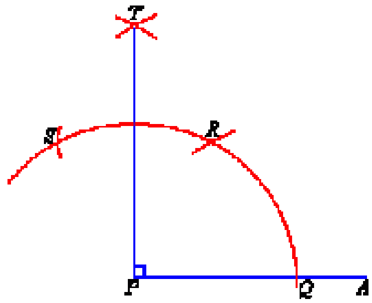
The line VC bisects the angle. Angles AVC and BVC are equal.



Constructing a 90° Angle

We can construct a 90° angle either by bisecting a straight angle or using the following steps.

Step 1: Draw the arm *PA*.
Step 2: Place the point of the compass at *P* and draw an arc that cuts the arm at *Q*.
Step 3: Place the point of the compass at *Q* and draw an arc of radius *PQ* that cuts the arc drawn in Step 2 at *R*.
Step 4: With the point of the compass at *R*, draw an arc of radius *PQ* to cut the arc drawn in Step 2 at *S*.
Step 5: With the point of the compass still at *R*, draw another arc of radius *PQ* near *T* as shown.
Step 6: With the point of the compass at *S*, draw an arc of radius *PQ* to cut the arc drawn in step 5 at *T*.
Step 7: Join *T* to *P*. The angle *APT* is 90°.

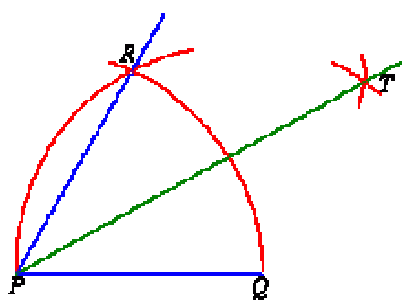


Constructing a 30° Angle

We know that: ½ of 60° = 30°

So, to construct an angle of 30°, first construct a 60° angle and then bisect it. Often, we apply the following steps.

Step 1: Draw the arm *PQ*.
Step 2: Place the point of the compass at *P* and draw an arc that passes through *Q*.
Step 3: Place the point of the compass at *Q* and draw an arc that cuts the arc drawn in Step 2 at *R*.
Step 4: With the point of the compass still at *Q*, draw an arc near *T* as shown.
Step 5: With the point of the compass at *R*, draw an arc to cut the arc drawn in Step 4 at *T*.
Step 6: Join *T* to *P*. The angle *QPT* is 30°.



Constructing a 60° Angle

We know that the angles in an equilateral triangle are all 60° in size. This suggests that to construct a 60° angle we need to construct an equilateral triangle as described below.

Step 1: Draw the arm *PQ*.
Step 2: Place the point of the compass at *P* and draw an arc that passes through *Q*.
Step 3: Place the point of the compass at *Q* and draw an arc that passes through *P*. Let this arc cut the arc drawn in Step 2 at *R*.

Step 4: Join P to R. The angle QPR is 60^{0}, as the ΔPQR is an equilateral triangle.

Try your understanding regarding the explanations above over and over again.

ASSESSMENT

1. What are the important steps to take when doing geometric construction?
2. Construct a **90^{0}** angle.

Triangle: Drawing and Bisection of Line Segment; Construction and Bisection of Angles

Bisection means to divide the line segment in two equal parts. In the real world, the majority of lines we see are line segments since they all have an end and a beginning. We can define a **line segment** as a line with a beginning and an end point. Below are some of instructions and procedures to follow when constructing a line segment-

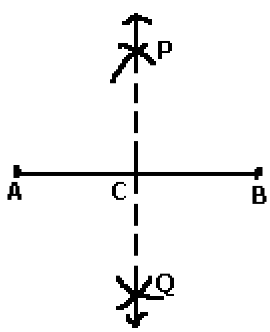
Draw a line segment measuring 5 cm and its perpendicular bisector. Write the steps of construction.

Step 1 : Draw a line segment AB of length 5 cm.

Step 2 : Mark two arcs with radius more than half of AB with centers A and B respectively.

Step 3 : These arcs intersect at P and Q respectively.
Step 4 : Join P and Q.

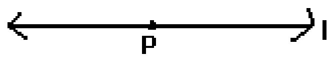
Step 5 : PQ is required perpendicular bisector of line segment AB.



Construction of a line perpendicular to a given line at a given point using Ruler and Compass

Draw any line segment AB . Mark any point P on it. Through P, draw a perpendicular to segment AB with the help of ruler and compasses.

Step 1: Given a point P on a line l.

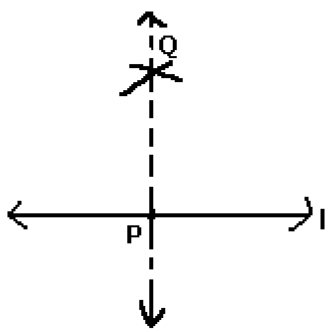


Step 2: With P as center and a convenient radius, construct an arc intersecting the line l at two points A and B.

Step 3: With A and B as centers and a radius greater than AP construct two arcs, which cut each other at Q.

Step 4: Join PQ.

Thus PQ is perpendicular to l.



Construction of a line perpendicular to a given line and passing through a given point not lying on it by using ruler and compasses.

Draw any line segment AB . Take any point P outside it. Through P, draw a perpendicular to segment AB.

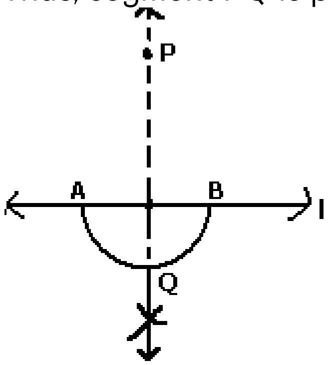
Step 1: Given a line l and a point P outside it.

Step 2: With P as center, draw an arc, which intersects line l at two points A and B.

Step 3: Using the same radius and with A and B as centers, construct two arcs that intersect at a point, say Q, on the other side.

Step 4: Join PQ.

Thus, segment PQ is perpendicular to line l.



Geometrical Constructions

- Basic Geometric Constructions
- Construction of Line Segment
- Bisecting a Line Segment
- Constructing Angles
- Bisecting Angles
- Constructing Parallel Lines
- Construction of Triangle (SSS)
- SAS Triangle Construction
- ASA Triangle Construction
- HL Triangle Construction (Rhs -construction)
- Constructing Quadrilaterals
- Constructing Triangles(when sum of sides or perimeter is given)

<u>Construction and Bisect of Angles</u>

Definition The **bisector of an angle** is a ray whose end point is the vertex of the angle and which divides the angle into two equal angles.In the diagram to the right, the ray CD is the bisector of the angle ACB if and only if the angles ACD and BCD have equal measures.	

Bisect Angle. To construct the Angle Bisector of an angle follow the following steps.

Given. An angle to bisect. For this example, angle ABC.

Step 1. Draw an arc that is centered at the vertex of the angle. This arc can have a radius of any length. However, it must intersect both sides of the angle. We will call these intersection points **P** and **Q** This provides a point on each line that is an equal distance from the vertex of the angle.

Step 2. Draw two more arcs. The first arc must be centered on one of the two points **P** or **Q**. It can have any length radius. The second arc must be centered on whichever point (P or Q) you did NOT choose for the first arc. The radius for the second arc MUST be the same as the first arc. Make sure you make the arcs long enough so that these two arcs intersect in at least one point. We will call this intersection point **X**. Every intersection point between these arcs (there can be at most 2) will lie on the angle bisector.

Step 3. Draw a line that contains both the vertex and **X**. Since the intersection points and the vertex all lie on the angle bisector, we know that the line which passes through these points **must** be the angle bisector.

Now, try to do this construction yourself.

Applet Instructions

- **Drawing lines**. Start by depressing the ruler button. Then, click on the point where the line should begin. You can then move the mouse to the other point and click again.
- **Drawing arcs**. Start by depressing the compass button. Click on the center of the arc. Use the up and down arrow keys to increase or decrease the angle of the arc (or use the method listed below). Click again to place the arc.
- **Drawing arcs with same radius**. If you hold down the "Shift" key when you select the first point of the arc the radius of your new arc will be same as that of last arc drawn.
- **Selecting Items**. Make sure both the ruler and compass are not depressed. Then, you can select items by clicking on them. The color of the marks will change from red to green. To deselect something click on it again.
- **Adjusting lines and arcs**. After placing a line or arc you can make adjustments to them. First, selecting the object you want to change. Then, by clicking (not holding down mouse button) on different points you can make different adjustments.
    - Lines. Clicking on either endpoint of a line will release that point and thus allow you to move.

- Arcs. When you select an arc four points will be drawn in addition to the arc. By clicking on each of these points you can modify a different aspect of the arc.
    1. Center of the circle containing the arc. You can move the center while leaving the center of the curve in the same place. If you hold down the shift key while moving this point, the entire arc will move and keep the same relative position to the center. Be careful not to move the curve off of the screen.
    2. Center of the curve. You can move the position of the curve while leaving the center of the circle in the same place. If you hold down the shift key while you are moving, the radius is remain constant.
    3. Ends of the curve. You can adjust the length of the curve.

The best way to see what can be adjusted is to just try things.

- **Starting Over**. To start over click the reset button.
- **Checking construction**. To check your construction select the "Check" button. A message indicating the first error that is found or that the construction is correct will be shown in the window to the right.

ASSESSMENT

1. Define bisection
2. What is the bisector of an angle?

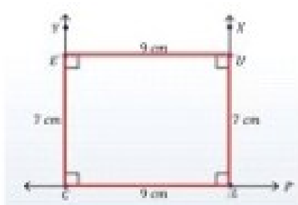
Week 4

Construction: Construction of Quadrilateral Polygon; Construction of Equilateral Triangle; Locus of Moving Points

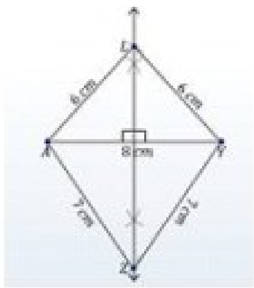
Topic: Construction of Quadrilateral Polygon

We can identify different **quadrilaterals** based on the properties sides, diagonals and angles.
Quadrilaterals are made up of ten parts. However, to construct them, you do not need to know the measurements of all of them.
In case of special **quadrilaterals**, like the **rectangle**, just two measurements, the lengths of its adjacent sides are enough to construct it.
A **kite** can be constructed if the lengths of its distinct adjacent sides and one diagonal are known.
Similarly, a **square** can be constructed with just the length of its side, while a **rhombus** can be constructed when the lengths of its diagonals are known.

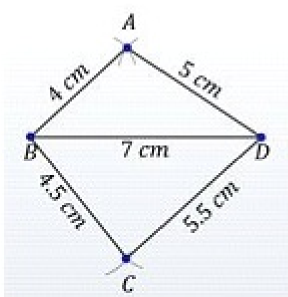
Steps to Construct a Rectangle:
Step 1: Draw a side of given length (say) CL
Step 2: Draw side LU (say) of given length perpendicular to CL at L.
Step 3: Draw side CE (say) of length equal to LU and perpendicular to CL at C.
Step 4: Draw side UE.



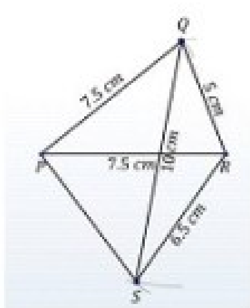
Steps to Construct a Kite:
Step 1: Draw diagonal (say) AY and its perpendicular bisector.
Step 2: Draw sides say AL and AZ of given length.
Step 3: Draw sides LY and YZ.



Constructing a Quadrilateral when four sides and one of its diagonals are given.
Step 1: Construct a triangle ABD (say).
Step 2: Find point C opposite to the vertex A as follows. With B as the centre and given radius, draw an arc on the other side of BD. Similarly with D as the centre and given radius, draw another arc intersecting the previous arc. The point of intersection of these arcs is marked as C.
Step 3: join points B and C, and D and C.

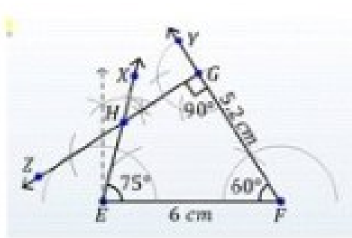


Constructing a quadrilateral when lengths of its three sides and two diagonals are given.
Step 1: Construct a triangle PQR (say).

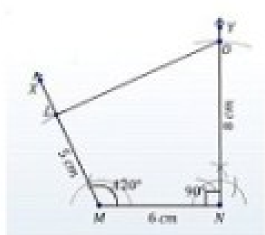


Constructing a quadrilateral when lengths of its adjacent sides and three angles are given.
Step 1: Draw a line segment of EF (say) of given length.
Step 2: Construct a given angle at E.
Step 3: Construct a given angle at F.

Step 4: Locate point G.
Step 5: Locate point H.



Constructing a quadrilateral when lengths of its three sides and two included angles.
Step 1: Draw a line segment MN (say) of given length.
Step 2: Construct a given angle at M.
Step 3: Construct an angle 90° at N.
Step 4: Locate vertices L and O.
Step 5: Join L and O.



Constructing an equilateral triangle

Constructing an equilateral triangle also known as drawing an equilateral triangle using only a straightedge and a compass is what I will show you here

Step #1:

Take your ruler and a pencil and construct a segment of any length on a piece of paper as shown below

Then, you will try to set your compass opening to match the length of segment AB

Take your compass. Make your sure that the pencil is included in it.

Put the needle of the compass at endpoint A and adjust your compass so that the tip of your pencil touches endpoint B

Step #2:

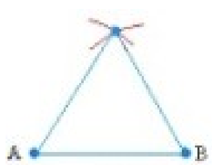
Put the needle of your compass at A and draw an arc

Put the needle of your compass at B and draw an arc

The two arcs should meet as shown below:

Step #3:

Draw the segments from the two endpoints to the point where the two arcs intersect



Locus of Moving Points Including Equidistance from Two Lines of Two Points and Constant Distant From the Point

When a point moves in a plane according to some given conditions the path along which it moves is called a **locus.** (Plural of locus is **loci.**).

CONDITION 1:
A point *P* moves such that it is always *m* units from the point *Q*

Locus formed: A circle with centre Q and radius *m.*

Example :

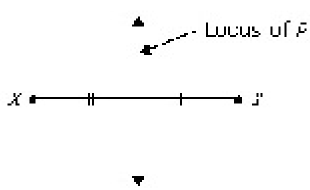
Construct the locus of a point *P* at a constant distance of 2 cm from a fixed point *Q.*

Solution:

Construct a circle with centre *Q* and radius 2 cm.

CONDITION 2:
A point *P* moves such that it is equidistant from two fixed points *X* and *Y*

Locus formed: A perpendicular bisector of the line *XY.*

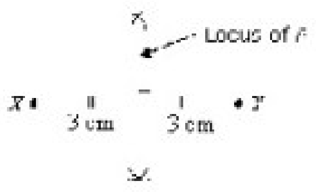


Example:

Construct the locus of point *P* moving equidistant from fixed points *X* and *Y* and *XY* = 6 cm.

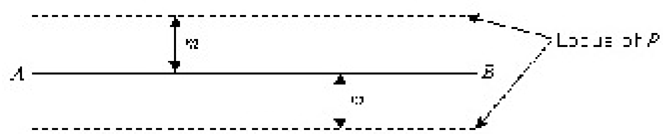
Solution:

Construct a perpendicular bisector of the line XY.



CONDITION 3:
A point P moves so that it is always m units from a straight line AB

Locus formed: A pair of parallel lines *m* units from *AB*.

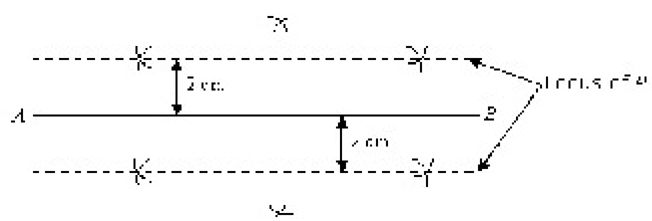


Example:

Construct the locus of a point *P* that moves a constant distant of 2 cm from a straight line *AB.*

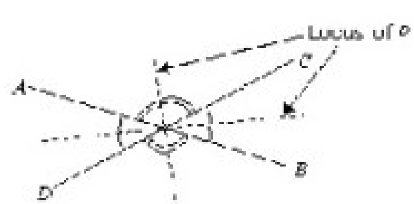
Solution:

Construct a pair of parallel lines 2 cm from *AB.*



CONDITION 4:
A point P moves so that it is always equidistant from two intersecting lines AB and CD

Locus formed: Angle bisectors of angles between lines *AB* and *CD*.

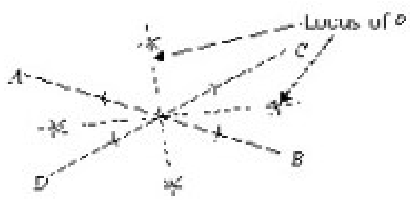


Example:

The following figure shows two straight lines *AB* and *CD* intersecting at point *O*. Construct the locus of point *P* such that it is always equidistant from *AB* and *CD.*

Solution:

Construct angles bisectors of angles between lines *AB* and *CD.*



ASSESSMENT

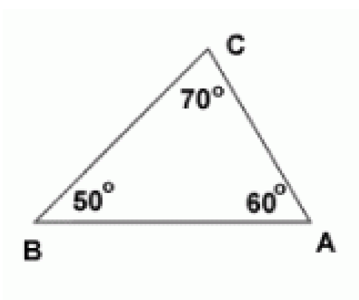
1. Define a quadrilateral
2. Construct a quadrilateral

Week 5

Deductive Proof: Sum of Angles of a Triangle; Revision of Angles on Parallel Line cut by a Transversal Line

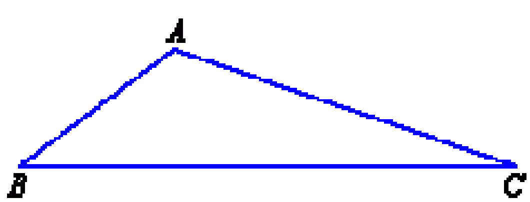
Angles can be Added

Just like regular numbers, angles can be added to obtain a sum, perhaps for the purpose of determining the measure of an unknown angle. Sometimes we can determine a missing angle because we know that the sum must be a certain value. Remember — the sum of the degree measures of angles in any triangle equals 180 degrees. Below is a picture of triangle ABC, where angle A = 60 degrees, angle B = 50 degrees and angle C = 70 degrees.



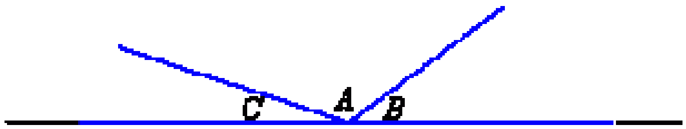
If we add all three angles in any triangle we get 180 degrees. So, the measure of angle A + angle B + angle C = 180 degrees. This is true for any triangle in the world of geometry. We can use this idea to find the measure of angle(s) where the degree measure is missing or not given.

Draw a triangle *ABC* and cut out the three angles.



Step 2:

Rearrange the three angles to form a straight angle on a straight line.



Angle A + Angle B + Angle $C = 180°$

So, the angle sum of a triangle is 180°.

Task 2

To investigate if this works for all triangles, repeat the above process for four different triangles.

Task 3

Copy and complete the following statements:

a. The three angles of a triangle can be arranged to form a .... angle.

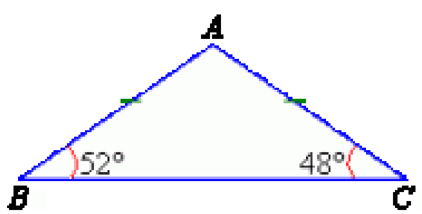
b. The sum of three angles in a triangle is ....

Finding the Third Angle of a Triangle

If the measurements of two angles of a triangle are known, then the third angle can be calculated.

Example 3

Calculate the size of the missing angle in the following triangle.



Solution:

Let the missing angle be *x*.

$\therefore x + 52° + 48° = 180°$ ($\because$ Angle sum of a triangle is 180°)

$x + 100° = 180°$

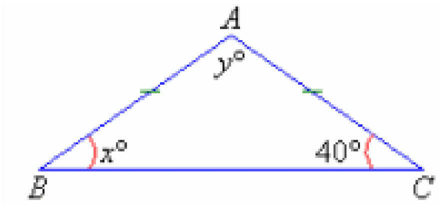
$x + 100° - 100° = 180° - 100°$

$x = 80°$

So, the missing angle is 80°.

Example 4

Find the values of the pronumerals x and y in the following diagram:



Solution:

In $\Delta\ ABC$, $AB = AC$

$\therefore\ x = 40$ (Base angles of isosceles triangle)

Also, $x + 40 + y = 180$ {∵ Angle sum of a triangle is 180°}

$40 + 40 + y = 180$ {∵ $x = 40$ as obtained above}

$80 + y = 180$

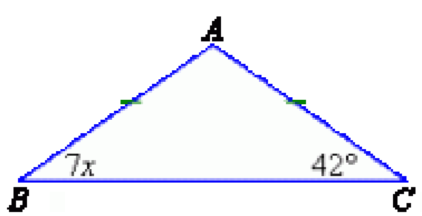
$80 + y - 80 = 180 - 80$

$y = 100$

So, $x = 40$, $y = 100$.

Example 5

Find the value of the pronumeral x in the following diagram:



Solution:

In $\Delta\ ABC$, $AB = AC$

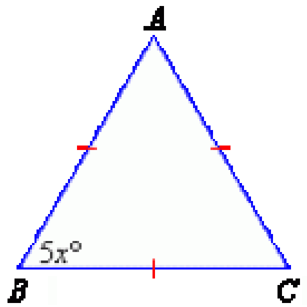
$\therefore 7x = 42°$ (Base angles of isosceles triangle)

$$\frac{7x}{7} = \frac{42°}{7}$$

$$x = 6°$$

Example 6

Find the value of the pronumeral *x* in the following diagram:



Solution:

In $\Delta\ ABC$, $AB = BC = CA$ (Given)

$\therefore \angle A = \angle B = \angle C = 60°$ ($\because$ Each angle $= 60°$ in an equilateral triangle)

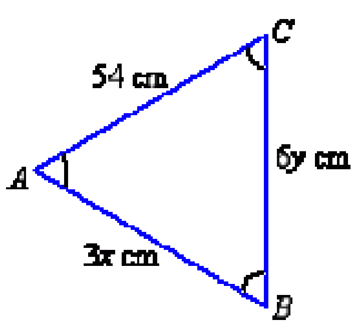
$\therefore 5x = 60$

$$\frac{5x}{5} = \frac{60}{5}$$

$$x = 12$$

Example 7

Find the values of the pronumerals *x* and *y* in the following diagram:



Solution:

In $\triangle ABC, \angle A = \angle B = \angle C$ (Given)

$\therefore AB = AC$ {$\because$ ABC is an equilateral triangle}

$\therefore 3x = 54$

$\frac{3x}{3} = \frac{54}{3}$

$x = 18$

Also, $BC = AC$ {$\because$ ABC is an equilateral triangle}

$\therefore 6y = 54$

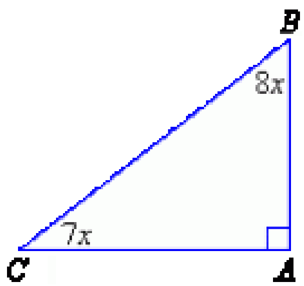
$\frac{6y}{6} = \frac{54}{6}$

$y = 9$

So, $x = 18, y = 9.$

Example 8

Find the value of the pronumeral x in the following diagram:



Solution:

$7x + 8x + 90^\circ = 180^\circ$ {$\because$ Angle sum of a triangle is 180°}

$15x + 90^\circ = 180^\circ$ {Subtract 90° from both sides}

$15x + 90^\circ - 90^\circ = 180^\circ - 90^\circ$

$15x = 90^\circ$ {Divide both sides by 15}

$\frac{15x}{15} = \frac{90^\circ}{15}$

$x = 6^\circ$

<u>Revision of Angles on Parallel Line cut by a Transversal Line</u>

A Line Crosses A Pair of Parallel Lines

If a set of 2 parallel lines, line l and line m, are crossed or cut by another line, line n, we say "a set of parallel lines are cut by a transversal."

Each of the parallel lines cut by the transversal has 4 angles surrounding the intersection.

These are matched in measure and position with a counterpart at the other parallel line.

At each of the parallel lines, there are two pairs of vertical angle. Each angle in the pair is congruent to the other angle in the pair.

1 4, angle 1 is congruent to angle 4.
2 3, angle 2 is congruent to angle 3.
5 8, angle 5 is congruent to angle 8.
6 7, angle 6 is congruent to angle 7.

Adjacent Supplementary Angles At each of the parallel lines adjacent angles are supplementary.

Names for the Matched Angles

The angles have special names identifying their positions with respect to the parallel lines and transversal.

They are corresponding angles, alternate interior angles, or alternate exterior angles.

An angles is congruent to its matched angle.

ASSESSMENT

- What is the total angles in a triangle?

Week 6

Collection, Tabulation and Presentation of Grouped Data

In some investigations you may collect an awful lot of information. How can you use this raw data and make it meaningful? This section will help you to collect, organise and interpret the data efficiently.

Explaining your results

Imagine that you are asked to carry out a survey to find the number of pets owned by pupils in your school. You decide to ask 50 people, and record your results as follows:

0 2 1 2 0 4 1 0 2 2 1 6 1 1 2 8 0 12

2 1 2 0 3 2 0 1 3 0 1 4 0 3 0 2 3 6

3 3 0 1 2 0 1 1 3 0 2 0 3 2

You now have the information you need, but is this the most efficient way to collect and display the data?

Tallying

Tallying is a method of counting using groups of five.

|= 1

||= 2

||| = 3

|||| = 4

卌= 5

卌| = 6

卌|| = 7

卌||| = 8

卌|||| = 9

卌卌 = 10

Because we have used groups of five, it is easy to find the total.

Question

Which numbers do the following tally marks represent?

a.) 𝍸𝍸 |||

b.) 𝍸𝍸 𝍸 𝍸 ////

Answer

𝍸𝍸 ||| =13

𝍸𝍸𝍸𝍸 |||| =24

Using the tally system to record results is much faster than writing out words or figures all the time. For example, if you had to investigate the most popular type of vehicle that passed the school gates, it would be easier to draw tally marks in one of three columns than write: car, car, lorry, bike, car, car, and so on.

By using a tally chart, the data is already collected into groups, and will not require further grouping at a later date.

Collecting data

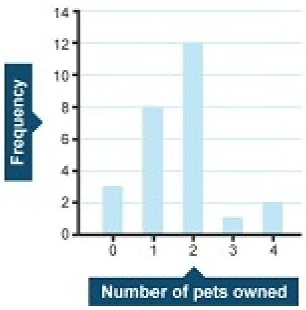
The easiest way to collect data is to use a tally chart.

When collecting data for the number of pets survey, it would have been useful to draw a table similar to this one.

As each person answers the question, we put a tally next to the appropriate number of pets. The frequency column is completed once all of the data has been collected. The table below shows the results of a new pets survey.

Number of Pets	Tally	Frequency
0	III	3
1	𝍸 III	8
2	𝍸𝍸 II	12
3	I	1
4	II	2

These frequencies can be displayed in a bar chart, as shown.



Frequency means the 'number of times it occurs'.

In this example, three people had no pets, so the frequency of 0 pets was three.

Remember that the total frequency should be the same as the number of people in your survey. Always check that this is correct.

In this example, we know that 26 people were questioned in the survey. Check this by adding up the frequency totals: 3 + 8 + 12 + 1 + 2 = 26

Here is the same information but this time we have two tables, one for the number of pets owned by boys and one for the number of pets owned by girls.

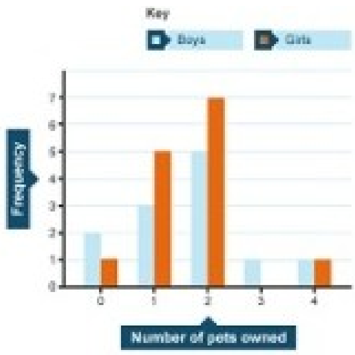
Number of pets owned by boys

Number of Pets	**Tally**	**Frequency**
0	II	2
1	III	3
2	卌	5
3	I	1
4	I	1

Number of pets owned by girls

Number of Pets	**Tally**	**Frequency**
0	|	1
1	卌	5
2	卌||	7
3	|	0
4	|	1

These frequencies can be displayed in a dual bar chart.



We can find more information from looking at this graph.

Question

How many pets were owned by the same number of boys as for girls?

Answer

Four pets.

We can see that the height of the bar is the same for both boys and girls for four pets.

Question

How many more girls than boys were there in the survey?

Answer

Two more girls.

Adding the bars for girls and for boys we find:

the total for girls is: 1 + 5 + 7 + 0 + 1 = 14

the total boys is: 2 + 3 + 5 + 1 + 1= 12

Grouping data

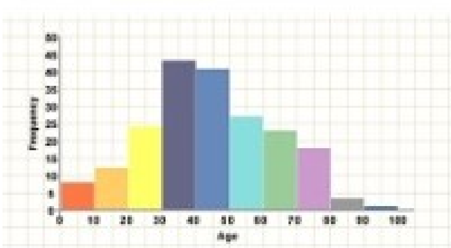
When a large amount of data has to be collected, use a **grouped** frequency distribution.

The following tally chart represents the ages of 200 people entering a park on a Saturday afternoon.

The ages have been grouped into the classes 0-9, 10-19, 20-29, and so on.

Age	**Tally**	**Frequency**
0-9	𝍸𝍸 III	8
10-19	𝍸𝍸 II	12
20-29	𝍸𝍸𝍸𝍸 IIII	24
30-39	𝍸𝍸𝍸𝍸𝍸𝍸 𝍸𝍸 III	43
40-49	𝍸𝍸𝍸𝍸𝍸𝍸 𝍸𝍸 I	41
50-59	𝍸𝍸𝍸𝍸𝍸II	27
60-69	𝍸𝍸𝍸𝍸 III	23
70-79	𝍸𝍸𝍸 III	18
80-89	III	3
90-99	I	1

These frequencies can also be shown in a histogram.



ASSESSMENT

1. What is tallying?
2. What is frequency?

Week 7

<u>Calculation of Range, Median and Mode of Ungrouped Data</u>

<u>Range of Ungrouped Data</u>

The Range is the difference between the lowest and highest values.

Example: In **{4, 6, 9, 3, 7}** the lowest value is 3, and the highest is 9.

So the range is 9 − 3 = **6**.

It is that simple!

But perhaps too simple …

<u>The Range Can Be Misleading</u>

The range can sometimes be misleading when there are extremely high or low values.

Example: In **{8, 11, 5, 9, 7, 6, 3616}**:

- the lowest value is 5,
- and the highest is 3616,

So the range is 3616-5 = **3611**.

The single value of 3616 makes the range large, but most values are around 10.

So we may be better off using Interquartile Range or Standard Deviation.

Range of a
Function

Range
can **also** mean
all the output
values of a
function, see
Domain,

Range and
Codomain.

Problem: Cheryl took 7 math tests in one marking period. What is the range of her test scores?

89, 73, 84, 91, 87, 77, 94

Solution: Ordering the test scores from least to greatest, we get:

73, 77, 84, 87, 89, 91, 94

highest – lowest = 94 – 73 = 21

Answer: The range of these test scores is 21 points.

Definition: The **range** of a set of data is the difference between the highest and lowest values in the set.

In the problem above, the set of data consists of 7 test scores. We ordered the data from least to greatest before finding the range. We recommend that you do this, too. This is especially important with large sets of data. Let's look at some more examples.

Example 1: The Jaeger family drove through 6 midwestern states on their summer vacation. Gasoline prices varied from state to state. What is the range of gasoline prices?

N1.79, N1.61, N1.96, N2.09, N1.84, N1.75

Solution: Ordering the data from least to greatest, we get:

N1.61, N1.75, N1.79, N1.84, N1.96, N2.09

highest – lowest = N2.09 – N1.61 = N0.48

Answer: The range of gasoline prices is $0.48.

Example 2: Ms. Kaiser listed 9 integers on the blackboard. What is the range of these integers?

14, $^{-}12$, 7, 0, $^{-}5$, $^{-}8$, 17, $^{-}11$, 19

Solution: Ordering the data from least to greatest, we get:

$^{-}12$, $^{-}11$, $^{-}8$, $^{-}5$, 0, 7, 14, 17, 19

highest – lowest = 19 – $^{-}12$ = 19 + $^{+}12$ = $^{+}31$

Answer: The range of these integers is $^{+}31$.

Example 3: A marathon race was completed by 5 participants. What is the range of times given in hours below?

2.7 hr, 8.3 hr, 3.5 hr, 5.1 hr, 4.9 hr

Solution: Ordering the data from least to greatest, we get:

2.7, 3.5, 4.9, 5.1, 8.3

highest – lowest = 8.3 hr – 2.7 hr = 5.6 hr

Answer: The range of swim times is 5.6 hr.

Summary: The range of a set of data is the difference between the highest and lowest values in the set. To find the range, first order the data from least to greatest. Then subtract the smallest value from the largest value in the set.

Exercises

Directions: Find the range of each set of data. Click once in an ANSWER BOX and type in your answer; then click ENTER. After you click ENTER, a message will appear in the RESULTS BOX to indicate whether your answer is correct or incorrect. To start over, click CLEAR.

1. **Find the range of these distances run by 6 marathon runners:**
10 km, 15 km, 12 km, 14 km, 8 km, 16 km

ANSWER BOX: km RESULTS BOX:

2. **Find the range of these quiz scores:**
93, 79, 83, 89, 90, 71, 85

ANSWER BOX: RESULTS BOX:

3. **Find the range of these race times given in seconds:**
7.3 s, 8.4 s, 8.0 s, 7.5 s, 9.4 s, 8.7 s, 9.1 s

ANSWER BOX: s RESULTS BOX:

4. **Employees at a retail store are paid the hourly wages listed below. What is the range of these hourly wages?**
N7.50, N9.25, N8.75, N9.50, N7.25, N8.50

ANSWER BOX: N RESULTS BOX:

5. **In a game, points were won and lost, represented by integers. What is the range of points given below?**
$^{-}14$, $^{+}21$, $^{-}17$, $^{+}25$, 0, $^{-}19$, $^{+}11$, $^{-}20$, $^{+}18$

ANSWER BOX:

Median of Ungrouped Data

Median is defined as the mid value of the data set. It is a value that falls in the middle-most position of the whole data. Median of an ungrouped data is determined by arranging the given numbers in ascending order and then selecting exactly middle value. In other words, the median is the value that divides the observations (in ascending order) into two equal divisions. The median is a kind of measures of central tendency.
The formulas for calculating the median of an ungrouped data which has total "n" number of observations arranged in increasing order, are:

Case I: When n is odd

Median = *[Math Processing Error](n+12)th* observation
Case II: When n is Even

Median = *[Math Processing Error](n2)th [Math Processing Error]observation* + (*[Math Processing Error]n2* + 1)*[Math Processing Error]th* observation

Mode of Ungrouped Data

The mode is defined the value that most frequently occurs in the given data; i.e. the number whose frequency is more than others, is called the mode. It is usually denoted by "Z".

In order to find the mode of an ungrouped data, we have to find the frequency of each number in the given data set. Then, we have to choose the number having the highest frequency as the mode. The mode is also one of the three measures of central tendency. We can write as:
Mode = Value with highest frequenc

ASSESSMENT

1. Define Range
2. Define Mode
3. Define Median

Week 8

Mean deviation, Variance and Standard Deviation

Mean Deviation

This is the average sum of the deviations from the arithmetic mean (i.e. the sum of the differences between the scores and the mean) divided by the total frequency.

Example 1

Find the mean deviation in the following test scores,

Score (x)	Frequency (f)
95	1
85	1
80	1
75	4
70	1
65	3
55	1
40	3

Solution

The mean of the score X = 66.7

1
Scores (X)2

Frequency (f)3

X – X̅ = x = d4

Fx = fd95195-66.7 = 28.31 X 28.3 = 28.385185-66.7 = 18.31 X 18.3 = 18.380180-66.7 = 13.31 X 13.3 = 13.375475-66.7 = 8.34 X 8.3 = 33.270170-66.7 = 3.31 X 3.3 = 3.365365-66.7 = -1.73 X (-1.7) = -5.155155-66.7 = -11.71(-11.7) = -11.740340-66.7 = -26.73(-26.7) = -80.1 ∑F = 15 ∑FD = -0.5

So we subtract each score (X) from the mean (66.7) as shown in column above (3). Hence column (3) shows the deviation of each score form the mean. Column (4) shows the product of the deviations and their frequencies which we then sum as ∑fd. Hence the mean deviation is given by the formula ∑fd/∑f

where ∑fd = the sum of the products of deviations and the frequencies

∑f = total frequency

Mean deviation = ∑fd/∑f = –05/15 = –0.0333

We always take the positive sign regardless of the sign. The value of the mean deviation calculated above is so small that it is ignored usually in computing measures of dispersion which is not the true situation. So we talk about the Mean Absolute Deviation to give the true picture.

Mean Absolute Deviation

From the above mean deviation, the negative sign in column 3 were not ignored so we had an insignificant value for the mean deviation. This leads to the finding of the mean **absolute** deviation where the modulus (or absolute) of the values are considered in the computation.

Mean absolute deviation is defined as the arithmetic mean of the absolute (modulus) value of the difference of each score form the mean (that is adding the values of the deviations from the mean as if they were all positive) and dividing by their total frequency. That is the sum of the modulus of the deviations from the arithmetic mean divided by the total frequency. Hence the mean absolute deviation is given by the formula

MD = $\sum f|x|/\sum f$ or $\sum f|d|/\sum f$

where $|x|$ or $|d|$ = modulus of the deviations

$\sum f|x|$ or $\sum f|d|$ = sum of the product of the modulus of the deviation and their frequencies

$\sum f$ = total frequency

Standard Deviation

The standard deviation is the most important of all the measures of dispersion. Equally important is the variance which is the square of standard deviation. The standard deviation tells us how far or near a score is to the mean score. The standard deviation is denoted with s when we consider only a sample of a group to study or with σ (sigma) when we study the entire group. At times it is shortened as S.D. meaning standard deviation.

The formula for its computation is as follows:

S.D = $\sqrt{\sum fd^2/\sum f}$ or $\sqrt{\sum fx^2/\sum f}$ or $\sqrt{\sum x^2/N}$

and variance usually denoted by

s^2 or $\sigma^2 = \sum fd^2/\sum f$ or $\sum fx^2/\sum f$

In other words S.D. = $\sqrt{v}$

where $\sum f$ = sum of frequencies

N = sum of the scores, if each score occurs once only

d^2 or x^2 = square of the deviations from the area

i.e. d^2 or $x^2 = (X - \bar{X})^2$, where X = score and $\bar{X}$ = arithmetic mean.

Examples

Compute (i) the variance and (ii) the standard deviation of the following test scores:

Score	95	85	80	75	70	65	55	40
Frequency	1	1	1	4	1	3	1	3

Solution

The scores can be arranged in a frequency table as follows:

(1)
Scores (X)(2)

Derivation from the mean $d = X - \bar{X} = x$(3)

Frequency (f)(4)

$fd^2 = f(X - \bar{X})^2$9595-66.7 = 28.311 x $(28.3)^2$ = 806.898585-66.7 = 18.311 x $(18.3)^2$ = 334.848080-66.7 = 13.311 x $(13.3)^2$ = 176.897575-66.7 = 8.344 x $(8.3)^2$ = 275.567070-66.7 = 3.311 x $(3.3)^2$ = 8.696565-66.7 = -1.733 x $(-1.7)^2$ = 8.675555-66.7 = -11.711$(-11.7)^2$ = 136.894040-66.7 = -26.733$(-26.7)^2$ = 2138.67

In the above column (2) we use the value of the already computed mean $\bar{X}$ = 66.6, hence so applying the formula for computing the standard deviation, $\sqrt{\sum fd^2/\sum f} = \sqrt{\sum (X - \bar{X})^2/\sum f}$

where $\sqrt{\sum fd^2}$ = 3883.35 and $\sqrt{\sum f}$ = 15

we have variance = 3883.25/15 = 258.89

SD = $-\sqrt{258.89}$ = 16.09

S the standard deviation for the rest scores is 16.09 while the variance, which is the square of the standard deviation = 258.89 i.e. $(16.09)^2$= 258.89 i.e. $(S.D)^2$ = variance.

Question

Score	50-54	55-59	60-64	65-69	70-74	75-79	80-84	85-89	90-94
Frequency	3	5	8	10	7	6	3	2	1

Calculate for questions 1 – 5 using the table above

1. Determine $\sum f$

A. 45 B. 50 C. 55 D. 60

2. $\sum fd$, if d = X – A

A. -160 B. -167 C. -155 D. -201

3. Mean $\bar{X}$, as $\bar{X} = A + \sum fd/\sum f$

A. 70.05 B. 68.56 C. 23.55 D. 65.75

4. What is the Variance?

A. 97.03 B. 87.75 C. 92.03 D. 50.08

5. What is the Standard Variation?

A. 12.54 B. 9.59 C. 25.75 D. 32.23

Answers

1. A. 2. C 3. B 4. C 5. B

SS1
MATHEMATICS
SECOND TERM

TABLE OF CONTENT

Week 1

Logarithms

Understanding Logarithms

From indices we have that $2^3 = 8$ where 2 is the base and 3 is the power. On the other hand, we can write that the log of 8 to base 2 is equal to 3, denoted thus

$\text{Log}_2 8 = 3$

Also $5^2 = 25$ which means that $\log_5 25 = 2$.

The log of any number N to base M is the index or power to which the base M must be raised to equal the number N.

i.e. if x is the logarithm of a number N to base b then $N = b^x$

i.e. if $\log_b N = x$ then $N = b^x$

e.g. If (i) $\log_2 16 = 4$, then $16 = 2^4$

(ii) $\log_5 125 = 3$, then $125 = 5^3$

(iii) $\log_9 81 = 2$, then $81 = 9^2$

(iv) $\log_{25} 125 = 3/2$, then $125 = 25^{3/2}$

(v) $\log_{10} 1000 = 3$, then $1000 = 10^3$

(iv) $\log_{10} (1/10) = -1$, then $1/10 = 10^{-1}$

Conversely, if (i) $16 = 2^4$, then $\log_2 16 = 4$

(ii) $81 = 9^2$, then $\log_9 81 = 2$

and (iii) $125 = 25^{3/2}$, then $\log_{25} 125 = 3/2$ and so on.

With the above expressions, we can say that logarithms and indices are inter-related.

Laws of Logarithms

These are similar to all the laws of indices.

It shows clearly that a logarithm is a mirror image of an index.

e.g. $100 = 10^2$; $2 = \log_{10}100$

$1000 = 10^3$; $3 = \log_{10}1000$

$0.01 = 10^{-2}$; $-2 = \log_{10}{}^{0.01}$

Evaluate the following:

(1) If

1. Product

Given that $100 = 100^2$ $\log_{10} 100 = 2$ and $1000 = 10^3$ $\log_{10} 1000 = 3$ then

$100 \times 1000 = 10^2 \times 10^3 = 10^{2+3} = 10^5$ (1st law, indices)

$\log_{10}(100 \times 1000) = 5$ which is equal to 2+3

$= \log_{10} 100 + \log_{10} 1000$

Example

Given that $\log_{10} 2 = 0.0310$, $\log_{10} 3 = 0.4771$ and $\log_{10} 7 = 0.8451$, evaluate

$\text{Log}_{10} 42$

$\text{Log}_{10} 42 = \log_{10} (7 \times 6) = \log_{10} (7 \times 2 \times 3)$

$= 0.8451 + 0.0310 + 0.4771 = 1.6232$.

The logarithm of a product is the sum of the logarithms of the factors that make up the product.

2. **Quotient**

Since $1000 \div 100 = 10^3 \div 10^2 = 10^{3-2} = 10^1$

$\log_{10}(1000 \div 100) = 1 = \log_{10}(1000) - \log_{10}(100)$. This is the 2nd law of indices.

Example

$Log_{10}(14/3)$

$Log_{10}(14/3) = log_{10} 14 - log_{10} 3$

$= log_{10} (2 \times 7) - log_{10} 3$

$= log_{10} 2 + log_{10} 7 - log_{10} 3$

= (0.3010 + 0.8451) – 0.4771

= 0.6690

The logarithm of a quotient is the difference of the logarithms of the dividend and the divisor.

3. **Raising to a Power**

Example

$Log_{10}(100)^2 = 2log_{10} 100 = 2 \times 2$

Evaluate $log_{10} 8$, if $log_{10} 2 = 0.3010$

$log_{10} 8 = log_{10} 2^3 = 3log_{10} 2$

= 3 x 0.3010

= 0.9030.

4. **Roots**

The logarithms of the n^{th} root of a number is the logarithm of the number, divided by n

e.g. ^{3} 1000 = ^{3} 10^3 = ^{3} 10 x 10 x 10 = 10 or 10^1

log ^{3} 1000 = 1 or (log 1000) ¸ 3 = 1

log_b ^{n} x = 1/n log_bX.

Use of Log Tables

Express each number in the standard form. That is put a decimal after the first digit and multiply by the appropriate power of 10. The logarithm of a number has two parts the first of which is the exponent of 10, called the characteristic, and the second part is a decimal, called the mantissa, which is read from the log tables. The characteristic can be positive or negative. The negative characteristic is written with a bar above the number. The mantissa is always a positive decimal less than 1.

To multiply two numbers, find the logarithm of each number and add them. Then find the antilogarithm of the mantissa (the fraction part) from anti-log table (which will be a decimal between 1 and 10) and multiply by 10 raised to the characteristic to get the product.

The numbers	From the integral part, No. of digits -1	The std form of the Nos.	Characteristic part of the logarithm
389	3 – 1 = 2	3.99×10^2	2
458312	6 – 1 = 5	4.58312×10^5	5
17	2 – 1 = 1	1.7×10^1	1
1.4532	1 – 1 = 0	1.452×10^0	0

The mantissa part of the logarithm tables. Thus for the number 399 we look up for the first two digits 39 in the left hand column of the log table and follow the line across till we come under the third 9. Here we get 6010. Remember that the mantissa is always decimal so this is .6010. Now joining both the characteristics above with this mantissa part we have log 399 = 2.6010.

Similarly for 458312. Since our table is a four figure table only, this number becomes (to 4 significant figures) 458300. We look up 45 along the left hand column in the table and across under 8 we get 6609 and then the four digit 3 in the difference column. Across from 45 under 3 in the difference column is 3, we add this to 6609 to get 6612. Remember this is a decimal, i.e. .6612. Combining this with the characteristic 5 above we get that the logarithm of 458312 is 5.6612 and so on.

NB it is important to have logarithm table there with you for better understanding of the topic.

Antilogarithm

If the $\log_{10} 1000$ = then the antilogarithm of base 3 to 10 is 1000

Also if $\log_3 81 = 4$ then $\text{antilog}_3\ 1.3010 = 20$.

Therefore the antilog of a number is that number whose logarithm is given (i.e. the conversion of the logarithm). Also in common logarithms we usually drop the base 10 and just write log and not $\log_{10}$.

Example

Find the number whose logarithm is 0.4771

Solution

We are looking for the antilog of the given log. From the antilog table we look for the position of the decimal parts in the antilogs, fro 0.4771 look for 47 under 7 in the main body of the antilog table we get 2999 then look under 1 in the difference column which gives 1, and 1 to 2999 to get 3000. Since the characteristic is 0 we add 1 to the characteristic when looking up in the antilog to get the number of digits before the decimal points in the antilog.

0 + 1 = 0.

There is only one digit before the decimal point here. Putting the decimal point we have 3.000. The antilog of 0.4771 = 3.000 which is the required number

Multiply 256 by 768

$256 = 2.56 \times 10^2$ and $768 = 2.56 \times 10^2$

Log (256 x 768) = log 256 + log 768

$= \log (2.56 \times 10^2) + \log (2.56 \times 10^2) = 2.4082 + 2.8854 = 5.2963 =$

$256 \times 768 = \text{antilog } 5.2963 = 1.966 \times 10^5 = 196600$

Multiply 67846 and 0.0839

$67846 = 6.7846 \times 10^4$, $0.0839 = 8.39 \times 10^{-2}$

Log (67846 x 0.0839) = log 67846 +log 0.0839

$= \log (6.7846 \times 10^4) + \log (8.39 \times 10^{-2})$

= 4.8315 + 2.9283 = 3. 7553

6.7846 x 0.0839 = antilog 3.7553

$= 5.6925 \times 10^3 = 5692.5$

The bar above the characteristic is written to show that only the characteristic part is negative and the mantissa part is positive.

Now let us do some problems on division.

4. Divide 826 by 347

826 = 8.26 x 10^2, 347 = 3.47 x 10^2

Log (826 ¸ 347) = log (8.26 x 10^2) – log (3.47 x 10^2)

= 2.9170 – 2.5403

= 0.3767

826¸347 = antilog 0.3767

= 2.381 x 10^0 = 2.381.

To divide a number by another number, find their logarithm and subtract the logarithm of the divisor from the logarithm of the dividend. Then find the antilogarithm of the mantissa from anti-log table and multiply by 10 raised to the characteristic to get the result.
5. Divide 273 by 9876

We can find square root of a number using log tables

6. Find the square root of 5468

5468 = $5468^{1/2}$

Log 5468 = log $5864^{1/2}$

= ½ log 5468 = ½ log (5.468 x 10^3) = ½ 3.7378 = 1.8689

5468 = antilog 1.8689 =7.394 x 10^1

= 73.94

To find the square root of a number, find its logarithm and divide it by 2 and then find its antilogarithm.

Hope you like this cool method of how to use log tables to multiply or divide two numbers or find the roots of numbers?

<u>ASSESSMENT</u>

1. Solve $\log_{10} 6$
   A. 0.7778   B. 0.7781 C. 0.8778 D. 1.7778
2. Evaluate $\log_{10}(2/7)$
   A. 0.5441 B. 1.5442 C. 1.4327 D. 0.6551

Find from the tables of logarithms of

3. 1820
   A. 7.2564 B. 2.6210 C. 3.2601 D. 3.5210
4. 236.9
   A. 3.2674 B. 2.3749 C. 3.3749 D. 2.7359
5. Find the number whose logarithm is 1.3010
   A. 21.00 B. 22.00 C. 23.00 D. 20.00
6. Find the square root of 7548
7. Divide 646 by 231

Answers

1. B
2. A
3. C
4. B
5. D

Week 2

Quadratic Equation by Factorization and Completing the Square Methods

What is Quadratic Equation by Factorization?

To factorize an expression means to write the expression as the product of its factors which usually involves the introduction of brackets. So we can say that factorization is the opposite of expansion and vice-versa. There are various ways of factorization. These methods depend on the nature of the expression to be factorized. These methods are:

Factorizing by taking out common factors

Factorizing by grouping of terms of expression

Factorizing quadratic expressions

Factorizing perfect squares

Factorizing difference of two squares

Factorizing by taking out common factors

Factorize the following

$12x - 3y$, $2x + bx - 3x$

$12x - 3y$

The H C F of 12x and 3y is 3

Taking out the H C F: $12x + 3y = 3(4x - y)$

$2x + bx - 3x$

H C F of 2x, bx and 3x is x

$2x + bx - 3x = x(2 + b - 3) = x(b - 1)$

Factorizing by grouping of Terms of Expression

This is done by grouping the terms of the expressions in pairs and then factoring out the common quantities.

Factorize the following expressions

$2dx + 2dy + cx + cy$, $ab + b^2 - ay - yb$

Solution:

The first and second terms, 2dx and 2dy are grouped (since 2d is common), while the third and fourth terms, cx and cy are grouped (since c is common).

i.e. $2dx + 2dy + cx + cy = (2dx + 2dy) + (cx + cy)$

Hence the method is to look for terms that have common factor in them and group them together

$= 2d(x + y)+(x + y)$

$(x + y)$ is common so we bring it out: $(x + y)(2d + c)$

= Hence $2dx + 2dy + cx + cy = (x + y)(2d + c)$

$ab + b^2 - ay - yb$

Grouping $= (ab + b^2) - (ay - yb)$

Factorizing $= b(a + b) - y(a + b)$

$= (a + b)(b - y)$

$ab + b^2 - ay - yb = (a + b)(b - y)$

<u>Factorization of Quadratic Equation</u>

Expression of the form $ax^2 + bx + c$ is called general quadratic expression in x where a, b and c are real constants, a ¹ O. In this expression $ax^2 + bx + c$, ax^2 is the first term with a, the coefficient of x^2. Bx is the second or middle term with b the coefficient of x and c the third or constant term.

In a nutshell, a quadratic equation must have a squared term as its highest power.

Now consider the product of two linear expressions $(x + a)$ and $(x + b)$ where a and b are constants.

i.e. $(x + a)(x + b) = x(x + b) + (x + B)$

$= x^2 + bx + ax + ab$

$= x^2 + (a + b)x + ab$

Observe that the coefficient of x in the result is the sum of a and b, i.e. $(a + b)$ and the constant term is the product of a and b, ab.

The factors of $x^2 + 3x - 4$ are:

$(x+4)$ and $(x-1)$

Let us multiply them to see:

$(x + 4)(x - 1) = x(x - 1) + 4(x - 1)$

$= x^2 - x + 4x - 4$

$= x^2 + 3x - 4$

Example: $6x^2 + 5x - 6$

List the positive factor Example: $6x^2 + 5x - 6$

Step 1: $a \times c$ is $6\times (-6) =$ **-36**, and b is **5**

One of the numbers has to be negative to make -36, so by playing with a few different numbers I find that -4 and 9 work nicely:

$-4\times9 = -36$ and $-4+9 = 5$

Step 2: Rewrite **5x** with $-4x$ and $9x$:

$6x^2 - 4x + 9x - 6$

Step 3: Factor first two and last two:

$2x(3x - 2) + 3(3x - 2)$

Step 4: Common Factor is $(3x - 2)$:

$(2x+3)(3x - 2)$

Check: $(2x+3)(3x - 2) = 6x^2 - 4x + 9x - 6 =$ $\mathbf{6x^2 + 5x - 6}$

Factorization of perfect squares

In some cases recognizing some common patterns in the trinomial will help you to factor it faster. For example, we could check whether the trinomial is a perfect square.

A perfect square trinomial is of the form:

$(ax)^2 + 2abx + b^2$

Take note that

1. The first term and the last term are perfect squares
   2. The coefficient of the middle term is twice the square root of the last term multiplied by the square root of the coefficient of the first term.

When we factor a perfect square trinomial, we will get

$(ax)^2 + 2abx + b^2 = (ax + b)^2$

The perfect square trinomial can also be in the form:

$(ax)^2 - 2abx + b^2$

In which case it will factor as follows:

h$(ax)^2 - 2abx + b^2 = (ax - b)^2$

Example

1. $x^2 + 2x + 1 = 0$

$(x + 1)^2 = 0$

2. $x^2 + 6x + 9 = 0$

$X^2 + 2(3)x + 3^2 = 0$

$(x + 3)^2 = 0$

Factor the following trinomials:

1. a) $x^2 + 8x + 16$
   b) $4x^2 - 20x + 25$

Solution:

1. a) $x^2 + 8x + 16$
   $= x^2 + 2(x)(4) + 4^2$
   $= (x + 4)^2$
2. b) $4x^2 - 20x + 25$
   $= (2x)^2 - 2(2x)(5) + 5^2$
   $= (2x - 5)^2$

Factorization of Difference of Two Squares

Remember from your translation skills that "difference" means "subtraction". So a difference of squares is something that looks like $x^2 - 4$. That's because $4 = 2^2$, so you really have $x^2 - 2^2$, a difference of squares. To factor this, do your parentheses, same as usual:

$x^2 - 4 = (x \quad)(x \quad)$

You need factors of −4 that add up to zero, so use −2 and +2:

$x^2 - 4 = (x - 2)(x + 2)$

Note that we had $x^2 - 2^2$, and ended up with $(x - 2)(x + 2)$. Differences of squares (something squared minus something else squared) always work this way:

For $a^2 - b^2$, do the parentheses:

$(\quad)(\quad)$

...put the first squared thing in front:

$(a \quad)(a \quad)$

...put the second squared thing in back:

$(a \quad b)(a \quad b)$

...and alternate the signs in the middles:

$(a - b)(a + b)$

Factor $x^2 - 16$

This is $x^2 - 4^2$, so I get:

$x^2 - 16 = x^2 - 4^2 = (x - 4)(x + 4)$

Factor $4x^2 - 25$

This is $(2x)^2 - 5^2$, so I get:

$4x^2 - 25 = (2x)^2 - 5^2 = (2x - 5)(2x + 5)$

Factor $9x^6 - y^8$

This is $(3x^3)^2 - (y^4)^2$, so I get:

$9x^6 - y^8 = (3x^3)^2 - (y^4)^2 =$ (3×3 − y4)(3×3 + y4)

Factor 4x – 1

This is $(x^2)^2 - 1^2$, so I get:

$x^4 - 1 = (x^2)^2 - 1^2 = (x^2 - 1)(x^2 + 1)$

Note that I'm not done yet, because $x^2 - 1$ is itself a difference of squares, so I need to apply the formula again to get the fully-factored form. Since $x^2 - 1 = (x - 1)(x + 1)$, then:

$x^4 - 1 = (x^2)^2 - 1^2 =$ (x2 − 1)$(x^2 + 1)$

= ((x)2 − (1)2)$(x^2 + 1)$

= (x − 1)(x + 1)(x2 + 1)

ASSESSMENT

Factorize this trinomial by grouping method

1. -8x + 60x – 28
   A. −4[(x−7)(2x+1)]
   B.−4[(x+7)(2x−1)]
   C. −4[(x−7)(2x−1)]
   D. −4[(x+7)(2x+1)]
2. Factorize by perfect square method – $V^2 + 14V + 49$
   A. (V + 7) (V − 7)
   B. (V − 7) (V − 7)

C. (V + 7) (V +7)
D. (V − 7) (V + 7)

3. Factorize by perfect square method $x^2 - 10x + 25$
   A. $(x - 5)^2$
   B. $(x + 5)^2$
   C. $(x - 10)^2$
   D. $(x + 10)^2$
4. Factorize by common factors $5x^2 + 15x$
   A. 5x(x + 3)
   B. 3x(x − 3)
   C. 5x(x − 5)
   D. 5x(x − 3)
5. Factorize by perfect square method $x^2 + 2x + 1 = 0$
   A. $(x + 3)^3 = 0$
   B. $(x + 3)^2 = 0$
   C. $(x - 3)^2 = 0$
   D. $(x - 3)^3 = 0$

Answers

1. C 2. A 3. A 4. D 5. B

Week 3

General Forms of Quadratic Equation to Leading to Formula Method

What is Quadratic Equation?

Quadratic equation refers to polynomial equation that have a general form of $ax^2+bx+c=0$, where a, b and c are co-efficient. $a \neq 0$ otherwise it would be a linear equation and c is constant. The quadratic formula is defined as $x= (-b\pm\sqrt{b^2-4ac})/2a$, where "a" is the co-efficient of the x^2, b is the linear co-efficient of the x and c is the constant term. Therefore, the quadratic formula involves substituting the co-efficient from a given quadratic equation into the formula.

Derivation of the Quadratic Formula

For all quadratic equations, we have the general form:

$ax^2 + bx + c = 0$

1. Moving the "non x" to the right,

we get: $ax^2 + bx = -c$

2. Dividing by 'a' (the coefficient of x^2),

we get: $x^2 + bx/a = -c/a$

3. We take the coefficient of x, divide it by 2, square the result and then add that to both sides of the equation. The coefficient of x is b/a, one half of that is (b/2a) and squaring that,

we get $b^2/4a^2$.

Adding that both sides of the equation,

we have $x^2 + bx/a + b^2/4a^2 = -c/a + b^2/4a^2$

4. Taking the square roots of both sides,

we get: $x + b/2a = \sqrt{-c/a + b^2/4a^2}$

Moving b/2a to the right ,

$x = -b/2a \pm \sqrt{-4ac/4a^2 + b^2/4a^2}$

Simplifying $x = -b/2a \pm \sqrt{b^2 - 4ac}/2a$

Finally, $x = -b \pm \sqrt{b^2 - 4ac}/2a$

ASSESSMENT

Solve for the roots for the general form of quadratic equation

1. $4x^2 - 5x + 1 = 0$

A. x = 1, ¼ B. x = -1, ¼ C. 1, 1 D. 1/4, 1

2. $2x^2 - 14x - 13 = 0$

A. $5+5\sqrt{3}/2$, $7+5\sqrt{3}/2$ B. $7+5\sqrt{3}/2$, $5+5\sqrt{3}/2$ C. $7+5\sqrt{3}/2$, $7-5\sqrt{3}/2$ D. $7+5\sqrt{6}/2$, $7+\sqrt{3}/2$

3. $3x^2 + x - 2 = 0$

A. x = 2/3, -1 B. x = -2/3, 1 C. x = -2/3, -1 D. x = -2/3, 1

4. 3×2+4x+1=0

A. x =-1, -1/3 B. x = 1, -1/3 C. x=-1, 1/3 D. x=1, 1/3

5. $2x^2 - 3x - 6$

A. ¾, 4/9 B. ¾, -9/4 C. 9/4, ¾ D. 9/5, ¾

Answers

1. A 2. C 3. C 4. A 5. B

Week 4

Solutions of Quadratic Equations by Graphical Methods

Quadratic Equation with Two Solutions

We can solve a quadratic equation by factoring, completing the square, using the quadratic formula or using the graphical method.

Compared to the other methods, the graphical method only gives an estimate to the solution(s). If the graph of the quadratic function crosses the x-axis at two points then we have two solutions. If the graph touches the x-axis at one point then we have one solution. If the graph does not intersect with the x-axis then the equation has no real solution.

We will now graph a quadratic equation that has two solutions. The solutions are given by the two points where the graph intersects the x-axis.

Example:

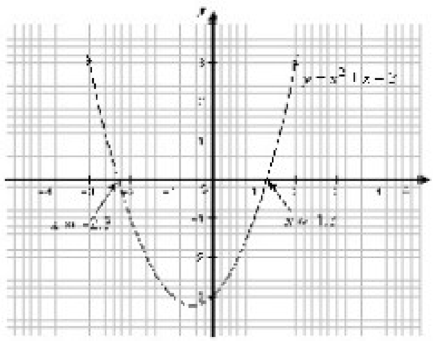
Solve the equation $x^2 + x - 3 = 0$ by drawing its graph for $-3 \leq x \leq 2$.

Solution:

Rewrite the quadratic equation $x^2 + x - 3 = 0$ as the quadratic function $y = x^2 + x - 3$

Draw the graph for $y = x^2 + x - 3$ for $-3 \leq x \leq 2$.

x	−3	−2	−1	0	1	2
y	3	−1	−3	−3	−1	3



The solution for the equation $x^2 + x - 3$ can be obtained by looking at the points where the graph $y = x^2 + x - 3$ cuts the x-axis (i.e. $y = 0$).

The graph $y = x^2 + x - 3$, cuts the x-axis at $x = 1.3$ and $x = -2.3$

So, the solution for the equation $x + x - 3$ is $x = 1.3$ or $x = -2.3$.

Recall that in the quadratic formula, the discriminant $b^2 - 4ac$ is positive when there are two distinct real solutions (or roots).

Graphing Quadratic Equations

$ax^2 + bx + c + 0$

A Quadratic Equation in Standard Form, (**a**, **b**, and **c** can have any value, except that **a** can't be 0.)

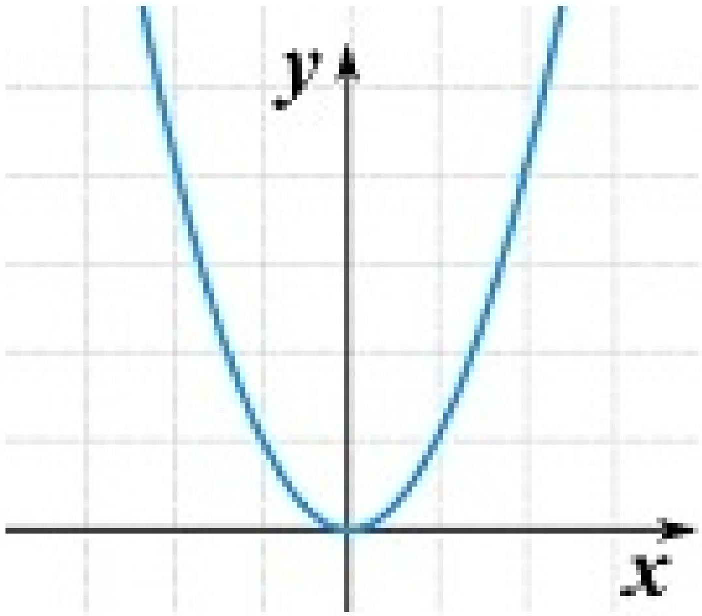
is an example:

$5x^2 - 3x + 3 = 0$

Graphing

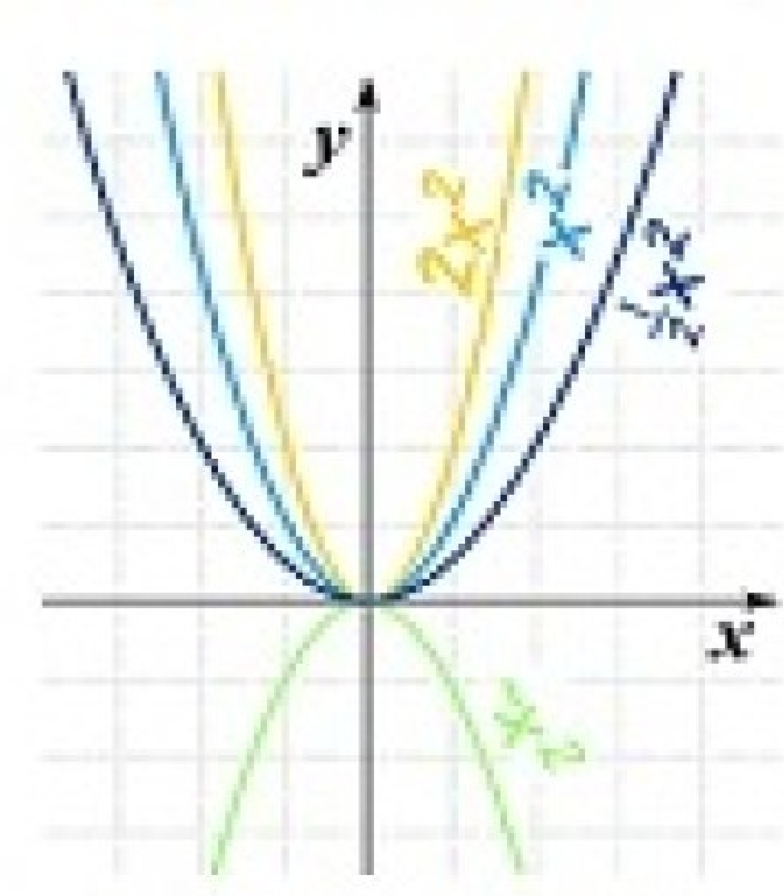
The simplest Quadratic Equation is $f(x) = x^2$

And its graph is simple too:



This is the curve $f(x) = x^2$, It is a parabola.

Now let us see what happens when we introduce the “a” value: $f(x) = ax^2$



Larger values of **a** squash the curve

Smaller values of **a** expand it

And negative values of **a** flip it upside down

The "General" Quadratic

Before graphing we **rearrange** the equation, from this:

$f(x) = ax^2 + bx + c$

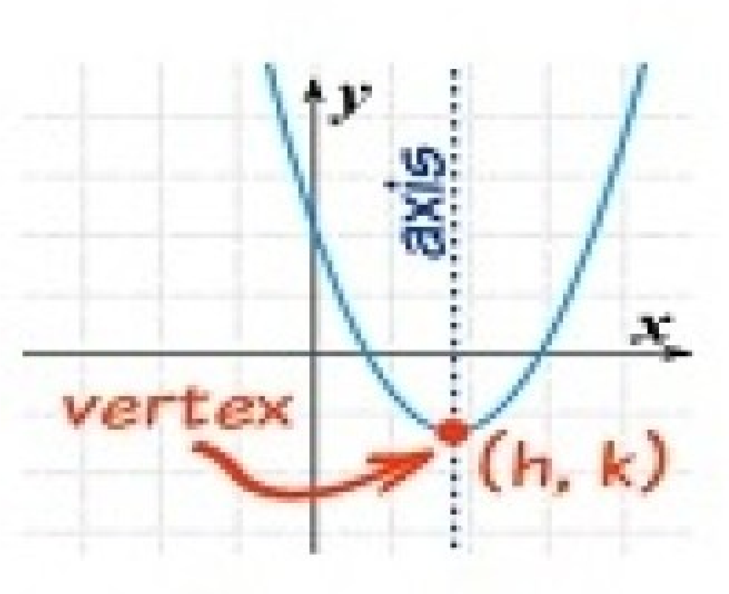
To this:

$f(x) = a(x-h)^2 + k$

Where: $h = -b/2a$ and $k = f(h)$

In other words, calculate **h** (=–b/2a), then find **k** by calculating the whole equation for **x=h**

First of all … Why?



Well, the wonderful thing about this new form is that h and k show you the very lowest (or very highest) point, called the **vertex**:

And also the curve is symmetrical (mirror image) about the **axis** that passes through **x=h**, making it easy to graph

h shows you how far left (or right) the curve has been shifted from x=0

k shows you how far up (or down) the curve has been shifted from y=0

Let us see an example of how to do this:

Example: Plot $f(x) = 2x^2 - 12x + 16$

First, let's note down:

a = 2, b = -12, and **c = 16**

Now, what do we know? a is positive, so it is an "upwards" graph ("U" shaped), a is 2, so it is a little "squashed" compared to the x^2 graph

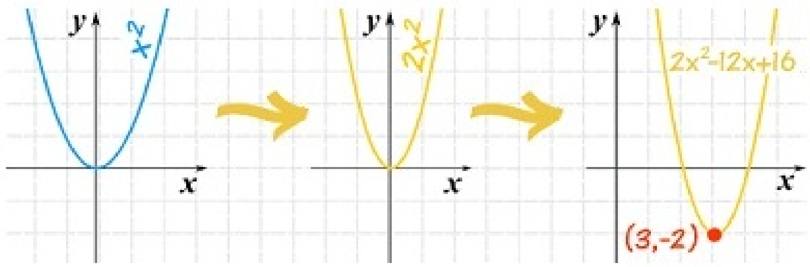
Next, let's calculate h:

h = -b/2a = -(-12)/(2×2) = **3**

And next we can calculate k (using h=3):

k = *f(*3*)* = $2(3)^2 - 12{\cdot}3 + 16 = 18-36+16 =$ **-2**

So now we can plot the graph (with real understanding!):

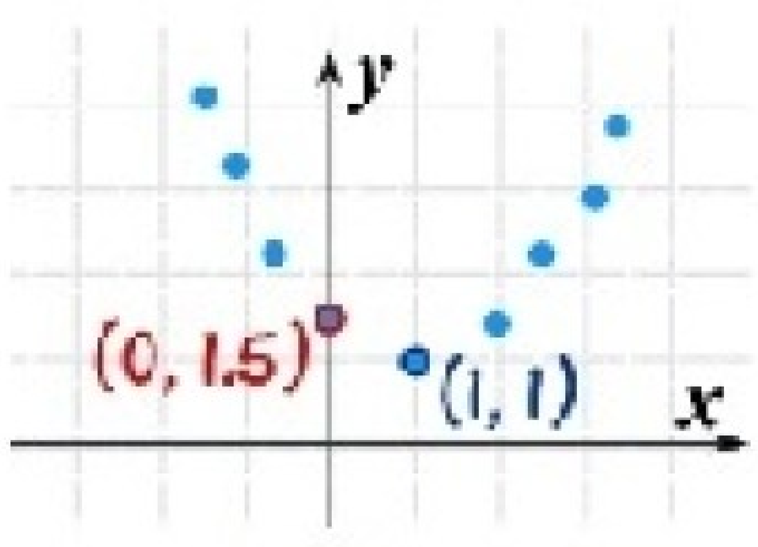


We also know: the **vertex** is (3,-2), and the **axis** is x=3

From A Graph to the Equation

What if you have a graph, and want to find an equation?

Example: you have just plotted some interesting data, and it looks Quadratic:



Just knowing those two points we can come up with an equation

Firstly, we know **h** and **k** (at the vertex): (h, k) = (1,1)

So let's put that into this form of the equation:

$f(x) = a(x-h)^2 + k$

$f(x) = a(x-1)^2 + 1$

Then we calculate "a":

And so here is the resulting Quadratic Equation:

$f(x) = 0.5(x-1)^2 + 1$

we know (0, 1.5) so: f(0) = 1.5 and we know the function (except for a):

$f(0) = a(0-1)^2 + 1 = 1.5$

Simplify f(0) = a+1 = 1.5, a = 0.5.

Note: This may not be the **correct** equation for the data, but it's a good model and the best we can come up with.

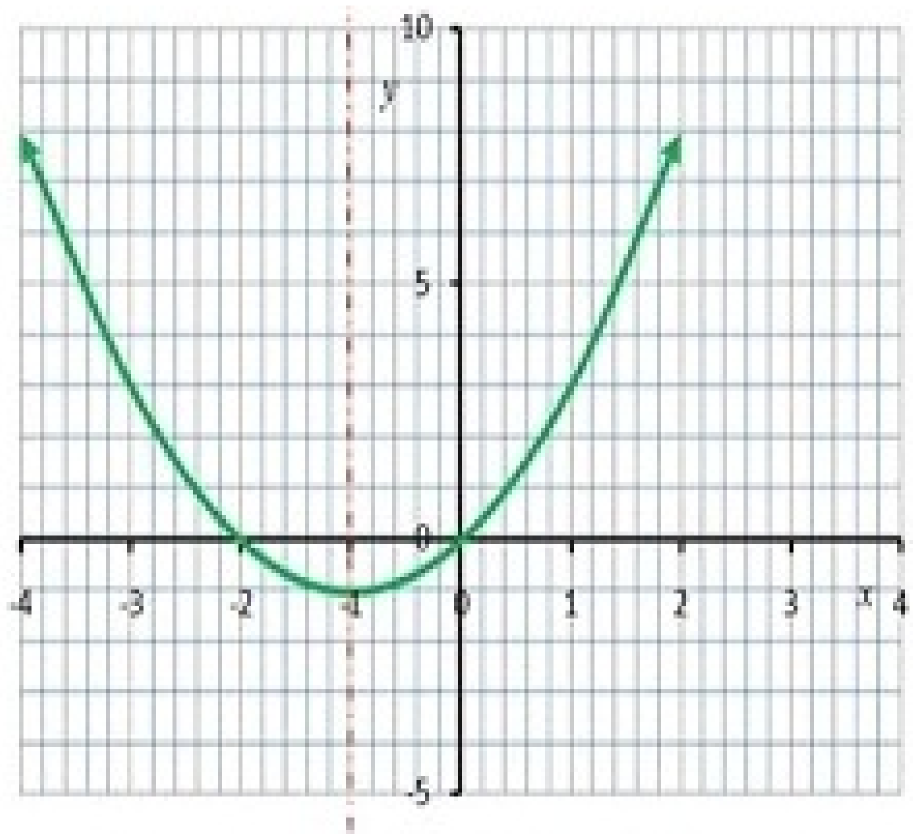
And so here is the resulting Quadratic Equation:

$f(x) = 0.5(x-1)^2 + 1$

Note: This may not be the **correct** equation for the data, but it's a good model and the best we can come up with.

Line of Symmetry

This is obtained by finding the value of x at the turning points where the curve is a maximum or minimum.



Green parabola: $y = x^2 + 2x$; Red line is the Line of Symmetry

Find the line of symmetry of $y = x^2 + 2x$ with 3 steps.

Find the vertex, which is the lowest or highest point of a parabola. *Hint*: The line of symmetry touches the parabola at the vertex. **(–1,–1)**

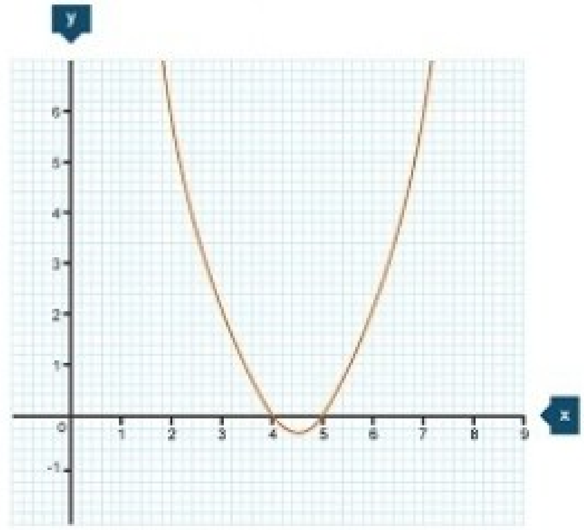
What is the x-value of the vertex? **–1**

The line of symmetry is $\boldsymbol{x = -1}$

ASSESSMENT

1. Symmetry is that point on the parabola where it is

A. Maximum B. Maximum C. Middle D. Maximum and Minimum



2. From the above graph to factor out the roots.

A. $x^2 - 9x + 20$ B. $2x^2 - 9x + 20$ C. $x^2 - x + 20$ D. $x^2 - 9x + 9$

3. What makes this equation $2x^2 - 12x + 16$ quadratic?

A. The coefficient of x^2 B. The coefficient of x which is 12 C. The squared power of x whose coefficient is 2 D. None of the options is correct.

4. What is a parabola?

A. The x – axis B. y – axis C. The curve on the graph D. None of the options.

5. $a < 0$ and in the quadratic equation, means that the curved end parabola is

A. Up on the graph B. Down on the graph C. Left side on the graph D. Right side on the graph.

Answer:

1. D 2. A 3. C 4. C 5. A

Week 5

Idea of Sets

Definition and Notation

A set can be defined as a collection of objects according to a well defined common elements or property. The main purpose of this their common property is foe easy identification. For instance, we hear of under 13 football players, meaning “the set of football players” whose ages fall below 13 years; “set of school uniform {of the dresses and sandals.

Notation and Methods of Describing Sets

A set is usually represented by a capital letter, for example,

A = The set of even numbers less than 110

B = The set of Nigerian Presidents since Independence in 1960

C = The set of months of the year

There are basically three ways of representing the sets, namely;

1. The **set builder** or **property form**: This describes the elements of the set by referring to their common property. This method of describing a set is called the **set-builder method**.

e.g. W = {x : x is the day of the week}

Y = {even numbers between 0 and 10}

or Y = {x : x is even number and 0 < x < 19}

2. Also the above example can be given by the **rule method** as W = {the days of the week}

This is using the rule method since any day of the week is in set W

3. The roster of tabular or listing method: this method actually lists all the members of the set

e.g. w = {Sunday, Tuesday, Wednesday, Thursday, Friday, Saturday}

Y = {2,4,6,8}

Note: The elements in 3 above and the descriptions in 1 and 2 are usually enclosed in curly brackets or braces as in the examples above. In the case of roster or tabular listing form, the elements are separated by commas as shown above.

Using the example above, Monday "is a member of" W. This is denoted as Monday Î W read as

Monday "belongs to" *W* or Monday "is an element of" *W* or Monday "is in" *W*. If a member "does not belong to" the set of days in the week, this can be denoted by Ada Ï W read as 'Ada is not in' *W* or Ada "is not an element of" *W* or Ada "is not a member of" *W*. Generally, the elements or members of set are usually the lower case letters such as a, b, c, d, ... , while the whole set itself is usually represented with a capital letter A, B, W, Y, ...

Finite and Infinite Set

As already mentioned above, some collections of objects according to a well defined common property can be large or small and their members or elements definite or infinite. If the members of a set have a definite numbers like the days of the week, we term this as finite, otherwise it is infinite like the set of natural numbers or counting numbers.

Examples of finite and infinite sets are as follows:

a. W = {days in the week} – W is finite

b. N = {x | x is a multiple of 2} – N is infinite

These can be written in tabular form as

W = {Sunday, Monday, Tuesday, Wednesday, Thursday, Friday, Saturday} which contains seven elements and therefore is finite.

N = {2,4,6,8, ...} which contains indefinite number elements and therefore is infinite.

Others examples are

i. M = {months in the year} – M is finite

ii. F = {y | y is odd} – F is infinite.

Null Set or Empty Set

Wherefore there is no body in the classroom (like after school hours or during the holidays), the classroom is empty of persons at that time. We refer to it as an empty or null set. A null set is therefore defined as a set that contains no elements. This set is denoted by { } or ∅. (Note that when this ∅ is used it is not enclosed in a bracket. Hence the set {.} or {∅} is not an empty set rather it is a set containing an element "." Or "∅").

Example of a Null Set

i. The set of students in Nigeria secondary schools below 5 years is empty. That is A = {Students in secondary schools in Nigeria below 5 years} = { } or because no student below five years in Nigerian secondary school.

ii. B = {x | x^2 = 4, x odd}. \the solution is 2 and -2 neither of which is an odd number, so B = { }.

iii. C = {women priests in Roman Catholic Church} is empty because there is no woman priest in Roman Catholic Church, so C = { } or ∅.

Subsets and Supersets

Subsets are sets which are contained in another set or given two sets A and B, set A is said to be a subset of a set B if and only if all the elements of set A are contained ion set B. This is denoted by A ⊂ B.

Examples

i. If two sets D = {0, 2, 4, 6, 8} and E = {2, 4, 6}, the E is a subset of D because all the elements of E are in D.

ii. If A = {Garri, cassava,yam , beans}, B = {Garri, beans} then B is a subset of A. Hence if A set P is contained another set Q, the relationship is denoted by "P ⊂ Q" which is read as "P is contained in Q" or "P is a subset of Q".

iii. X = {a, b, c, d, e} and Y = {a, b, c}. Y is a subset of X

iv. If F = {x | x is a positive integer}, F = {1, 2, 3, 4, 5, 6,....}, G = {y | y is a multiple of 3} i.e. G = {3, 6, 9, 12....} then G is contained in F i.e. G ⊂ F.

i. Since the null or empty set contains no elements, it can be contained in any set without changing the set. Hence the null set or empty is a subset of every set.

ii. Every set is a subset of itself because all the elements in the set are contained in the set itself. These two sets, the null set or the set itself are called improper subsets because they are special case s of subsets while all the others are called proper subset.

Universal Set

The set under consideration are likely to be the subsets of a fixed or a global set. This fixed or global set is called the **Universal Set.**

A universal set is the set of all objects of interest in a particular discussion. This set can be very big or small depending on the context.

For example if we are discussing medical students at Nnamdi Azikwe University, the whole students of the Nnamdi Azikwe University can be our universal set. If our reference is to persons, every person in the Nnamdi Azikwe University can be the universal set. Therefore, the universal set varies as our group of reference varies. The universal set is usually denoted by U or e, so

U = {students in Nnamdi Azikwe University} or {every person in Nnamdi Azikwe University}

A = {a, b, c, d} can have a universal set of all letters of the English alphabet, i.e. U = {letters of the English alphabets}; or the sets of {first 13 letters of the English alphabet}; or the {set of first 5 letters of the English alphabets}. This implies that the universal set may not necessarily be unique.

Example

If set U = {1, 2, 3, 4,....}, describes the following sets by the listing method.

a. $A = \{x \mid x^2 - 3x + 2 = 0\}$ b. $C = \{x \mid x = 2 \text{ or } x = 3 \text{ or } x = 20\}$

Solution

$A = \{x \mid x^2 - 3x + 2 = 0\}$ here $x^2 - 3x + 2 = 0$ can be factorized into $(x - 2)(x - 1) = 0$, hence x = 2 or 1 so A = {1, 2}.

b. C = {2, 3, 10}. This kind of question requires nothing other than listing the solutions in the set 2, 3,20.

ASSESSMENT

1. Describe the set in a set builder form A = {January , June, July}, put ∉ or ∈ in the spaces provided

January ……… A

A. ∈ B. ∉ C. ⊂ D. ∪

2. B = {SS1, SS2, SS3}; where SS denotes senior secondary class.

25 ……….. B

A. A. ∈ B. ∉ C. ⊂ D. ∪

3. Which of these sets is not null?

A. The set of all female presidents in Nigeria between 1960 and 1998

B. F = {d | d is a letter before 'f' in the English alphabets}

C. D = {x | x^2 = 4 and 2x = 6}

D. A = {y | y y}

4. Let the universal set U be the set of all real numbers. If S = {x | $x^2 - 3x + 2 = 0$}, decide which of following statements is correct

A. 1 ∈ S B. 2 ∉ S C. 4 ∈ S D. 3 ∈ S

5. Which of the symbols denotes a null set?

A. ∉ B. ⊂ C. ∪ D. { }

Answers

1. A 2. B 3. B 4. A 5. D

Week 6

Complement of Sets

Introduction to Complement of Sets

Let's say that we have a set *A* that is a subset of some universal set *U*.
The **complement** of *A* is the set of elements of the universal set that are not elements of *A*. In our example above, the complement of {-2, -1, 0, 1} is the set containing all the integers that do not satisfy the inequality.
We can write A^c

You can also say complement of A in U

Example #1.

Take a close look at the figure above. d and f are in U, but they are not in A.

Therefore A^c = {d, f}

Sometimes, instead of looking at a the Venn Diagrams, it may be easier to write down the elements of both sets

Then, we show in bold the elements that are in U, but not in A

A = { a, b, c}

U = { a, b, c, **d**, **f**}

Example #2.

Let B = {1 orange, 1 pinapple, 1 banana, 1 apple}

Let U = {1 orange, **1 apricot**, 1 pinapple, 1 banana, **1 mango**, 1 apple, **1 kiwifruit** }

Again, we show in bold all elements in U, but not in B

B^c = {1 apricot, 1 mango, 1 kiwifruit}

Example #3.

Find the complement of B in U

B = { 1, 2, 4, 6}

U = {1, 2, 4, 6, **7**, **8**, **9** }

Complement of B in U = { 7, 8, 9}

Example #4.

Find the complement of A in U

A = { x / x is a number bigger than 4 and smaller than 8}

U = { x / x is a positive number smaller than 7}

A = { 5, 6, 7} and U = { 1, 2, 3, 4, 5, 6}

A^c = { 1, 2, 3, 4}

Or A^c = { x / x is a number bigger than 1 and smaller than 5 }

The graph below shows the shaded region for the complement of set A

ASSESSMENT

1. If A = { 1, 2, 3, 4} and U = { 1, 2, 3, 4, 5, 6, 7, 8} then find A complement (A').
2. If B = { x | x is a book on Algebra in your library} . Find B'.
3. If A = { 1, 2, 3, 4, 5 } and U = N , then find A'.
4. If A = { x | x is a multiple of 3, x ∉ N }. Find A'

ANSWERS

1. A = { 1, 2, 3, 4} and Universal set = U = { 1, 2, 3, 4, 5, 6, 7, 8}Complement of set A contains the elements present in universal set but not in set A.

   Elements are 5, 6, 7, 8.

   ∴ A complement = A' = { 5, 6, 7, 8}.

2. B' = { x | x is a book in your library and x ∉ B }
3. A = { 1, 2, 3, 4, 5 }U = N

   ⇒ U = { 1, 2, 3, 4, 5, 6, 7, 8, 9,10,… }

   A' = { 6, 7, 8, 9, 10, … }

4. As a convention, x ∉ N in the bracket indicates N is the universal set.N = U = { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10,11, … }

   A = { x | x is a multiple of 3, x ∉ N }

   A = { 3, 6, 9, 12, 15, … }

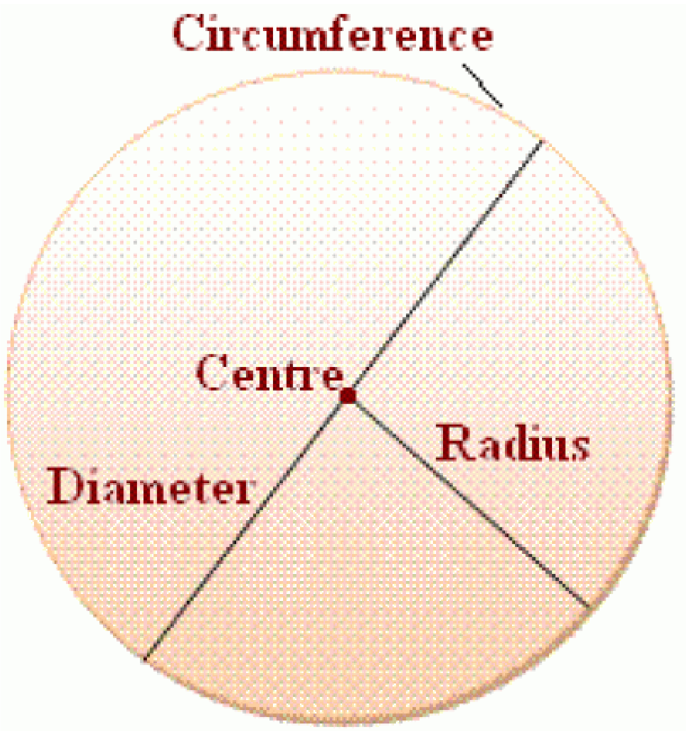
   So, A' = { 1, 2, 4, 5, 7, 8, 10,11, … }

Week 7

Circle and its Properties

Introduction

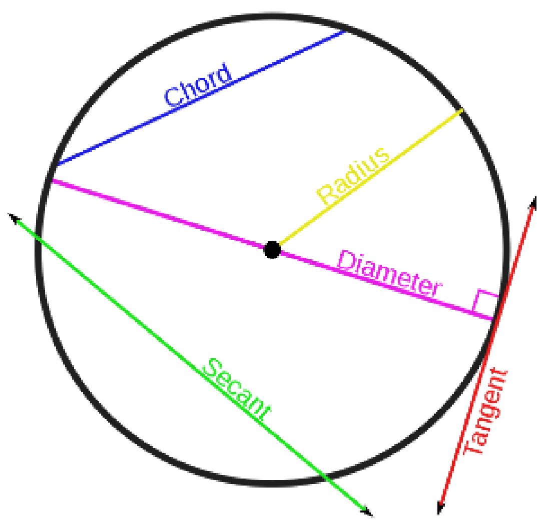
A circle is a simple, beautiful and symmetrical shape. When a circle is rotated through any angle about its centre, its orientation remains the same. When any straight line is drawn through its centre, it divides the circle into two identical semicircles. The line is known as the diameter. The common distance of the points of a circle from its centre is called radius. The perimeter or length of the circle is also known as the circumference.



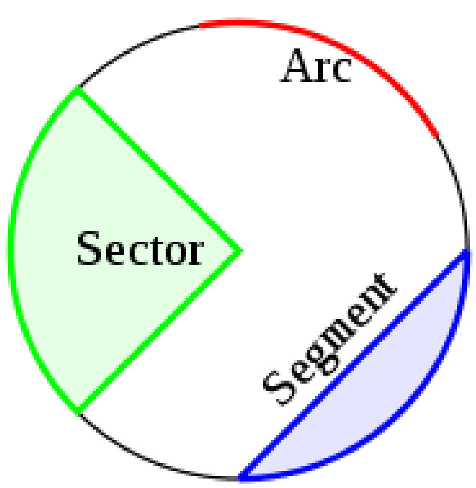
Diameter

The diameter of a circle is the length of a line segment whose endpoints lie on the circle and which passes through the centre of the circle.
This is the largest distance between any two points on the circle.
The diameter of a circle is twice its radius.

In other words, two radius make one diameter.
Chord
A chord of a circle is a line segment whose two endpoints lie on the circle.
The diameter, passing through the circle's centre, is the largest chord in a circle.
Tangent
A tangent to a circle is a straight line that touches the circle at a single point.
Secant
A secant is an extended chord: a straight line cutting the circle at two points.



Arc
An arc of a circle is any connected part of the circle's circumference.
Sector
A sector is a region bounded by two radius and an arc lying between the radius.
Segment
A segment is a region bounded by a chord and an arc lying between the chord's endpoints.



Angle Properties in a circle

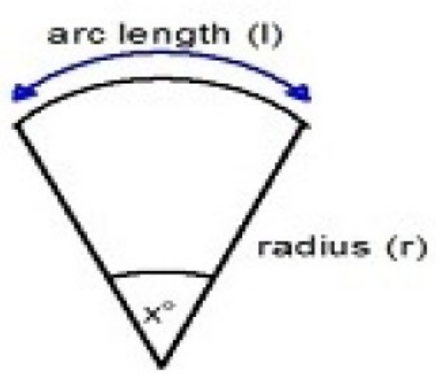
Arc length of a sector

If you take a part of the circumference of a circle then the distance along this arc is called the arc length. So since the arc length is part of the circumference of a circle, then the arc length of the sector can be found by using the following formula:

$L = (q/360) \times \pi \times d$

Where L is the arc length, q is the angle inside the sector and d is the diameter of the sector.

So make sure that you use the diameter of the circle if you are calculating the arc length of a sector.



The angle inside the sector is 41º, so x = 41º.

Now the diameter of the whole circle is 24cm (as the 12cm is the radius), so d = 24cm.

All you need to do now is substitute these values into the above formula so you can find the arc length.

$l = (x/360) \times \pi \times d$

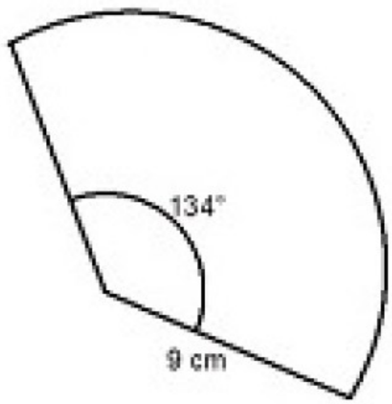
$L = (41/360) \times \pi \times 24$

L = 8.6 cm rounded to 1 decimal place.

1. Work out the arc length of this sector:

Example

Work out the arc length of this sector:



The angle inside the sector is 134º, so x = 134º.

Now the diameter of the whole circle is 18cm (as the 9cm is the radius), so d = 18cm.

All you need to do now is substitute these values into the above formula so you can find the arc length.

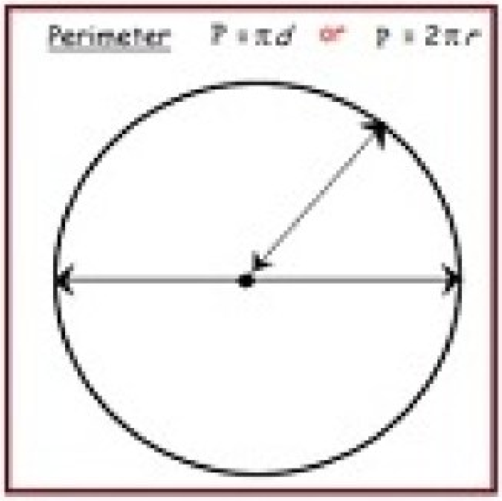
$l = (x/360) \times \pi \times d$

$l = (134/360) \times \pi \times 18$

l = 21.0 cm rounded to 1 decimal place.

Perimeter of a Circle

The perimeter of a circle is the whole length of a circle. It is the total length that is measured around and outside the circle. Perimeter of a circle is also referred as Circumference of a circle.



The length of the thread that winds around the circle exactly once, gives the perimeter of the circle.

The perimeter of a circle depends on its radius of the circle.

If ‘ r ‘ is the radius of a circle, then its perimeter “ P “ is given by the formula

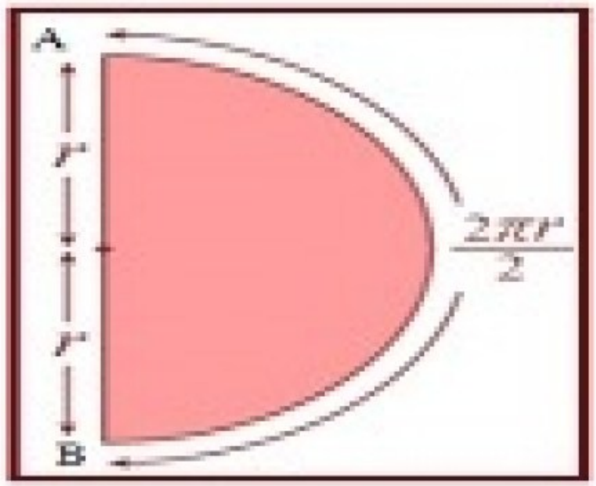
$P = 2\pi r$

If ‘ d ‘ is the diameter of a circle, then its perimeter “ P “ is given by the formula

$P = \pi d$

Perimeter of a Half Circle

The diameter divides the circle into two equal arcs called semi circle or half circle.



We know that, If ‘ r ‘ is the radius of a circle, then its perimeter “ P “ is given by the formula

$P = 2\pi r$

Therefore, Perimeter of a semi circular arc AB = π* r units.

If ‘ d ‘ is the diameter of a circle, then its perimeter “ P “ is given by the formula

$P = \pi * d$

Therefore, Perimeter of the semi circular arc AB in terms of diameter is $(d2)$ units.

Perimeter of a Half Circle Formula

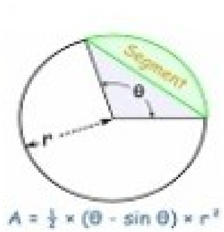
If ‘ r ‘ is the radius of a circle, then the perimeter “ P “ of the half circle is given by the formula

$P = \pi r + 2r$

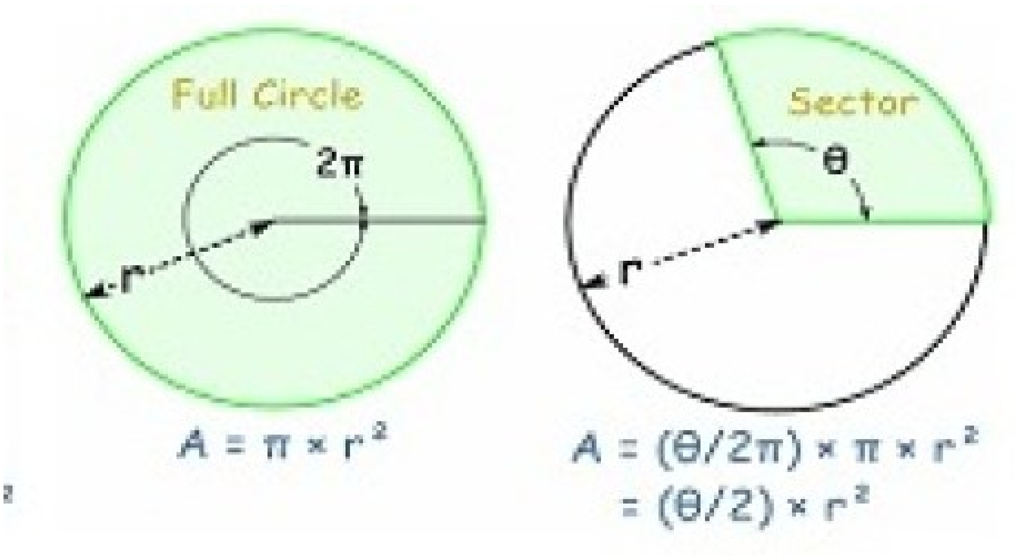
If ‘ d ‘ is the diameter of a circle, then the perimeter “ P “ is the half circle is given by the formula

$P = (d2) + d$

Area of Sector s and Segments



Area Of a Sector



You can work out the Area of a sector by comprising its angle to the angle of a full circle.

Note: I am using radians for the angles. This is the reasoning.

A circle has an angle of 2π and an Area of: πr^2
So a Sector with an angle of θ (instead of 2π) must have an area of: $(\theta/2\pi) \times \pi r^2$

Which can be simplified to: $(\theta/2) \times r^2$

Area of Sector = $\frac{1}{2} \times \theta \times r^2$ *(when θ is in radians)*

Area of Sector = $\frac{1}{2} \times (\theta \times \pi/180) \times r^2$ *(when θ is in degrees)*

<u>ASSESSMENT</u>

1. Find the area of triangle *PQR* if p = 6.5 cm, r = 4.3 cm and Ð Q = 39°. Give your answer correct to 2 decimal places.

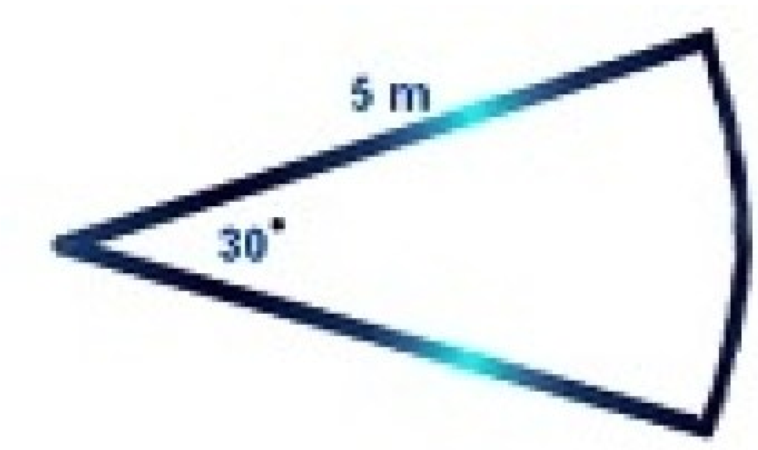
A. 5.7 cm^2 B. 8.79 cm^2 C. 8.01 cm^2 D. 9.8 cm^2

2. If the Ferris wheel's **diameter is 120 metres**, how far would a person riding the ferris wheel travel in one rotation? (Use π= 3.142)

A. 407.1 m B. 546 m C. 376.8 m D. 307.3 m

3. If the Earth's **radius is 6371 metres**, what is its circumference? (Use π = 3.14.)

A. 36 000.23 km B. 43 000.88 km 9800 km D. 40 009.88 km



4. Calculate the perimeter around this sector of the image. (Use π = 3.142.)

A. 16 m B. 12.62 m C. 23.7 m D. 45 m

5. Find the arc length of the sector.

A. 10 cm B. 8.6 cm C. 14.3 cm D. 12 cm

Answer

1. B 2. C 3. B 4. B 5. B

Week 8

Trigonometric Ratios

What are Trigonometric Ratios?

Trigonometry is the study of triangles in relation to their sides and angles and many other areas which find applications in many disciplines. In particular, trigonometry functions have come to play great roles in science. For example, in physics, it is used when we want to analyse different kinds of waves, like the sound waves, radio waves, light waves, etc. Also trigonometric ideas are of great importance to surveying, navigation and engineering.

We shall, however at this stage be concerned with the elementary ideas of trigonometry and their application.

Trigonometrical Ratios (Sine, Cosine and Tangent)

The basic trigonometric ratios are defined in terms of the sides of a right-angled triangle. It is necessary to recall that in a right-angled ΔABC, with ∠C = 90^0, the side AB opposite the 90^0 is called the hypotenuse.

The notion that there should be some standard correspondence between the lengths of the sides of a triangle and the angles of the triangle comes as soon as one recognizes that similar triangles maintain the same ratios between their sides. That is, for any similar triangle the ratio of the hypotenuse (for example) and another of the sides remains the same. If the hypotenuse is twice as long, so are the sides. It is these ratios that the trigonometric functions express.

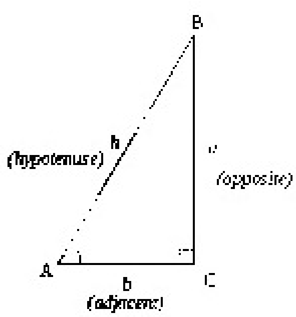
To define the trigonometric functions for the angle *A*, start with any right triangle that contains the angle *A*. The three sides of the triangle are named as follows:

- The *hypotenuse* is the side opposite the right angle, in this case side **h**. The hypotenuse is always the longest side of a right-angled triangle.
- The *opposite side* is the side opposite to the angle we are interested in (angle *A*), in this case side **a**.
- The *adjacent side* is the side having both the angles of interest (angle *A* and right-angle *C*), in this case side **b**.

In ordinary Euclidean geometry, according to the triangle postulate, the inside angles of every triangle total 180° (π radians). Therefore, in a right-angled triangle, the two non-right angles total 90° (π/2 radians), so each of these angles must be in the range of (0°, 90°) as expressed in interval notation. The following definitions apply to angles in this 0° – 90° range. They can be extended to the full set of real arguments by using the unit circle, or by requiring certain symmetries and that they be periodic functions. For example, the figure shows $\sin\theta$ for angles θ, $\pi - \theta$, $\pi + \theta$, and $2\pi - \theta$ depicted on the unit circle (top) and as a graph (bottom). The value of the sine repeats itself apart from sign in all four quadrants, and if the range of θ is extended to additional rotations, this behavior repeats periodically with a period 2π.

Rigorously, in metric space, one should express angle, defined as scaled arc length, as a function of triangle sides. It leads to inverse trigonometric functions first and usual trigonometric functions can be defined by inverting them back.

The trigonometric functions are summarized in the following table and described in more detail below. The angle θ is the angle between the hypotenuse and the adjacent line – the angle at A in the accompanying diagram.



Function	Abbreviation	Description	Identities (using radians)
Sine	sin	opp/hyp	$\sin\theta = (\pi/2 - \theta) = 1/\csc\theta$
Cosine	cos	adj/hyp	$\cos\theta = (\pi/2 - \theta) = 1/\sec\theta$
Tangent	tan	opp/adj	$\tan\theta = (\sin\theta/\cos\theta = \cot(\pi/2 - \theta) = 1/\cot\theta$
Cotangent	cot	adj/opp	$\cot\theta = \cos\theta/\sin\theta = \tan(\pi/2 - \theta) = 1/\tan\theta$
Secant	sec	hyp/adj	$\sec\theta = \csc(\pi/2 - \theta) = 1/\cos\theta$
Cosecant	cosec	hyp/opp	$\csc\theta = \sec(\pi/2 - \theta) = 1/\sin\theta$

Sine, Cosine and Tangent

The sine of an angle is the ratio of the length of the opposite side to the length of the hypotenuse. (The word comes from the Latin *sinus* for gulf or bay, since, given a unit circle, it is the side of the triangle on which the angle *opens*.) In our case sin A = opposite/hypotenuse = a/h

This ratio does not depend on the size of the particular right triangle chosen, as long as it contains the angle *A*, since all such triangles are similar.

The **cosine** of an angle is the ratio of the length of the adjacent side to the length of the hypotenuse: so called because it is the sine of the complementary or co-angle. In our case

cos A = adjacent/hypotenuse = b/h

The **tangent** of an angle is the ratio of the length of the opposite side to the length of the adjacent side: so called because it can be represented as a line segment tangent to the circle, that is the line that touches the circle, from Latin *linea tangens* or touching line (cf. *tangere*, to touch). In our case

tan A = opposite/adjacent = a/b

The acronyms "SOHCAHTOA" ("Soak-a-toe", "Sock-a-toa", "So-kah-toa") and "OHSAHCOAT" are commonly used mnemonics for these ratios.

Reciprocal functions

The remaining three functions are best defined using the above three functions.

The **cosecant** csc (*A*), or cosec (*A*), is the reciprocal of sin (*A*); i.e. the ratio of the length of the hypotenuse to the length of the opposite side:

csc A = 1/sin A = hypotenuse/opposite = h/a

The **secant** sec (*A*) is the reciprocal of cos (*A*); i.e. the ratio of the length of the hypotenuse to the length of the adjacent side:

sec A = 1/cos A = hypotenuse/adjacent = h/b.

It is so called because it represents the line that *cuts* the circle (from Latin: *secare*, to cut).

The **cotangent** cot (*A*) is the reciprocal of tan (*A*); i.e. the ratio of the length of the adjacent side to the length of the opposite side:

cot A = 1/tan A = adjacent/opposite = b/a.

Use of Trigonometric Ratio Tables

Apart from finding the trigonometric ratios of angles by construction, the tables of trigonometric ratios in the mathematical tables can be used. These tables consist of both the natural and logarithmic sine, cosine and tangent of angles between 0^o and 90^o at intervals of 6^1 or $0.1^o$and having the difference column on the extreme right for intermediate values. Also the table of trigonometric ratio of angles measured in radians are given, for the moment we will consider the natural trigonometric ratios (sine, cosine and tangent) of angles measured in degrees. The tables are usually supplied to save time, since finding the trig ratios of angles or the angles whose trig ratios are given takes a long time.

Example

Find sin 32^o

Solution: from the sine table, to find sin 32^o, we look for 32^o under $0'$ which gives 0.5299. i.e. sin 32^o = 0.5299.

Find sin 43^o $30'$

Solution: for sin 43^o $30'$, we look for sin 43^o under sin $30'$ and get 0.6884.

i.e. sin 43^o $30'$ = 0.6884.

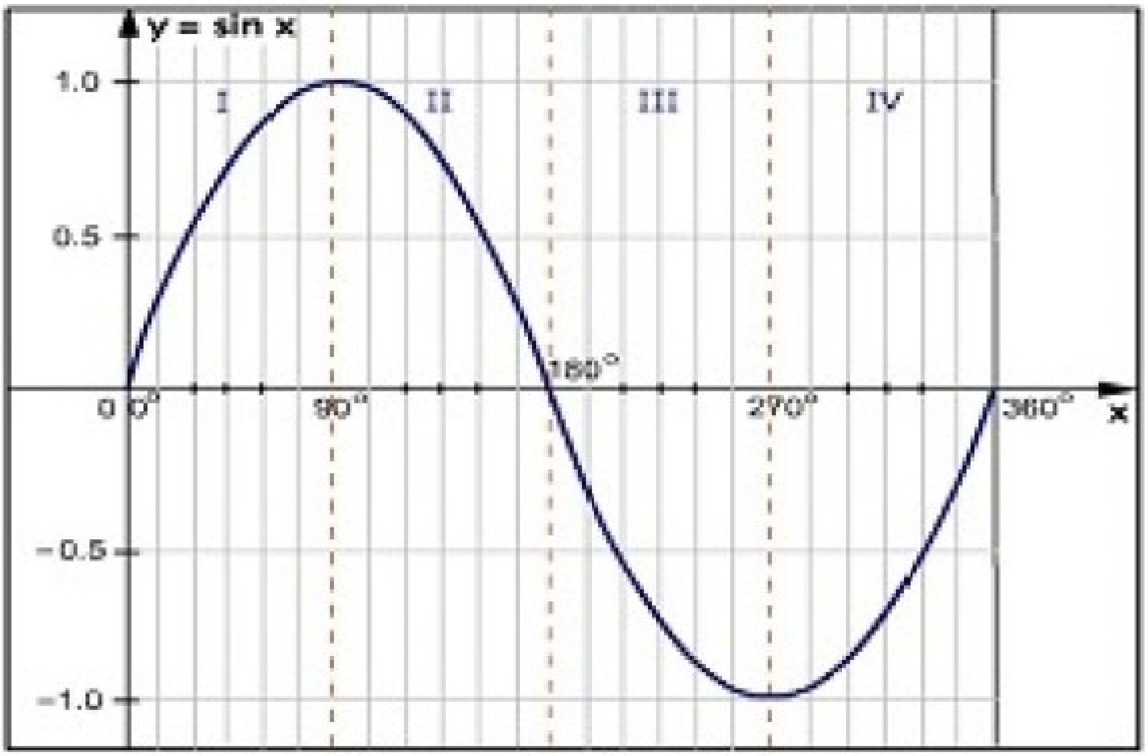
Graph of Sine and Cosine for Angles $0^o \leq x \leq 360^o$

Introduction

We shall consider the graphs of the following functions: sin x and cos x. We usually put y = sin x, y = cos x, to be able to plot points and draw the graphs.

The graph graph of y = sin x for $0^o \leq x \leq 360^o$

x	0^o	30^o	45^o	60^o	90^o	120^o	135^o	150^o	180^o	210^o	225^o	240^o	270^o	300^o	315^o	330^o	360^o
y=sin x	0	0.50	0.71	0.87	1	0.87	0.71	0.50	0	-0.50	-0.71	-0.87	-1	-0.87	-0.71	-0.50	0



From the figure it is evident that the curve repeats itself every 360° or 2p. This fact is expressed by saying that the function has a period of 360° or 2p.

In symbols we write sin (x + n.360°) or sin (x + 2np), sin x = sin (x + n.360°) = sin (x + 2np), where n is any positive or negative integer. This infers that sin x varies and takes a complete ordered range of values once and that sin x is periodic has the period 2p. From the figure we observe that as x increases from 0° to 90°, sin x increases from 0 to 1 and as x increases from 90° to 180°, sin x decreases from 1 to 0.

[A function f(x) is periodic with period T if f (x+T) = f(x) for all values of x]

As x increases from 180° to 270°, sin x decreases from 0 to -1 and as x increases from 270° to 360°, sin x increases from -1 to 0. The maximum absolute value of sin x = 1.

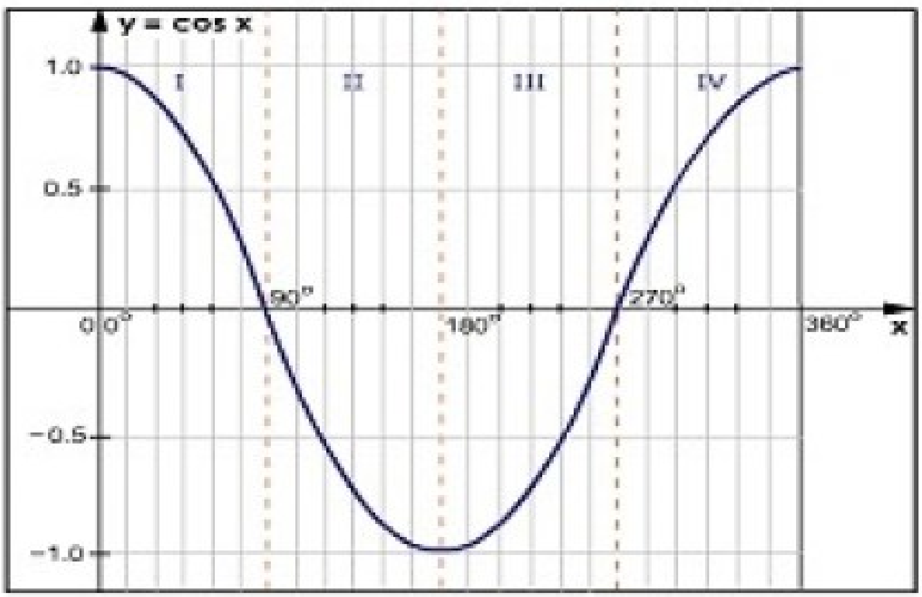
The graph of y = cos x, 0° ≤ x ≤ 360°

In this page we are going to discuss about **Graph of cosine function.** The trigonometric function cosine is defined as the ratio of adjacent side to the hypotenuse. The value of cosine always lies between 1 and -1.Trignometric functions are cyclic functions. It repeats the shape again and again. Study of trigonometric functions are used in many other fields of physics, chemistry and biology like simple harmonic motion, electronics

Properties

Satisfies point symmetry, Domain is all Real Numbers, Range is lies between -1 and 1, Period is 2Pi, From 0 to Pi it decreases and from Pi to 2Pi it increases.

x	0°	30°	45°	60°	90°	120°	135°	150°	180°	210°	225°	240°	270°	300°	315°	330°	360°
y=cosx	1	0.87	0.71	0.50	0	-0.50	-0.71	-0.87	-1	-0.87	-0.71	-0.50	0	0.50	0.71	0.87	1



From the graph it is clear that the curve repeats itself every 360° (2p rad). This fact is expressed by the statement that the function has a period of 360° (2p radians). In symbols we write cos (x + 360°. n) = cos (x + 2pn) = cos x where n is a positive or negative integer.

From the graph we also observe that cos x does not pass through the origin. The maximum and minimum values of cos x are +1 and -1 respectively. As x increases from 0° to 90° cos x decreases from 1 to 0, as x increases from 90° to 180° cos x decreases from 0 to -1, as x increases from 180° to 270° cos x increases from -1 to 0, as x increases from 270° to 360° cos x increases from 0 to 1. cos x is period and has a period 2p.

ASSESSMENT

1. Consider a right angle triangle PQR, if PQ is 27 and QR is 17 then value of angle P is

   (a) 32.19°

   (b) 45.19°

   (c) 49.58°

   (d) 62.46°

2. Cos P of triangle PQR with respect to P is calculated as

   (a) QR/PQ

   (b) QR/PR

   (c) PQ/PR

   (d) PR/PQ

3. Corporate office building is 50m high and angle of elevation at top of building is 52° when seen from a point on level ground. distance between point and foot of building is

   (a) 21.35 m

   (b) 52 m

   (c) 25 m

   (d) 31.25

4. Answer of tan 40° up to three significant figures is

   (a) 0.583

   (b) 0.839

   (c) 1.839

   (c) 2.839

5. Consider a right angle triangle XYZ, XY is 17 and XZ is supposed as unknown number ‘a’ then X 54.26° up to three significant figures is

   (a) 25.1

   (b) 26.1

   (c) 27.1

   (d) 29.1

ANSWERS

1. a
2. c
3. d
4. b
5. d

Week 9

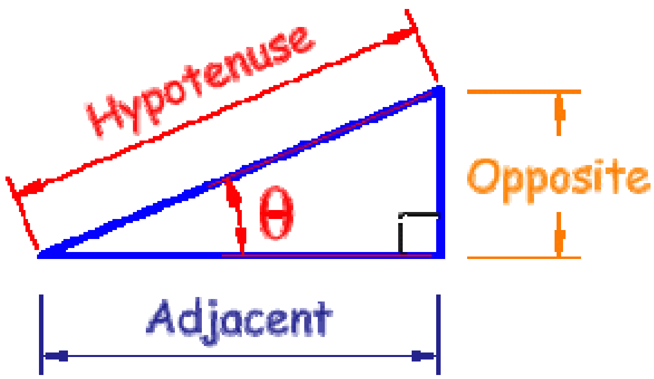
Application Of Sine, Cosine And Tangent

TOPIC: APPLICATION OF SINE, COSINE AND TANGENT SIMPLE PROBLEMS WITH RESPECT TO RIGHT ANGLE TRIANGLES

Right Triangle

Sine, Cosine and Tangent are all based on a Right-Angled Triangle

Before getting stuck into the functions, it helps to give a **name** to each side of a right triangle:



"Opposite" is opposite to the angle θ

"Adjacent" is adjacent (next to) to the angle θ

"Hypotenuse" is the long one

Sine, Cosine and Tangent

Sine, Cosine and Tangent are the three main functions in trigonometry.They are often shortened to sin, cos and tan.

To calculate them:

Divide the length of one side by another side. But you must know which sides!

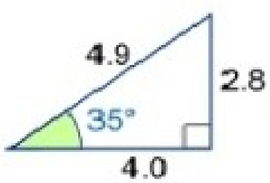
For a triangle with an angle θ, the functions are calculated this way:

Sine Function: sin (θ) = Opposite / Hypotenuse

Cosine Function: cos (θ) = Adjacent / Hypotenuse

Tangent Function: tan (θ) = Opposite / Adjacent

Example: What is the Sine of 35°



Using this triangle (lengths are only to one decimal place):

Sin (35°) = Opposite / Hypotenuse = 2.8 / 4.9 = 0.57…

How to remember? Think "SohCahToa"! It works like this:

Soh: Sine = Opposite / Hypotenuse

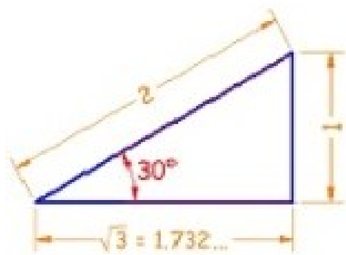
Cah: Cosine = Adjacent / Hypotenuse

Toa: Tangent = Opposite / Adjacent

You can read more about SohCahToa … please remember it, it may help in an exam!

Example: what are the sine, cosine and tangent of 30°?

The classic 30° triangle has a hypotenuse of length 2, an opposite side of length 1 and an adjacent side of √(3):



Now we know the lengths, we can calculate the functions:

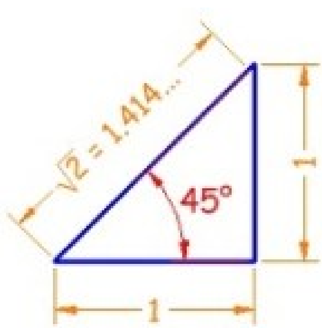
Sine: sin (30⁰) =1/2

Cosine: cos (30⁰) = 1.732/2 = 0.866

Tangent: tan (30⁰) = 1/1.732 = 0.577

Example: what are the sine, cosine and tangent of 45°?

The classic 45° triangle has two sides of 1 and a hypotenuse of √(2):



Sine: sin (45°) = 1 / 1.414 = 0.707...

Cosine: cos (45°) = 1 / 1.414 = 0.707...

Tangent: tan (45°) = 1 / 1 = 1

Angles of Elevation and Depression

We will now consider some practical applications of trigonometry in the calculation of angles of elevation and angles of depression.

The **angle of elevation** is the angle between a horizontal line from the observer and the line of sight to an object that is above the horizontal line.

In the diagram below, *AB* is the horizontal line. *q* is the angle of elevation from the observer at *A* to the object at *C*.

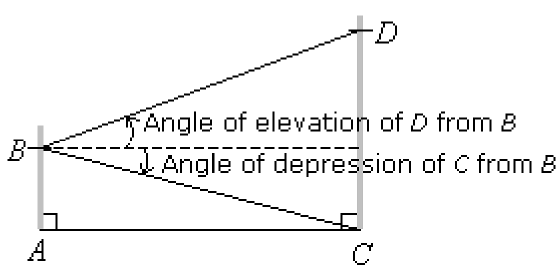
The **angle of depression** is the angle between a horizontal line from the observer and the line of sight to an object that is below the horizontal line.

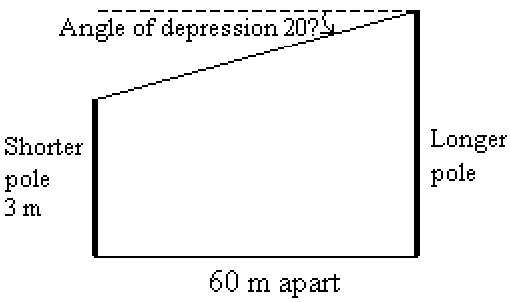
Example:

In the diagram below, *PQ* is the horizontal line. *q* is the angle of depression from the observer at *P* to the object at *R*.

How to define an angle of elevation or an angle of depression

Identify angles of depression and angles of elevation, and the relationship between them.



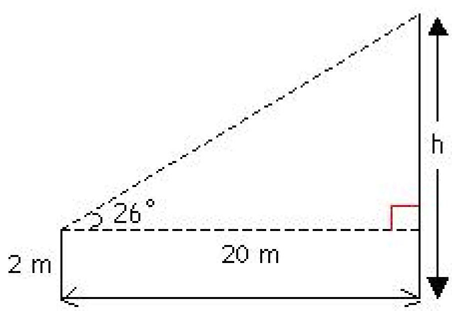


Example:

A man who is 2 m tall stands on horizontal ground 30 m from a tree. The angle of elevation of the top of the tree from his eyes is 28˚. Estimate the height of the tree.

Solution:

Let the height of the tree be *h*. Sketch a diagram to represent the situation.



tan 26˚ =

h – 2 = 20 tan 26˚

tan 26˚ = 1.179

h = (20 ´ 1.179) + 2

h = 23.58 + 2

= 25.58

The height of the tree is approximately **25.58 m**.

Bearing and Distances of Places Strictly by Application of Trigonometric Ratios

ASSESSMENT

1. $\sin(A+45°)\sin(A-45°) =$
   (a) $-1/2\cos(2A)$
   (b) $-1/2\sin(2A)$
   (c) $1/2\cos(2A)$
   (d) one of Above
2. $\sin 5\theta \cos 2\theta =$
   (a) $1/2[\sin 7\theta+\sin 3\theta]$
   (b) $[\sin 7\theta+\sin 3\theta]$
   (c) $1/2[\sin 7\theta-\sin 3\theta]$
   (d) None of Above
3. $2\sin 12° \sin 46° =$
   (a) $\cos 34° \cos 58°$
   (b) $\sin 34°+\sin 58°$
   (c) $\sin 34°-\sin 58°$
   (d) $\cos 34°-\cos 58°$
4. $(\cos(x)-\cos 3x)/(\sin 3x-\sin(x)) =$
   (a) $\cot 2x$
   (b) $\tan 2x$
   (c) $\csc 2x$
   (d) $\sec 2x$
5. $2\sin(x+45°)\sin(x-45°) =$
   (a) $\cos 2x$
   (b) $-\cos 2x$
   (c) $\sin 2x$
   (d) $-\sin 2x$

ANSWERS

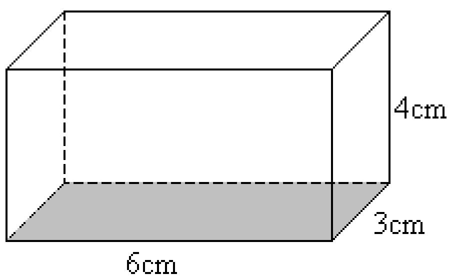
1. a
2. a
3. d
4. b
5. b

Week 10

Topic: Solids 2: Volume

The volume of a solid is a measure of the space it takes up. The cube is used as the shape for the basic unit of volume. A cube of edge 1 metre has a volume of **1 cube metre** or **1 m^3.** A cube of edge 1 centimetre has a volume of **1 cubic centimeter** or **1 cm^3.**

It is different to measure volume directly. One way is to build a copy of the solid using basic units. For example, to measure the volume of the 6 cm by 3 cm by 4 cm cuboid in the figure below, a copy can be built from 1 cm^3 cubes.



Units of volume

The **cubic metre, m^3,** is the basic unit of volume.

1 m = 100 cm

1 m^3 = (100 X 100 X 100) cm^3 = 1 000 000 cm^3

Similarly

1 cm = 10 mm

1 cm^3 = (10 X 10 x 10) mm^3 = 1 000 mm^3

When calculating problems about volume, make sure that all dimension are in the same units.

Volume of Cuboids and Cubes

Cuboid

Notice that

a. the 6 cm by 3 cm by 4 cm cuboid in the above figure has a volume of 72 cm^3

b. 6 X 3 X 4 – 72.

We can the volume of any cuboid by finding the product of its length, breadth and height:

volume of cuboid = length X breadth X height

= area of base X height

= area of end face X length

= area of side face X breadth

Cube

Similarly, for a cube of side s,

Volume of cube = (side) X (side) X (side)

= s X s X s

= s^3

Example 1

Calculate the volume of a rectangular box which measures 30 cm X 15 cm X 10 cm.

Volume of box = (30 X 15 X 10)cm^3

= 4 500 cm^3

Example 2

A rectangular room 4 m long by 3 m wide contains 30 m^3 of air.

Calculate the height of the room.

Volume of room = 30 m^3

area of floor (base) = 4 m X 3 m = 12 m^2

height of room = 30/12 m = 2½ m

Capacity of containers

The **capacity of a container** is the measure of the space inside it. The basic unit of capacity is the **litre.** 1 litre of water will just fill a 10 cm by 10 cm by 10 cm cubic container.

Therefore in practice,

1 liter = (10 X 10 X 10) cm^3 = 1 000 cm^3

The table below shows the relation between units of capacity and units of volume.

	capacity	Volume
Kilolitre	1 kl = 1 000l	= 1 000 000 cm^3 = 1 m^3
Litre	1 l	= 1 000 cm^3
Millimeter	1 ml = 0.001 l	= 1 cm^3

Example

How many litres of water dos a 5 m X 4 m X 3 m tank hold?

volume of tank = (5 X 4 X 3) m^3 = 60 m^3

but, 1 m^3 = 1 000 litres

capacity of tank = 60 X 1 000 litres

= 60 000 litres

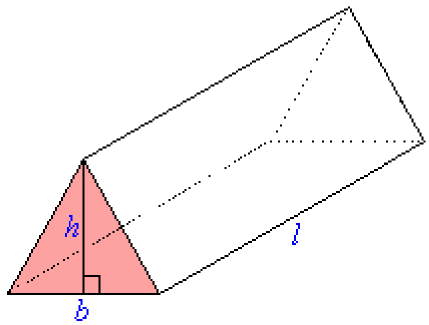
ASSESSMENT

1. A rectangular tin measure 10 cm by 10 cm by 20 cm. What is its capacity in litres?

2. Calculate the capacity in litres of a tin 20 cm by 2 cm by 10 cm.

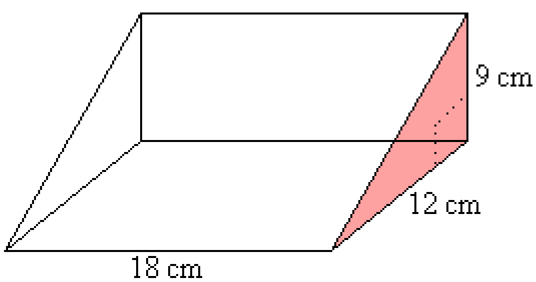
Volume of right-angled triangular prism

A triangular prism whose length is *l* units, and whose triangular cross-section has base *b* units and height *h* units, has a volume of *V* cubic units given by v = Al = ½ bhl



Example

Find the volume of the triangular prism shown in the diagram.



V = Al

= ½ bhl

= ½ X 12 X 9 X 18

= 972

ASSESSMENT

1. If diameter of cylinder is 8cm and its height is 16cm then volume of cylinder is
   (a) 804.352cm³
   (b) 1000cm³

(c) $900cm^3$
(d) $850cm^3$

2. If volume of cylinder is $900cm^2$ with height of 20cm then diameter of cylinder is
   (a) $24cm^2$
   (b) $7.57cm^2$
   (c) $9.57cm^2$
   (d) $12.23cm^2$
3. If a rectangular tank is 21cm long, 13cm wide and 18cm high and contains water up to a height of 11cm then total surface area is
   (a) $1450cm^2$
   (b) $1350cm^2$
   (c) $1200cm^2$
   (d) $1021cm^2$
4. If circular metal sheet is 0.65cm thick and of 50cm in diameter is melted and recast into cylindrical bar with 8cm diameter then length of bar will be
   (a) 24.41cm
   (b) 35.41cm
   (c) 40.41cm
   (d) 30.41cm
5. If a cuboid is 3.2 cm high, 8.9cm long and 4.7 wide then total surface area is
   (a) $170.7cm^2$
   (b) $180cm^2$
   (c) $205.7cm^2$
   (d) $325.8cm^2$

ANSWERS

1. a
2. b
3. d
4. a
5. a

SS1

MATHEMATICS

THIRD TERM

TABLE OF CONTENT

WEEK 1

Topic: MENSURATION

CUBES

A cube is a solid of uniform cross-section. It is formed by squares and has 8 vertices. An example is processed cubed sugar.

The length of a side of a cube is 'e' which is the length of all sides since a cube is formed with squares.

TOTAL SURFACE AREA OF A CUBE

A cube has 6 faces. The surface area of each side = e^2 as each side is a square.

Therefore, Total surface area of a cube (all 6 sides) = $6e^2$

The surface area of a cube is gotten by the formula

Surface area= $6e^2$ sq. units

VOLUME OF A CUBE

In a cube all sides are equal. Length=e, height=e and width=e

Therefore, Volume of a cube= length× width× height

Volume of a cube= e^3 cubic units

CYLINDERS

A Cylinder is a uniform circular cross-section. Examples of cylinders are unsharpened pencils like HB or 2B pencils, garden rollers, tins of milk or tomato et cetera

TOTAL SURFACE AREA OF A CYLINDER

There are two types of cylinders;

(1) A closed cylinder and

(2) An open cylinder

TOTAL SURFACE AREA OF A CLOSED CYLINDER

The total surface area of a closed cylinder consists of a sum of the areas of (i) the curve surface and (ii) The two circular end faces.

The curved surface when opened out is a rectangle. This rectangle has length equal to the length of the

Cylinder and the width are equal to the circumference of the circular end face.

Area of curved surface of a cylinder = area of rectangle of dimensions length (L) and width

(Circumference of base)

$= 2\pi rl$

Area of the two circular end faces= twice the area of one circular face

$= \pi r^2$

Hence the total surface area of the closed cylinder

$= 2\pi rl + 2\pi r2$ sq. units

TOTAL SURFACE AREA OF AN OPEN CYLINDER

The total surface area of an open cylinder is the area of the curved surface which is the area of the rectangle the cylinder forms when spread.

Sometimes we are given a thick hollow cylinder. The total surface area is the sum of;

(i) the area of the external curved surface

(ii) the area of the internal curved surface and

(iii) the area of the end annular faces which will be shaded.

VOLUME OF A CYLINDER

A right circular cylinder is a solid of uniform cross-section. If a paper is wrapped round a cylinder, on opening it, a rectangle will be found.

Thus if the height of a cylinder

= h units

And the base radius

= r units

Then the volume of a cylinder

= Area of base by height

= $\pi r^2 h$ cubic units

TRIANGULAR PRISMS

A prism is a solid with uniform cross-section of a shape of a triangle or a trapezium or any other polygon.

TOTAL SURFACE AREA OF A TRIANGULAR PRISM

In the case of a triangular shaped prism, the total surface area is the sum of the surface areas of the five faces that make up the prism.

VOLUME OF A TRIANGULAR PRISM

The volume of a prism is the area of its cross-section multiplied by the distance between the end faces. Examples are funnel, Chinese hat, cut periwinkle shell et cetera.

CONES

A cone is a figure with circular base and sides slanting to a common point or vertex. There are two types of cones (i) a right circular cone, where the line joining the vertex is symmetrical and perpendicular to the base of the cone and (ii) the non-right circular cone but this isn't in the syllabus.

SURFACE AREA OF A CONE

Since a cone is formed from a sector of circle, then the surface area of a cone is equal to the area of the sector that formed it. Let L be the radius of the sector, then L becomes the slant height of the cone. If r is the radius of the base of the cone, then the length of arc of the sector is equal to 2pi.r which equals the circumference of the base of the cone.

If the sector subtends an angle- which it always does- then the area of the sector will be equal to

= $\theta /360 \times \pi l^2$ = curved surface of a cone

But $2\pi r$ = length of arc of a sector.

Therefore, $2\pi r = \theta / 360 \times 2\pi l$

Finally, surface area of a cone = πrl

TOTAL SURFACE AREA OF A CONE

The total surface area of cone is the sum of (i) the curved surface area πrl sq. units and (ii) the area of the base of the cone πr^2 sq. units.

Therefore, total surface area of a cone = $\pi r^2 + \pi rl$

ASSESSMENT

1. How many vertices does a cube have?
2. What is the formula for the volume of a cylinder?
3. What are the two types of cylinders?

Week 2

Topic: VOLUME OF A CONE

Volume of a cone = $1/3 \times \pi r^2 h$

This is the formula for the volume of all cones.

RECTANGULAR BASED PYRAMID

A pyramid is a solid whose base is a polygon and has a common point or vertex. A pyramid is named according to its base, evidently the pyramid in question here has a rectangular base.

TOTAL SURFACE AREA OF A RECTANGULAR BASED PYRAMID

In the case of a pyramid, the total surface area is found by summing up the areas of the common shapes that make up the pyramid.

VOLUME OF A RECTANGULAR BASED PYRAMID

Since a pyramid is shaped like a cone with the pyramid having a polygonal base, the volume of a pyramid is also 1/3 the base area height.

Therefore, volume of pyramid= 1/3 × the product of base area and perpendicular height.

FRUSTUMS (FRUSTRA) OF CONES AND PYRAMIDS

A frustum is the remaining part of a cone or pyramid when the top part is cut off. Daily examples of frustums are buckets, lamp shades et cetera.

TOTAL SURFACE AREA OF A FRUSTUM

The total surface area of a frustum is obtained the same way as the total surface of solid objects. That is, we sum up all the areas of the surfaces that make up the frustum in the case of the frustum of a pyramid.

Therefore, Total surface area of a Closed frustum = π (height × sum of radii) + area of top and base circles, (or any other polygon as in the case of pyramids).

VOLUME OF FRUSTUMS OF CONES AND PYRAMIDS

Volume of the frustum = volume of the cone/pyramid – volume of the part cut off.

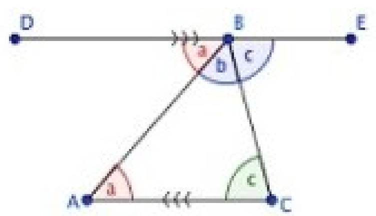
Proofs of Angles

Theorem. The sum of the interior angles of any triangle is 180°.

Here are three proofs for the sum of angles of triangles. Proof 1 uses the fact that the alternate interior angles formed by a transversal with two parallel lines are congruent. Proof 2 uses the exterior angle theorem. Proof 3 uses the idea of transformation specifically rotation.

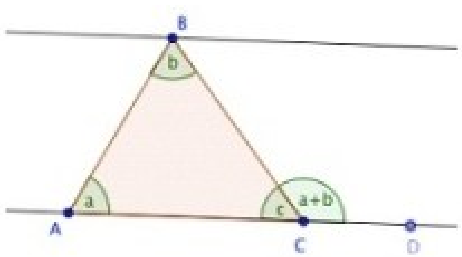
Proof 1

Construct a line through B parallel to AC. Angle DBA is equal to CAB because they are a pair of alternate interior angle. The same reasoning goes with the alternate interior angles EBC and ACB.



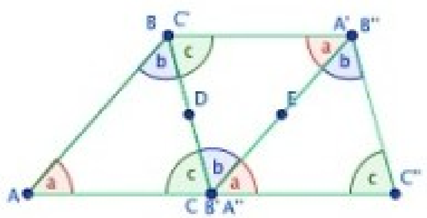
Proof 2

This is similar to Proof 1 but the justification used is the exterior angle theorem which states that the measure of the exterior angle of a triangle is the sum of the measures of the two remote interior angles. In the diagram, angle A and angle B are the remote interior angles and angle BCD is the exterior angle.



Proof 3

Rotate ΔABC 180° clockwise about D the midpoint of BC. This rotation produces the image ΔA'B'C'. Rotate the image again by 180° clockwise but this time about E, the midpoint of A'B'. In the figure you can see that a + b + c forms a straight line and heznce measures 180°.



In order to study geometry in a logical way, it will be important to understand key mathematical properties and to know how to apply useful postulates and theorems. A **postulate** is a proposition that has not been proven true, but is considered to be true on the basis for mathematical reasoning. **Theorems**, on the other hand, are statements that have been proven to be true with the use of other theorems or statements. While some postulates and theorems have been introduced in the previous sections, others are new to our study of geometry. We will apply these properties, postulates, and theorems to help drive our mathematical proofs in a very logical, reason-based way.

Before we begin, we must introduce the concept of congruency. Angles are **congruent** if their measures, in degrees, are equal. ***Note***: "congruent" does not mean "equal." While they seem quite similar, congruent angles do not have to point in the same direction. The only way to get equal angles is by piling two angles of equal measure on top of each other.

Properties

We will utilize the following properties to help us reason through several geometric proofs.

Reflexive Property

A quantity is equal to itself.

Symmetric Property

If $A = B$, then $B = A$.

Transitive Property

If $A = B$ and $B = C$, then $A = C$.

Addition Property of Equality

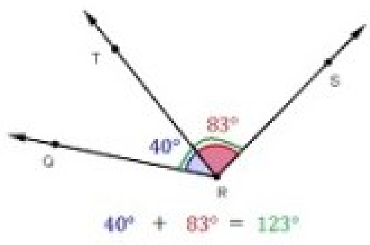
If $A = B$, then $A + C = B + C$.

Angle Postulates

Angle Addition Postulate

If a point lies on the interior of an angle, that angle is the sum of two smaller angles with legs that go through the given point.

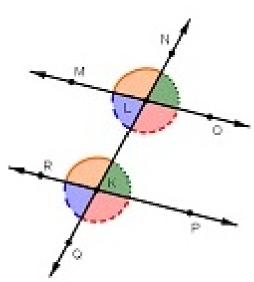
Consider the figure below in which point *T* lies on the interior of ∠ ***QRS***. By this postulate, we have that ∠ ***QRS* = ∠ *QRT* + ∠ *TRS***. We have actually applied this postulate when we practiced finding the complements and supplements of angles in the previous section.



Corresponding Angles Postulate

If a transversal intersects two **parallel** lines, the pairs of corresponding angles are congruent.

Converse also true: If a transversal intersects two lines and the corresponding angles are congruent, then the lines are parallel.

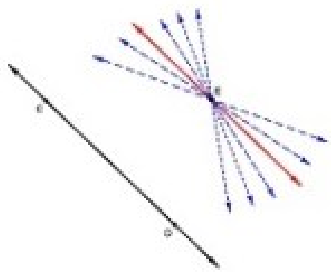


The figure above yields four pairs of corresponding angles.

Parallel Postulate

Given a line and a point not on that line, there exists a unique line through the point parallel to the given line.

The parallel postulate is what sets Euclidean geometry apart from non-Euclidean geometry.



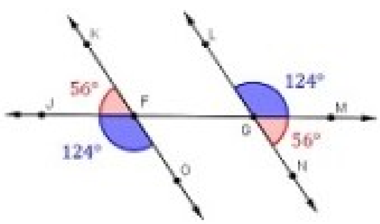
*There are an infinite number of lines that pass through point **E**, but only the red line runs parallel to line **CD**. Any other line through **E** will eventually intersect line **CD**.*

Angle Theorems

Alternate Exterior Angles Theorem

If a transversal intersects two **parallel** lines, then the alternate exterior angles are congruent.

Converse also true: If a transversal intersects two lines and the alternate exterior angles are congruent, then the lines are parallel.

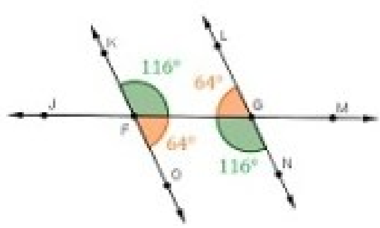


The alternate exterior angles have the same degree measures because the lines are parallel to each other.

Alternate Interior Angles Theorem

If a transversal intersects two **parallel** lines, then the alternate interior angles are congruent.

Converse also true*: If a transversal intersects two lines and the alternate interior angles are congruent, then the lines are parallel.*



The alternate interior angles have the same degree measures because the lines are parallel to each other.

Congruent Complements Theorem

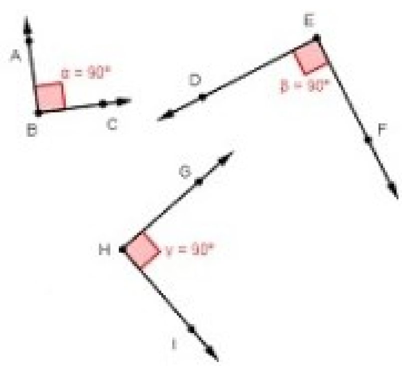
If two angles are complements of the same angle (or of congruent angles), then the two angles are congruent.

Congruent Supplements Theorem

If two angles are supplements of the same angle (or of congruent angles), then the two angles are congruent.

Right Angles Theorem

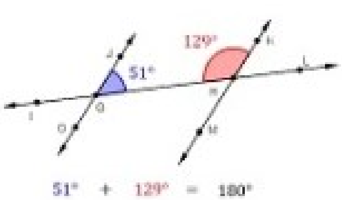
All right angles are congruent.



Same-Side Interior Angles Theorem

If a transversal intersects two **parallel** lines, then the interior angles on the same side of the transversal are supplementary.

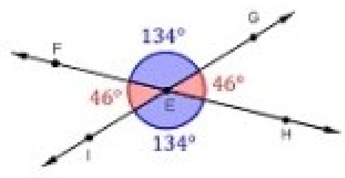
Converse also true: *If a transversal intersects two lines and the interior angles on the same side of the transversal are supplementary, then the lines are parallel.*



The sum of the degree measures of the same-side interior angles is 180°.

Vertical Angles Theorem

If two angles are vertical angles, then they have equal measures.

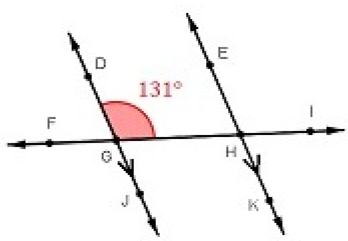


The vertical angles have equal degree measures. There are two pairs of vertical angles.

Exercises

(1) Given: *m∠ DGH = 131*

Find: *m∠ GHK*



First, we must rely on the information we are given to begin our proof. In this exercise, we note that the measure of ∠ *DGH* is ***131°***.

From the illustration provided, we also see that lines ***DJ*** and ***EK*** are parallel to each other. Therefore, we can utilize some of the angle theorems above in order to find the measure of ∠ *GHK*.

We realize that there exists a relationship between ∠ *DGH* and ∠ *EHI*: they are corresponding angles. Thus, we can utilize the **Corresponding Angles Postulate** to determine that ∠ *DGH* is ∠ *EHI*.

Directly opposite from ∠ *EHI* is ∠ *GHK*. Since they are vertical angles, we can use the **Vertical Angles Theorem**, to see that ∠ *EHI* ∠ *GHK*.

Now, by **transitivity**, we have that ∠ *DGH* is ∠ *GHK*.

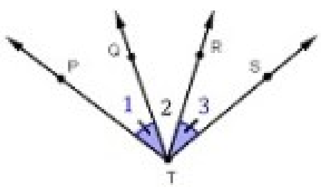
Congruent angles have equal degree measures, so the measure of ∠ *DGH* is equal to the measure of ∠ *GHK*.

Finally, we use **substitution** to conclude that the measure of $\angle GHK$ is $131°$. This argument is organized in two-column proof form below.

Statements	Reasons
1. $m\angle DGH = 131$	1. Given
2. $\angle DGH \cong \angle EHI$	2. Corresponding Angles Postulate
3. $\angle EHI \cong \angle GHK$	3. Vertical Angles Theorem
4. $\angle DGH \cong \angle GHK$	4. Transitive Property
5. $m\angle DGH = m\angle GHK$	5. Definition of congruent angles
6. $131 = m\angle GHK$	6. Substitution

(2) Given: $m\angle 1 = m\angle 3$

Prove: $m\angle PTR = m\angle STQ$



We begin our proof with the fact that the measures of $\angle 1$ and $\angle 3$ are equal.

In our second step, we use the **Reflexive Property** to show that $\angle 2$ is equal to itself.

Though trivial, the previous step was necessary because it set us up to use the **Addition Property of Equality** by showing that adding the measure of $\angle 2$ to two equal angles preserves equality.

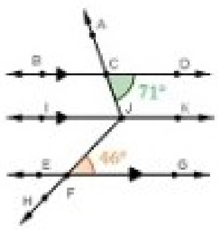
Then, by the **Angle Addition Postulate** we see that $\angle PTR$ is the sum of $\angle 1$ and $\angle 2$, whereas $\angle STQ$ is the sum of $\angle 3$ and $\angle 2$.

Ultimately, through **substitution**, it is clear that the measures of $\angle PTR$ and $\angle STQ$ are equal. The two-column proof for this exercise is shown below.

Statements	Reasons
1. $m\angle 1 = m\angle 3$	1. Given
2. $m\angle 2 = m\angle 2$	2. Reflexive Property
3. $m\angle 1 + m\angle 2 = m\angle 3 + m\angle 2$	3. Addition Property of Equality
4. $m\angle 1 + m\angle 2 = m\angle PTR$ $m\angle 3 + m\angle 2 = m\angle STQ$	4. Angle Addition Postulate
5. $m\angle PTR = m\angle STQ$	5. Substitution

(3) Given: *$m\angle DCJ = 71$, $m\angle GFJ = 46$*

Prove: *$m\angle AJH = 117$*



We are given the measure of $\angle$ *DCJ* and $\angle$ *GFJ* to begin the exercise. Also, notice that the three lines that run horizontally in the illustration are parallel to each other. The diagram also shows us that the final steps of our proof may require us to add up the two angles that compose $\angle$ *AJH*.

We find that there exists a relationship between $\angle$ *DCJ* and $\angle$ *AJI*: they are alternate interior angles. Thus, we can use the **Alternate Interior Angles Theorem** to claim that they are congruent to each other.

By the definition of **congruence**, their angles have the same measures, so they are equal.

Now, we **substitute** the measure of $\angle$ *DCJ* with *71* since we were given that quantity. This tells us that $\angle$ *AJI* is also *71°*.

Since $\angle$ *GFJ* and $\angle$ *HJI* are also alternate interior angles, we claim congruence between them by the **Alternate Interior Angles Theorem**.

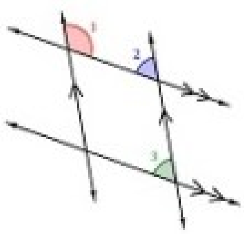
The definition of congruent angles once again proves that the angles have equal measures. Since we knew the measure of $\angle$ *GFJ*, we just **substitute** to show that *46* is the degree measure of $\angle$ *HJI*.

As predicted above, we can use the **Angle Addition Postulate** to get the sum of $\angle$ *AJI* and $\angle$ *HJI* since they compose $\angle$ *AJH*. Ultimately, we see that the sum of these two angles gives us *117°*. The two-column proof for this exercise is shown below.

Statements	Reasons
1. $m\angle DCJ = 71$	1. Given
2. $m\angle GFJ = 46$	2. Given
3. $\angle DCJ \cong \angle AJI$	3. Alternate Interior Angles Theorem
4. $m\angle DCJ = m\angle AJI$	4. Definition of congruent angles
5. $71 = m\angle AJI$	5. Substitution
6. $\angle GFJ \cong \angle HJI$	6. Alternate Interior Angles Theorem
7. $m\angle GFJ = m\angle HJI$	7. Definition of congruent angles
8. $46 = m\angle HJI$	8. Substitution
9. $m\angle AJH = m\angle AJI + m\angle HJI$	9. Angle Addition Postulate
10. $m\angle AJH = 71 + 46$	10. Substitution
11. $m\angle AJH = 117$	11. Simplify

(4) Given: $m\angle 1 = 4x + 9$, $m\angle 2 = 7(x + 4)$

Find: $m\angle 3$



In this exercise, we are not given specific degree measures for the angles shown. Rather, we must use some algebra to help us determine the measure of $\angle 3$. As always, we begin with the information given in the problem. In this case, we are given equations for the measures of $\angle 1$ and $\angle 2$. Also, we note that there exists two pairs of parallel lines in the diagram.

By the **Same-Side Interior Angles Theorem**, we know that that sum of $\angle 1$ and $\angle 2$ is *180* because they are supplementary.

After **substituting** these angles by the measures given to us and simplifying, we have $11x + 37 = 180$. In order to solve for x, we first subtract both sides of the equation by *37*, and then divide both sides by *11*.

Once we have determined that the value of x is *13*, we plug it back in to the equation for the measure of $\angle 2$ with the intention of eventually using the **Corresponding Angles Postulate**. Plugging *13* in for x gives us a measure of *119* for $\angle 2$.

Finally, we conclude that ∠ *3* must have this degree measure as well since ∠ *2* and ∠ *3* are **congruent**. The two-column proof that shows this argument is shown below.

Statements	Reasons
1. $m\angle 1 = 4x + 9$	1. Given
2. $m\angle 2 = 7(x + 4)$	2. Given
3. $m\angle 1 + m\angle 2 = 180$	3. Same-Side Interior Angles Theorem
4. $4x + 9 + 7(x + 4) = 180$	4. Substitution
5. $4x + 9 + 7x + 28 = 180$	5. Distributive Property
6. $11x + 37 = 180$	6. Simplify
7. $11x = 43$	7. Subtract 37 from each side
8. $x = 13$	8. Divide each side by 11
9. $m\angle 2 = 7((13) + 4)$	9. Substitution
10. $m\angle 2 = 119$	10. Simplify
11. $\angle 2 \cong \angle 3$	11. Corresponding Angles Postulate
12. $m\angle 2 = m\angle 3$	12. Definition of congruent angles
13. $119 = m\angle 3$	13. Substitution

<u>ASSESSMENT</u>

1. What is the Corresponding angles postulate?
2. What is the Vertical angle theorem?
3. What sets Euclidean geometry apart from non-Euclidean geometry?
4. What is the sum of all interior angles of a triangle?

Week 3

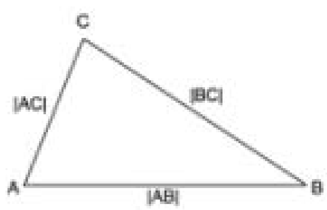
Topic: Content Geometrical Construction

Revision of Construction of Triangle

A triangle can be constructed with a compass, a ruler and a protractor. You also need some squared paper and a pencil.

At first you need an overview of the information about the triangle you have.

A good place to start is by drawing the base line, which is the basis of the further construction of the triangle.



In the following explanations are the vertices of a triangle called A, B and C, and the side lengths are called |AB|, |AC| and |BC|. We will explain how the construction of the triangle is carried out, depending on which of the mentioned information that are given.

Not all possible situations are explained, but hopefully it's enough to give you an idea of what to do, when the triangle's information is presented to you.

If all the side lengths are given:

First you draw the base line |AB|.
Then you set your compass to the length |AC|, put the needle point in the point A and draw a circle.
Then you set your compass to the length |BC|, put the needle point in the point B and draw a circle.
In the point of intersection between the two circles, you'll find the point C. The triangle is complete when you connect all three points with straight lines.

If the side length |AB|, the angle A and the side length |AC| is given:

First you draw the line |AB|. You put your protractor in the point A and measure the angle A from the line |AB|. You mark the angle with a small dot.

Then you draw a straight line passing through A and the small dot.
On this line you measure the length |AC|, and you will get the point C.
You can now connect B and C with a straight line.

If the length |AC|, the angle A and the angle C is given:

First you draw the line |AC|.
Then you measure the angle A and put a small mark.
You draw a straight line passing through A and the small dot.
You do the same with the angle C.
In the point of intersection between the two lines, you'll find the point B, and you can complete the triangle.

If the length |AC|, the angle A and the angle B is given:

First you calculate the angle C. It's possible, because you know that the angle sum of a triangle is 180˚. The angle C is therefore given by C = 180-A-B.
Now you have the length |AC|, the angle A and the angle C, and that is exactly the same as in the example above.

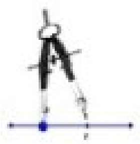
Construct an Isosceles Triangle Using Given Segment Lengths:

When constructing an isosceles triangle, you may be given pre-determined segment lengths to use for the triangle (such as in this example), or you may be allowed to determine your own segment lengths. Either way, the construction process will be the same.

Construct an isosceles triangle whose legs and base are of the pre-determined lengths given. Construct the new triangle on the reference line.

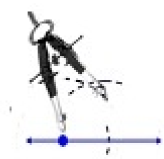
Using your compass, measure the length of the given "base".

Do not change the size of the compass. Place your compass point on the reference line point and scribe a small arc which will cross the line.

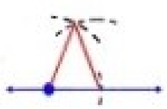


Using your compass, measure the length of the given “leg”. Place the compass point where the previous arc crosses the reference line and scribe another arc above the reference line

Without changing the size of the compass, move the compass point to the point on the reference line. Scribe an arc above the line such that it intersects with the previous arc.



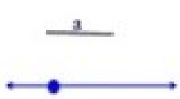
You now have three points which will define the isosceles triangle.



Construct an Equilateral Triangle Using a Given Segment Length:

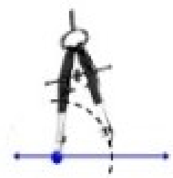
When constructing an equilateral triangle, you may be given a pre-determined segment length to use for the triangle (such as in this example), or you may be allowed to determine your own segment length. Either way, the construction process will be the same.

Construct an equilateral triangle whose sides are of given length “a”. Construct the new triangle on the reference line.

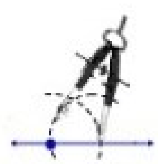


Using your compass, measure the length of the given segment, “a”.

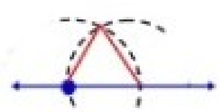
Do not change the size of the compass. Place your compass point on the reference line point and scribe an arc which will cross the line and will rise above the line.



Do not change the size of the compass. Place the compass point where the arc crosses the reference line and scribe another arc which crosses the previous arc.



You now have three points which will define the equilateral triangle.

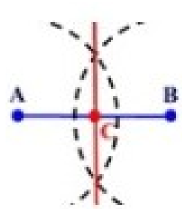


Drawing and Bisection of Line Segment

Given: (Line segment) $\overline{AB}$

Task: Bisect . $\overline{AB}$

Directions:



Place your compass point on A and stretched the compass MORE THAN half way to point B, but not beyond B.

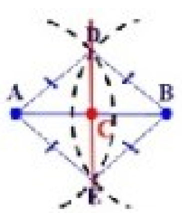
2. With this length, swing a large arc that will go BOTH above and below $\overline{AB}$.

(If you do not wish to make large continuous arc, you may simply place one small arc above $\overline{AB}$ and one small arc below $\overline{AB}$.)

3. Without changing the span on the compass, place the point on B and swing the arc again. The two arcs you have created should intersect.

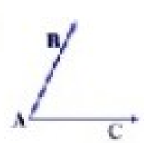
4. With your straightedge, connect the two points of intersection.
5. This new straight line bisects $\overline{AB}$. Label the point where the new line and $\overline{AB}$ cross as C. $\overline{AB}$ has now been bisected and bars AC ≅ CB. (It could also be said that the segments are congruent, .)

Explanation of construction: To understand the explanation you will need to label the point of intersection of the arcs above segment $\overline{AB}$ as *D* and below segment $\overline{AB}$ as *E.* Draw segments bar AD, bar AE , bar BD and bar BE . All four of these segments are of the same length since they are radii of two congruent circles. More specifically, *DA = DB* and *EA = EB*. Now, remember a locus theorem: The locus of points equidistant from two points, is the perpendicular bisector of the line segment determined by the two points. Hence, bar DE is the **perpendicular** bisector of $\overline{AB}$.
The fact that the bisector is also perpendicular to the segment is actually MORE than we needed for a simple "bisect" construction. Isn't this great! Free stuff!!!

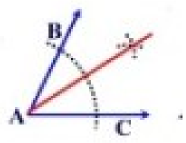


Construction and Bisection of Angles

Given: BAC
Task: Bisect BAC.



Directions:



1. Place the point of the compass on the vertex of BAC (point A).

2. Stretch the compass to any length so long as it stays ON the angle.

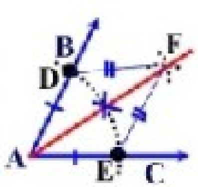
3. Swing an arc so the pencil crosses both sides BAC of . This will create two intersection points with the sides (rays) of the angle.

4. Place the compass point on one of these new intersection points on the sides of BAC.

If needed, stretch your compass to a sufficient length to place your pencil well into the interior of the angle. Stay between the sides (rays) of the angle. Place an arc in this interior – you do not need to cross the sides of the angle.
5. Without changing the width of the compass, place the point of the compass on the other intersection point on the side of the angle and make the same arc. Your two small arcs in the interior of the angle should be crossing.
6. Connect the point where the two small arcs cross to the vertex *A* of the angle.

You have now created two new angles that are of equal measure (and are each 1/2 the measure of BAC .)

Explanation of construction: To understand the explanation, some additional labeling will be needed. Label the point where the arc crosses side $\overline{AB}$ as *D*. Label the point where the arc crosses side bar AC as *E*. And label the intersection of the two small arcs in the interior as *F*. Draw segments bar DF and bar EF. By the construction, *AD = AE* (radii of same circle) and DF = EF (arcs of equal length), of course *AF = AF*. All of these sets of equal length segments are also congruent. We have congruent triangles by SSS. Since the triangles are congruent, any of their leftover corresponding parts are congruent which makes BAF equal (or congruent) to CAF.



In this lesson, we will learn how to construct a 45 degrees angle by bisecting a 90 degrees angle.

We can use a similar angle bisector method to construct some other angles from existing angles.

Example:

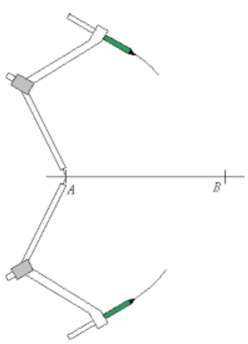
A 30° angle can be obtained by bisecting a 60° angle.
A 15° angle can be obtained by bisecting a 30° angle.
A 45° angle can be obtained by bisecting a 90° angle.
A 22.5° angle can be obtained by bisecting a 45° angle.
Example:

The figure shows a point A on a straight line. Construct an angle of 45° at point A.

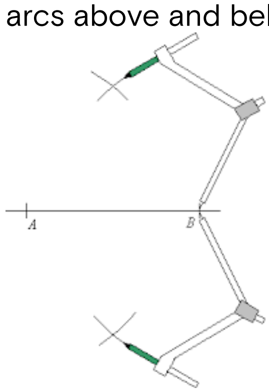
Solution:

Construct a 90° angle, and then construct an angle bisector to obtain a 45° angle.

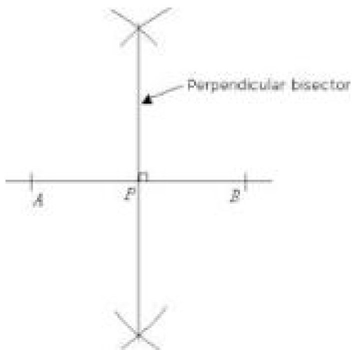
Step 1: Stretch your compasses until it is more then half the length of AB. Put the sharp end at A and mark an arc above and another arc below line segment AB.



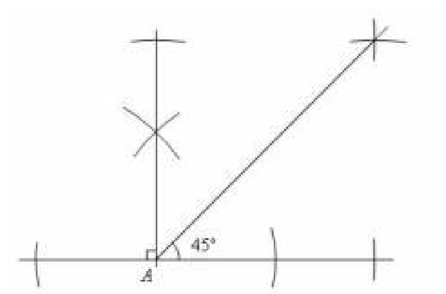
Step 2: Without changing the width of the compasses, put the sharp end at B and mark arcs above and below the line segment AB that will intersect with the arcs drawn in step 1.



Step 3: Join the two points where the arcs intersect with a straight line. This line is the perpendicular bisector of *AB*. *P* is the midpoint of *AB*.



Step 4: Bisect the 90 degree angle to form a 45 degree angle.

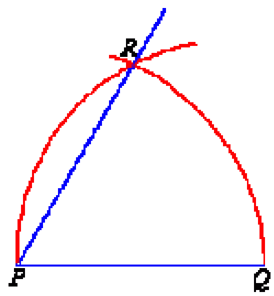


Constructing a 60° Angle

We know that the angles in an equilateral triangle are all 60° in size. This suggests that to construct a 60° angle we need to construct an equilateral triangle as described below.

Step 1: Draw the arm *PQ*.
Step 2: Place the point of the compass at *P* and draw an arc that passes through *Q*.
Step 3: Place the point of the compass at *Q* and draw an arc that passes through *P*. Let this arc cut the arc drawn in Step 2 at *R*.

Step 4: Join P to R. The angle QPR is 60^0, as the ΔPQR is an equilateral triangle.

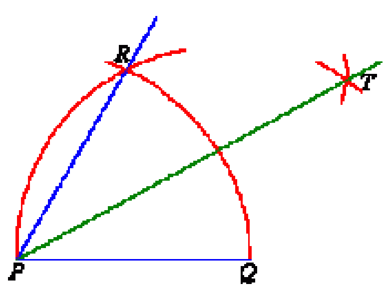


Constructing a 30° Angle

We know that: ½ of $60^{O} = 30^{O}$

So, to construct an angle of 30°, first construct a 60° angle and then bisect it. Often, we apply the following steps.

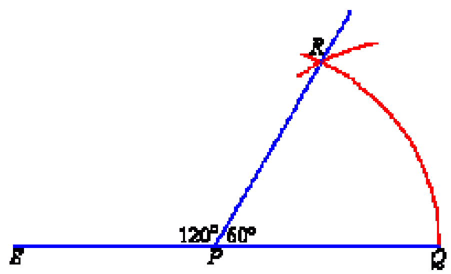
Step 1: Draw the arm *PQ*.
Step 2: Place the point of the compass at *P* and draw an arc that passes through *Q*.
Step 3: Place the point of the compass at *Q* and draw an arc that cuts the arc drawn in Step 2 at *R*.
Step 4: With the point of the compass still at *Q*, draw an arc near *T* as shown.
Step 5: With the point of the compass at *R*, draw an arc to cut the arc drawn in Step 4 at *T*.
Step 6: Join *T* to *P*. The angle *QPT* is 30°.



Constructing a 120° Angle

We know that $60^{O} + 120^{O} = 180^{O}$

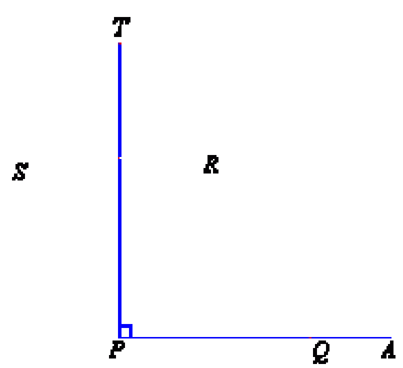
This means that 120° is the supplement of 60°. Therefore, to construct a 120° angle, construct a 60° angle and then extend one of its arms as shown below.



Constructing a 90° Angle

We can construct a 90° angle either by bisecting a straight angle or using the following steps.

Step 1: Draw the arm *PA*.
Step 2: Place the point of the compass at *P* and draw an arc that cuts the arm at *Q*.
Step 3: Place the point of the compass at *Q* and draw an arc of radius *PQ* that cuts the arc drawn in Step 2 at *R*.
Step 4: With the point of the compass at *R*, draw an arc of radius *PQ* to cut the arc drawn in Step 2 at *S*.
Step 5: With the point of the compass still at *R*, draw another arc of radius *PQ* near *T* as shown.
Step 6: With the point of the compass at *S*, draw an arc of radius *PQ* to cut the arc drawn in step 5 at *T*.
Step 7: Join *T* to *P*. The angle *APT* is 90°.



Example:

a. Use a ruler and compass only to construct a triangle ABC with AB = 5cm, ∠BAC = 60° and AC = 4.5 cm.

b. Measure the size of ∠ABC and the size of ∠ACB. Hence, calculate the angle sum of triangle ABC.

c. Measure BC to the nearest millimeter. Hence, find the perimeter of the triangle ABC in millimeters.

Solution:

a. Step 1: Draw a line, AB, 5cm long.

Step 2: Use the compass to construct a 60^0 angle at A.

Step 3: Use the ruler to find C such that AC is 4.5 cm long.

Step 4: Join B to C.

The Δ ABC is the required triangle.

b. Using a protractor, we find that:

$\angle ABC = 55^0$

$\angle ACB = 65^0$

∴ Angle sum of the triangle ABC = $60^0 + 55^0 + 65^0$

= 180^0

c. Using the ruler, we find that:

BC = 48 mm

∴ perimeter = AB + BC + CA

= 5 cm + 48 mm + 4.5 cm

= 50 mm + 48 mm + mm

= 143 mm

ASSESSMENT

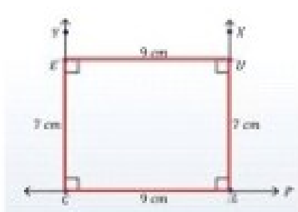
1. What are the instruments needed to construct a triangle?
2. Construct a 90 degree angle.
3. Construct a triangle assuming all sides are given.

Week 4

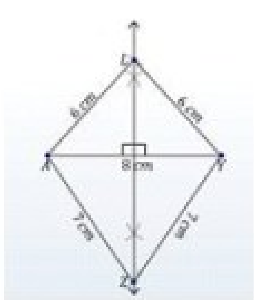
Topic: Construction of Quadrilateral Polygon

We can identify different **quadrilaterals** based on the properties sides, diagonals and angles.
Quadrilaterals are made up of ten parts. However, to construct them, you do not need to know the measurements of all of them.
In case of special **quadrilaterals**, like the **rectangle**, just two measurements, the lengths of its adjacent sides are enough to construct it.
A **kite** can be constructed if the lengths of its distinct adjacent sides and one diagonal are known.
Similarly, a **square** can be constructed with just the length of its side, while a **rhombus** can be constructed when the lengths of its diagonals are known.

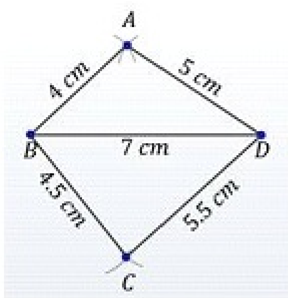
Steps to Construct a Rectangle:
Step 1: Draw a side of given length (say) CL
Step 2: Draw side LU (say) of given length perpendicular to CL at L.
Step 3: Draw side CE (say) of length equal to LU and perpendicular to CL at C.
Step 4: Draw side UE.



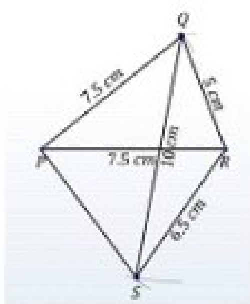
Steps to Construct a Kite:
Step 1: Draw diagonal (say) AY and its perpendicular bisector.
Step 2: Draw sides say AL and AZ of given length.
Step 3: Draw sides LY and YZ.



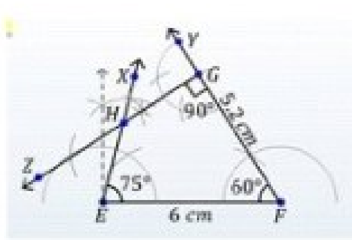
Constructing a Quadrilateral when four sides and one of its diagonals are given.
Step 1: Construct a triangle ABD (say).
Step 2: Find point C opposite to the vertex A as follows. With B as the centre and given radius, draw an arc on the other side of BD. Similarly with D as the centre and given radius, draw another arc intersecting the previous arc. The point of intersection of these arcs is marked as C.
Step 3: join points B and C, and D and C.



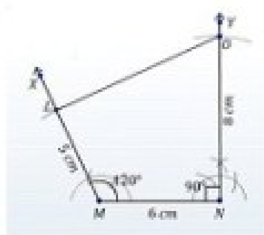
Constructing a quadrilateral when lengths of its three sides and two diagonals are given.
Step 1: Construct a triangle PQR (say).



Constructing a quadrilateral when lengths of its adjacent sides and three angles are given.
Step 1: Draw a line segment of EF (say) of given length.
Step 2: Construct a given angle at E.
Step 3: Construct a given angle at F.
Step 4: Locate point G.
Step 5: Locate point H.



Constructing a quadrilateral when lengths of its three sides and two included angles.
Step 1: Draw a line segment MN (say) of given length.
Step 2: Construct a given angle at M.
Step 3: Construct an angle 90° at N.
Step 4: Locate vertices L and O.
Step 5: Join L and O.



Constructing an equilateral triangle

Constructing an equilateral triangle also known as drawing an equilateral triangle using only a straightedge and a compass is what I will show you here

Step #1:

Take your ruler and a pencil and construct a segment of any length on a piece of paper as shown below

Then, you will try to set your compass opening to match the length of segment AB

Take your compass. Make your sure that the pencil is included in it.

Put the needle of the compass at endpoint A and adjust your compass so that the tip of your pencil touches endpoint B

Step #2:

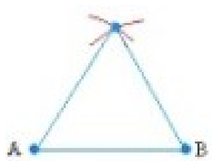
Put the needle of your compass at A and draw an arc

Put the needle of your compass at B and draw an arc

The two arcs should meet as shown below:

Step #3:

Draw the segments from the two endpoints to the point where the two arcs intersect



Locus of Moving Points Including Equidistance from Two Lines of Two Points and Constant Distant From the Point

When a point moves in a plane according to some given conditions the path along which it moves is called a **locus**. (Plural of locus is **loci.**).

CONDITION 1 :
A point *P* moves such that it is always *m* units from the point *Q*

Locus formed: A circle with centre Q and radius *m.*

Example :

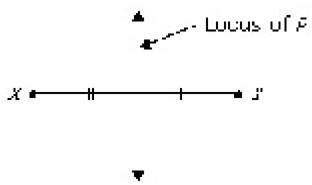
Construct the locus of a point *P* at a constant distance of 2 cm from a fixed point *Q.*

Solution:

Construct a circle with centre *Q* and radius 2 cm.

CONDITION 2:
A point *P* moves such that it is equidistant from two fixed points *X* and *Y*

Locus formed: A perpendicular bisector of the line *XY*.

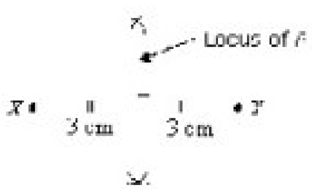


Example:

Construct the locus of point *P* moving equidistant from fixed points *X* and *Y* and *XY* = 6 cm.

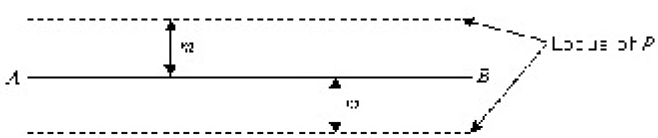
Solution:

Construct a perpendicular bisector of the line XY.



CONDITION 3:
A point P moves so that it is always m units from a straight line AB

Locus formed: A pair of parallel lines *m* units from *AB*.

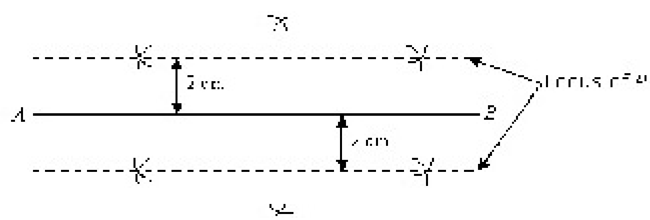


Example:

Construct the locus of a point *P* that moves a constant distant of 2 cm from a straight line *AB.*

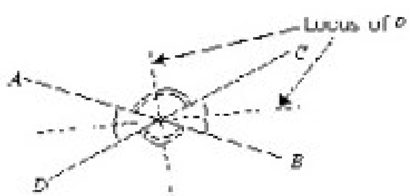
Solution:

Construct a pair of parallel lines 2 cm from *AB.*



CONDITION 4:
A point P moves so that it is always equidistant from two intersecting lines AB and CD

Locus formed: Angle bisectors of angles between lines *AB* and *CD*.

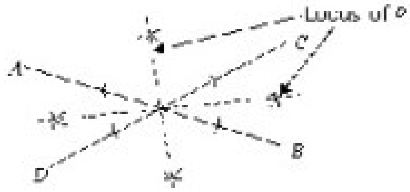


Example:

The following figure shows two straight lines *AB* and *CD* intersecting at point *O*. Construct the locus of point *P* such that it is always equidistant from *AB* and *CD.*

Solution:

Construct angles bisectors of angles between lines *AB* and *CD.*



ASSESSMENT

1. What are the properties used in identifying a quadrilateral?
2. Construct a kite.
3. Construct the locus of point *P* moving equidistant from fixed points *X* and *Y* and *XY* = 10 cm.

Week 5

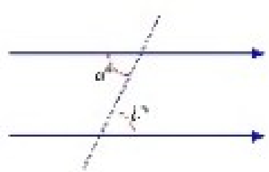
Topic: Deductive Proof

Sum of Angles of a Triangle

An **axiom** is a statement that is simply accepted as being true. We have accepted the following statements as facts:

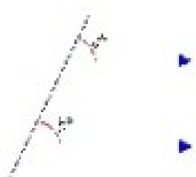
1. Alternate angles are equal. That is:

$a^\circ = b^\circ$



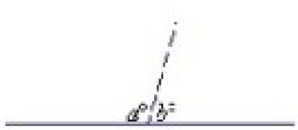
2. Corresponding angles are equal. That is:

$b^\circ = c^\circ$



3. The sum of adjacent angles forming a straight line is equal to 180°. That is:

$a^\circ + b^\circ = 180^\circ$



In deductive geometry, we do not accept any other geometrical statement as being true unless it can be proved (or deduced) from the axioms.

A statement that is proved by a sequence of logical steps is called a **theorem**.

To prove a theorem we start by using one or more of the axioms in a particular situation to get some true statements. We then have to apply logical reasoning to these statements to produce new statements that are true. The proof ends when we arrive at the statement of the theorem.

Properties of Equality

The **relation of equality** has the following properties. We will use these properties in the proofs of some theorems.

1. Transitive Property

If $a = b$ and $b = c$, then $a = c$.

2. Substitution Property

If a statement about *a* is true and $a = b$, then the statement formed by replacing *a* with *b* (throughout) is also true.

E.g. If the statement $a + c = 180$ is true and *a* is equal to *b*, then the statement $b + c = 180$ is also true.

Deductive Proofs of Theorems

To prove a theorem, draw a diagram. Write related statements and give the reasons for each (i.e. state the axioms used). Then use the transitive property and/or one of the other properties of equality.

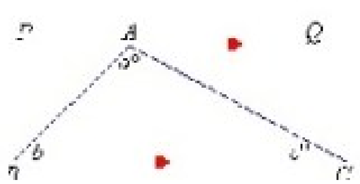
Angle Sum of a Triangle

Theorem 1

Prove that the angle sum of a triangle is 180°.

Proof:

Consider any triangle *ABC* in which the angles are *a*°, *b*° and *c*°. Draw a line through *A* parallel to *BC*.



∴ ∠PAB = b^o {Alternate angles}

∠QAC = c^o {Alternate angles}

Now, ∠PAB + ∠BAC + ∠QAC = 180^o {Angle sum of a straight line}

∴ $b^o + a^o + c^o = 180^o$ {∴ ∠PAB = b^o and ∠QAC = c^o, proved above}

∴ $a^o + b^o + c^o = 180^o$

Hence the angle sum of a triangle is 180^o.

Further theorems can now be deduced by using this theorem together with the axioms. This is how the body of knowledge is increased using the deductive method.

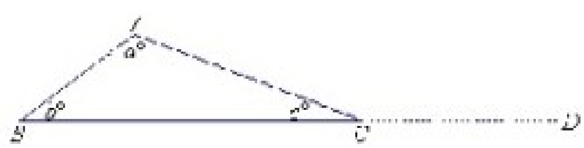
The Exterior Angle of a Triangle

Theorem 2

Prove that the exterior angle of a triangle is equal to the sum of the interior opposite angles.

Proof:

Consider any triangle *ABC* in which the angles are a^o, b^o and c^o. Extend the line *BC* to the point *D*.



$a^o + b^o + c^o = 180^o$ {Angle sum of a triangle}

∴ $a^o + b^o = 180^o - c^o$ {Subtract c from both sides}

But ∠ACD = $180^o - c^o$ {Angle sum of a straight line is 180^o}

∴ ∠ACD = $a^o + b^o$

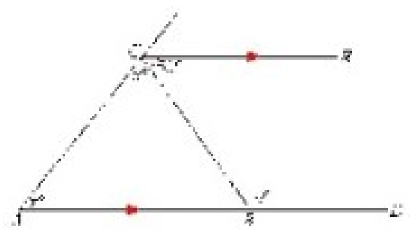
Hence the exterior angle of a triangle is equal to the sum of the interior opposite angles.

Applying Properties of Angles in Triangles

The theorems we have proved can be used to prove other theorems. They can also be used to find the values of the pronumerals in a problem.

Example 12

Find the values of the pronumerals *x* and *y* in the following diagram.



Solution:

y + 50 = 180   {Allied angles}

y + 50 – 50 = 180 – 50

y = 130

In ΔABD, y = x + y   {Exterior angle of a triangle}

130 = 2x   {∴y = 130}

2x = 130

2x/2= 130/2

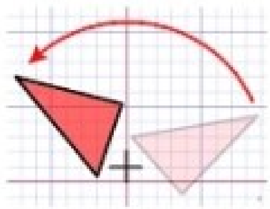
X = 65

Congruent Triangles

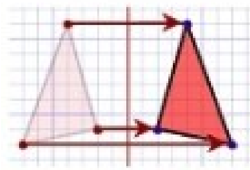
Triangles are congruent when they have
exactly the **same three sides** and exactly the **same three angles**.

What is “Congruent”?

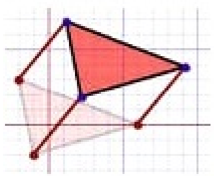
It means that one shape can become another using Turns, Flips and/or Slides:



Rotation Turn



Rotation Flip



Translation Slide

Congruent Triangles

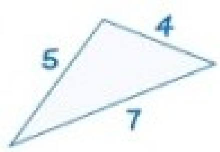
When two triangles are congruent they will have exactly **the same three sides** and exactly **the same three angles**.

The equal sides and angles may not be in the same position (if there is a turn or a flip), but they will be there.

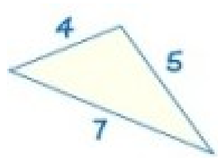
Same Sides

When the sides are the same then the triangles are congruent.

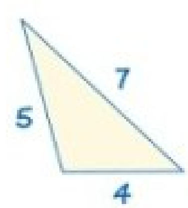
For example:



is congruent to:



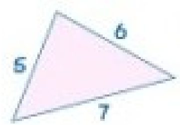
and



because they all have exactly the same sides.

But

is not congruent to:

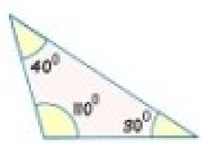


because the two triangles **do not have** exactly the same sides.

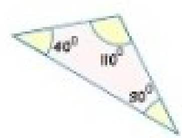
Same Angles

Does this also work with angles? Not always!

Two triangles with the same angles **might be** congruent:

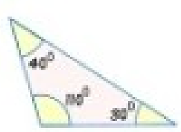


is congruent to:



only because they are the **same size**

But they might **NOT** be congruent because of **different sizes**:



is not congruent to:

because, even though all angles match, **one is larger than the other**.

So just having the same angles is no guarantee they are congruent.

Properties of Parallelograms

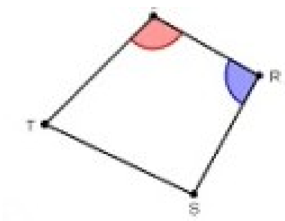
The broadest term we've used to describe any kind of shape is “polygon.” When we discussed quadrilaterals in the last section, we essentially just specified that they were polygons with four vertices and four sides. Still, we will get more specific in this section and discuss a special type of quadrilateral: the parallelogram. Before we do this, however, let's go over some definitions that will help us describe different parts of quadrilaterals.

Quadrilateral Terminology

Since this entire section is dedicated to the study of quadrilaterals, we will use some terminology that will help us describe specific pairs of lines, angles, and vertices of quadrilaterals. Let's study these terms now.

Consecutive Angles

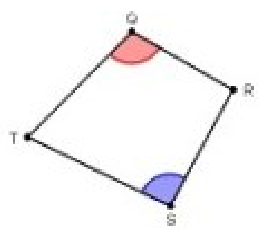
Two angles whose vertices are the endpoints of the same side are called consecutive angles.



Q and R are consecutive angles because Q and R are the endpoints of the same side.

Opposite Angles

Two angles that are not consecutive are called **opposite angles**.



Q and S are opposite angles because they are not endpoints of a common side.

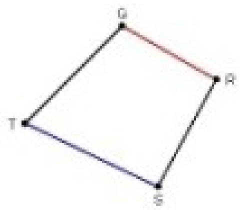
Consecutive Sides

Two sides of a quadrilateral that meet are called **consecutive sides**.

QR and RS are consecutive sides because they meet at point R.

Opposite Sides

Two sides that are not consecutive are called **opposite sides**.

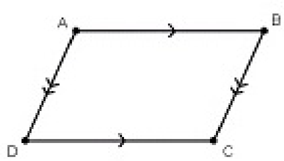


QR and TS are opposite sides of the quadrilateral because they do not meet.

Now, that we understand what these terms refer to, we are ready to begin our lesson on parallelograms.

Properties of Parallelograms: Sides and Angles

A parallelogram is a type of quadrilateral whose pairs of opposite sides are parallel.



Quadrilateral ABCD is a parallelogram because AB = DC and AD = BC.

Although the defining characteristics of parallelograms are their pairs of parallel opposite sides, there are other ways we can determine whether a quadrilateral is a parallelogram. We will use these properties in our two-column geometric proofs to help us deduce helpful information.

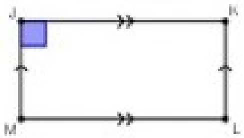
If a quadrilateral is a parallelogram, then:

(1) its opposite sides are congruent,

(2) its opposite angles are congruent, and

(2) its consecutive angles are supplementary.

Another important property worth noticing about parallelograms is that if one angle of the parallelogram is a right angle, then they all are right angles. Why is this property true? Let’s examine this situation closely. Consider the figure below.



Given that *J* is a right angle, we can also determine that *L* is a right angle since the opposite sides of parallelograms are congruent. Together, the sum of the measure of those angles is *180* because

m∠J + m∠ L

= 90 + 90

= 180

We also know that the remaining angles must be congruent because they are also opposite angles. By the **Polygon Interior Angles Sum Theorem**, we know that all quadrilaterals have angle measures that add up to ***360***. Since *J* and *L* sum up to *180*, we know that the sum of *K* and *M* will also be *180*:

(m∠J + m∠L) + m∠K + m∠ M = 360

180 + m∠K + m∠M = 360

m∠K + m∠M = 180

Since *K* and *M* are congruent, we can define their measures with the same variable, ***x***. So we have

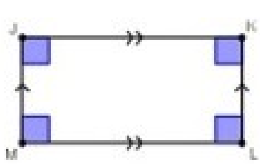
m∠K + m∠M = 180

x + x = 180

2x = 180

X = 90

Therefore, we know that *K* and *M* are both right angles. Our final illustration is shown below.



Intercept Theorem

The **intercept theorem**, also known as **Thales' theorem** is an important theorem in elementary geometry about the ratios of various line segments that are created if two intersecting lines are intercepted by a pair of parallels. It is equivalent to the theorem about ratios in similar triangles. Traditionally it is attributed to Greek mathematician Thales.

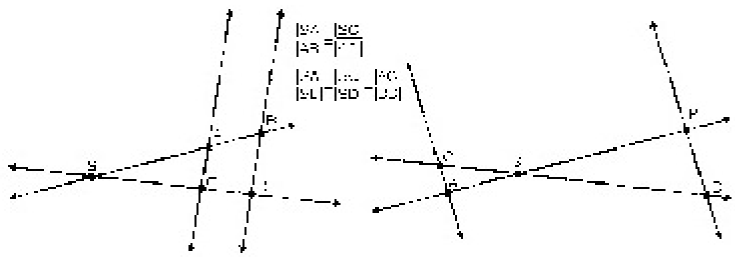
Suppose S is the intersection point of two lines and A, B are the intersections of the first line with the two parallels, such that B is further away from S than A, and similarly C, D are the intersections of the second line with the two parallels such that D is further away from S than C.

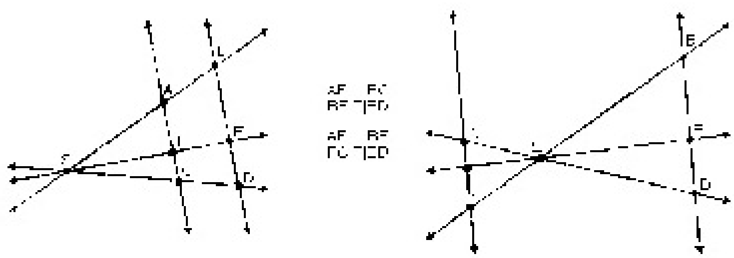
The ratios of any two segments on the first line equals the ratios of the according segments on the second line: |SA| : |AB| = |SC| : |CD| , |SA| : |AB| = |SD| : |CD , |SA| : |SB| = |SC| : |SD |

The ratio of the two segments on the same line starting at S equals the ratio of the segments on the parallels: |SA| : |AB| = |SC| : |CD| = |AC| : |BD|

The converse of the first statement is true as well, i.e. if the two intersecting lines are intercepted by two arbitrary lines and |SA| : |AB| = |SC| : |CD| holds then the two intercepting lines are parallel. However the converse of the second statement is not true.

If you have more than two lines intersecting in S, then ratio of the two segments on a parallel equals the ratio of the according segments on the other parallel. An example for the case of three lines is given in the second graphic below.





ASSESSMENT

1. What is a theorem?
2. List the equality properties
3. What is the intercept theorem?
4. What are the properties of a parallelogram?
5. What are congruent triangles?
6. What are two sides of a quadrilateral that meet called?

Week 6

Topic: Statistics

Collection and Presentation of Data

Introduction

To derive conclusions from data, we need to know how the data were collected; that is, we need to know the method(s) of data collection.

Methods of Data Collection

There are four main methods of data collection.

- **Census**. A census is a study that obtains data from every member of a population. In most studies, a census is not practical, because of the cost and/or time required.
- **Sample survey**. A sample survey is a study that obtains data from a subset of a population, in order to estimate population attributes.
- **Experiment**. An experiment is a controlled study in which the researcher attempts to understand cause-and-effect relationships. The study is "controlled" in the sense that the researcher controls (1) how subjects are assigned to groups and (2) which treatments each group receives.In the analysis phase, the researcher compares group scores on some dependent variable. Based on the analysis, the researcher draws a conclusion about whether the treatment (independent variable) had a causal effect on the dependent variable.
- **Observational study**. Like experiments, observational studies attempt to understand cause-and-effect relationships. However, unlike experiments, the researcher is not able to control (1) how subjects are assigned to groups and/or (2) which treatments each group receives.

Data Collection Methods: Pros and Cons

Each method of data collection has advantages and disadvantages.

- **Resources**. When the population is large, a sample survey has a big resource advantage over a census. A well-designed sample survey can provide very precise estimates of population parameters – quicker, cheaper, and with less manpower than a census.
- **Generalizability**. Generalizability refers to the appropriateness of applying findings from a study to a larger population. Generalizability requires random selection. If participants in a study are randomly selected from a larger population, it is appropriate to generalize study results to the larger population; if not, it is not appropriate to

generalize.Observational studies do not feature random selection; so generalizing from the results of an observational study to a larger population can be a problem.

- **Causal inference**. Cause-and-effect relationships can be teased out when subjects are randomly assigned to groups. Therefore, experiments, which allow the researcher to control assignment of subjects to treatment groups, are the best method for investigating causal relationships.

Tabulation of Data

Tabulation is the systematic arrangement of the statistical data in columns or rows. It involves the orderly and systematic presentation of numerical data in a form designed to explain the problem under consideration. Tabulation helps in drawing the inference from the statistical figures.

Tabulation prepares the ground for analysis and interpretation. Therefore a suitable method must be decided carefully taking into account the scope and objects of the investigation, because it is very important part of the statistical methods.

Types of Tabulation

In general, the tabulation is classified in two parts, that is a simple tabulation, and a complex tabulation.

Simple tabulation, gives information regarding one or more independent questions. Complex tabulation gives information regarding two mutually dependent questions.

ONE-WAY TABLE

DIVISION

	POPULATION	
Karachi	10.875968	
Hyderabad	14.186954	
Sukkur	12.994401	

This table gives us information regarding one characteristic information about the population in different divisions of Sindh.

All questions that can be answered in ONE WAY TABLE are independent of each other. It is therefore an example of a simple tabulation, since the information obtained in it is regarding one independent question, that is the number of persons in various divisions of Sindh in millions.

Two-Way Table

These types of table give information regarding two mutually dependent questions. For example, question is, how many millions of the persons are in the Divisions; the One-Way Table will give the answer. But if we want to know that in the population number, who are in the majority, male, or female. The Two-Way Tables will answer the question by giving the column for female and male. Thus the table showing the real picture of divisions sex wise is as under:

<u>Two-Way Table</u>

	POPULATION (Millions)		
DIVISION	Male	Female	Total
Karachi			
Hyderabad			
Sukkur			

Three-Way Table

Three-Way Table gives information regarding three mutually dependent and inter-related questions.

For example, from one-way table, we get information about population, and from two-way table, we get information about the number of male and female available in various divisions. Now we can extend the same table to a three way table, by putting a question, “How many male and female are literate?” Thus the collected statistical data will show the following, three mutually dependent and inter-related questions:

1. Population in various division.

2. Their sex-wise distribution.

3. Their position of literacy.

THREE-WAY TABLE

	POPULATION			
DIVISION		**Male**	**Female**	**Total**
Karachi	Literate			
	Illiterate			
	Total			
Hyderabad	Literate			
	Illiterate			
	Total			
Sukkur	Literate			
	Illiterate			
	Total			

This table gives information concerning the literacy of both male and female in various divisions of Sindh. From the table we can explain the sex which has more education in relation to division, and also, we can say whether literacy is low in rural areas than in urban areas.

Higher Order Tables

Higher order tables are those which provide information about a large number of inter related questions. Higher order tables may be of four-way, five-way, six-way etc. Such kind of tables are called manifold tables.

Presentation of Data

Introduction

Although tabulation is very good technique to present the data, but diagrams are an advanced technique to represent data. As a layman, one cannot understand the tabulated data easily but with only a single glance at the diagram, one gets complete picture of the data presented. According to M.J. Moroney, “diagrams register a meaningful impression almost before we think.

Importance or utility of Diagrams

Diagrams give a very clear picture of data. Even a layman can understand it very easily and in a short time.

We can make comparison between different samples very easily. We don’t have to use any statistical technique further to compare.

This technique can be used universally at any place and at any time. This technique is used almost in all the subjects and other various fields.

Diagrams have impressive value also. Tabulated data has not much impression as compared to Diagrams. A common man is impressed easily by good diagrams.

This technique can be used for numerical type of statistical analysis, e.g. to locate Mean, Mode, Median or other statistical values.

It does not save only time and energy but also is economical. Not much money is needed to prepare even good diagrams.

These give us much more information as compared to tabulation. Technique of tabulation has its own limits.

This data is easily remembered. Diagrams which we see leave their lasting impression much more than other data techniques.

Data can be condensed with diagrams. A simple diagram can present what even cannot be presented by 10000 words.

General Guidelines for Diagrammatic presentation

The diagram should be properly drawn at the outset. The pith and substance of the subject matter must be made clear under a broad heading which properly conveys the purpose of a diagram.

The size of the scale should neither be too big nor too small. If it is too big, it may look ugly. If it is too small, it may not convey the meaning. In each diagram, the size of the paper must be taken note-of. It will help to determine the size of the diagram.

For clarifying certain ambiguities some notes should be added at the foot of the diagram. This shall provide the visual insight of the diagram.

Diagrams should be absolutely neat and clean. There should be no vagueness or overwriting on the diagram.

Simplicity refers to love at first sight. It means that the diagram should convey the meaning clearly and easily.

Scale must be presented along with the diagram.

It must be Self-Explanatory. It must indicate nature, place and source of data presented.

Different shades, colors can be used to make diagrams more easily understandable.

Vertical diagram should be preferred to Horizontal diagrams.

It must be accurate. Accuracy must not be done away with to make it attractive or impressive.

Limitations of Diagrammatic Presentation

Diagrams do not present the small differences properly.

These can easily be misused.

Only artist can draw multi-dimensional diagrams.

In statistical analysis, diagrams are of no use.

Diagrams are just supplement to tabulation.

Only a limited set of data can be presented in the form of diagram.

Diagrammatic presentation of data is a more time consuming process.

Diagrams present preliminary conclusions.

Diagrammatic presentation of data shows only on estimate of the actual behavior of the variables.

For easy understanding the variations in data. After the collection and tabulation of data, it can be represented by a graph. Most common graphs are

Bar graphs

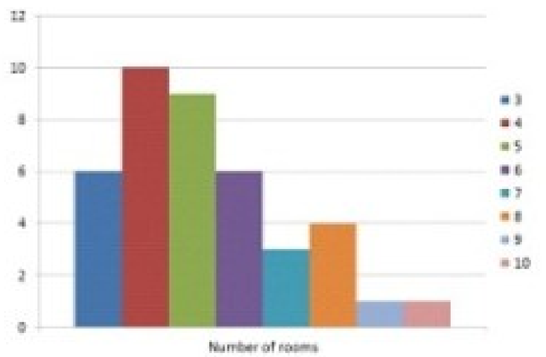
Pie graphs

Line graphs

Bar Graph

Bar graph represents the relation between class interval/group and frequency of that particular class interval.

Along x-axis is the class distribution and along y-axis is the number of times same event repeats and it is named as frequency of that class/group.



Pie Graph

It is a circular chart that is divided into a number of sectors. For representing data in a pie graph, circle is divided into sectors.

Example

The number of bedrooms on each floor of a 15 storey building is given as:

1 2 4 3 2 5 6 5 4 5 2 5 6 6 3

Solution

Tabulation Of Data

Bedrooms	Tally	Frequency
1	|	1
2	|||	3
3	||	2
4	||	2
5	||||	4
6	|||	3

For pie graph presentation, divide 360 degrees by 15. Because 360 is an angle of a full circle.

360/15 = 24 degrees

For 1-bedroom sector = 1*24 = 24 degrees

For 2-bedrooms sector = 3*24 = 72 degrees

For 3-bedrooms sector = 2*24 = 48 degrees

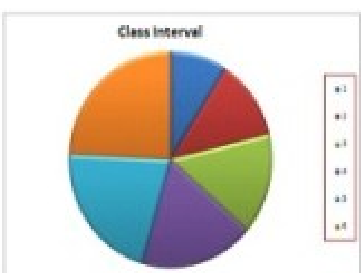
For 4-bedrooms sector = 2*24 =48 degrees

For 5-bedrooms sector =4*24 =96 degrees

For 6-bedrooms sector = 3*24 =72 degrees

By adding all the sector angle = 360 degrees

Graphically…



Line Graph

The simplest type of graphs is the line graph. Most commonly used when given data is in the form of grouped frequency.

Along x-axis is the group number and along y-axis is the frequency with respect to each separate group.

ASSESSMENT

1. What is statistics?
2. What are the four main methods of data collection?
3. What are the 3 most common types of graph?
4. What is Tabulation?
5. What are the two classifications of Tabulation?

WEEK 7

Topic: Calculation of Mean, Median and Mode of Ungrouped Data

Mean, median, and mode are three basic ways to look at the value of a set of numbers. You will start by learning about the mean.

The **mean**, often called the average, of a numerical set of data, is simply the sum of the data values divided by the number of values. This is also referred to as the arithmetic mean. The mean is the balance point of a distribution.

Mean = sum of the values/the number of values

For instance, take a look at the following example. Use the formula to calculate the mean number of hours that Stephen worked each month based on the example below.

Example

Stephen has been working on programming and updating a Web site for his company for the past 15 months. The following numbers represent the number of hours Stephen has worked on this Web site for each of the past 7 months:

24, 25, 31, 50, 53, 66, 78

What is the mean (average) number of hours that Stephen worked on this Web site each month?

Step 1: Add the numbers to determine the total number of hours he worked.

24 + 25 + 33 + 50 + 53 + 66 + 78 = 329

Step 2: Divide the total by the number of months.

329/7 = 47

The mean number of hours that Stephen worked each month was 47.

The calculations for the mean of a sample and the total population are done in the same way. However, the mean of a population is constant, while the mean of a sample varies from sample to sample.

Example

Mark operates Technology Titans, a Web site service that employs 8 people. Find the mean age of his workers if the ages of the employees are as follows:

55, 63, 34, 59, 29, 46, 51, 41

Step 1: Add the numbers to determine the total age of the workers.

55 + 63 + 34 + 59 + 29 + 46 + 51 + 41 = 378

Step 2: Divide the total by the number of months.

378/8 = 47.25

The mean age of all 8 employees is 47.25 years, or 47 years and 3 months.

Look at another approach. If you were to take a sample of 3 employees from the group of 8 and calculate the mean age for these 3 workers, would the results change?

Use the ages 55, 29, and 46 for one sample of 3, and the ages 34, 41, and 59 for another sample of 3:

Mean =55 + 29 + 46/3 Mean = 34 + 41 + 59/3

Mean = 130/3 Mean = 134/3

Mean = 43.33 Mean = 44.66

The mean age of the first group of 3 employees is 43.33 years.

The mean age of the second group of 3 employees is 44.66 years.

The mean age for a sample of a population depends upon the values that are included in the sample. From this example, you can see that the mean of a population and that of a sample from the population are not necessarily the same.

In addition to calculating the mean for a given set of data values, you can apply your understanding of the mean to determine other information that may be asked for in everyday problems.

Example

Two weeks before Mark opened Technology Titans, he launched his company Web site. During those 14 days, Mark had an average of 24.5 hits on his Web site per day. In the first two days that Technology Titans was open for business, the Web site received 42 and 53 hits respectively. Determine the new average for hits on the Web site.

Step 1: Multiply the given average by 14 to determine the total number of hits on Mark's Web site.

24.5 x 14 = 343

Step 2: Add the hits for the first two days his business was open.

343 + 42 + 53 = 438

Step 3: Divide this new total by 16 to determine the new average.

Mean = 438/16 = 27.375

The average number of hits Mark's Web site has received per day since it was launched is 27.375.

All values for the means you have calculated so far have been for ungrouped, or listed, data. A mean can also be determined for data that is grouped, or placed in intervals. Unlike listed data, the individual values for grouped data are not available, and you are not able to calculate their sum. To calculate the mean of grouped data, the first step is to determine the midpoint of each interval or class. These midpoints must then be multiplied by the frequencies of the corresponding classes. The sum of the products divided by the total number of values will be the value of the mean.

The following example will show how the mean value for grouped data can be calculated.

Example

In Tim's office, there are 25 employees. Each employee travels to work every morning in his or her own car. The distribution of the driving times (in minutes) from home to work for the employees is shown in the table below.

Driving Time (minutes)	Number of Employees
0 to less than 10	3
10 to less than 20	10
20 to less than 30	6
30 to less than 40	4
40 to less than 50	2

Calculate the mean of the driving times.

Step 1: Determine the midpoint for each interval.

For 0 to less than 10, the midpoint is 5.

For 10 to less than 20, the midpoint is 15.

For 20 to less than 30, the midpoint is 25.

For 30 to less than 40, the midpoint is 35.

For 40 to less than 50, the midpoint is 45.

Step 2: Multiply each midpoint by the frequency for the class.

For 0 to less than 10, (5)(3) = 15

For 10 to less than 20, (15)(10) = 150

For 20 to less than 30, (25)(6) = 150

For 30 to less than 40, (35)(4) = 140

For 40 to less than 50, (45)(2) = 90

Step 3: Add the results from Step 2 and divide the sum by 25.

15 + 150 + 150 + 140 + 90 = 545

Mean = 545/25 = 21.8

Each employee spends an average (mean) time of 21.8 minutes driving from home to work each morning.

The mean is often used as a summary statistic. However, it is affected by extreme values (outliers): either an unusually high or low number. When you have extreme values at one end of a data set, the mean is not a very good summary statistic.

The Median

What is the Median?

The **median** is the number that falls in the middle position once the data has been organized. Organised data means the numbers are arranged from smallest to largest or from largest to smallest.

The median for an odd number of data values is the value that divided the data into two halves. If n represents the number of data values and n is an odd number, then the median will be found in the n+1 /2 position.

This measure of central tendency is typically used when the mean value is affected by an unusually low number or an unusually high number in the data set **(outliers)**. Outliers distort the mean value to the extent that the mean value no longer accurately depicts the set of data.

For example: If one of the houses in your neighborhood was broken down and maintained a low property value, then you would not want to include this property when determining the value of your own home. However, if you are purchasing a home in that neighborhood, you may want to include the outlier since it would drive down the price you would have to pay.

Try a few examples to follow the steps needed to calculate the median.

Example

Find the median of the following data:

12, 2, 16, 8, 14, 10, 6

Step 1: Organize the data, or arrange the numbers from smallest to largest.

2, 6, 8, 10, 12, 14, 16

Step 2: Since the number of data values is odd, the median will be found in the n+1 /2 position.

n+1 /2 = 7 + 1 / 2 = 8/2 = 4

Step 3: In this case, the median is the value that is found in the fourth position of the organized data.

2, 6, 8, 10, 12, 14, 16

The mean is 10.

Another way to look at the example is to narrow the data down to find the middle number.

2, 6, 8, 10, 12, 14, 16

X, 6, 8, 10, 12, 14, **X**

X, **X**, 8, 10, 12, **X**, **X**

X, **X**, **X**, 10, **X**, **X**, **X**

Here is another example of how to calculate the median of a set of numbers.

Example

Find the median of the following data:

7, 9, 3, 4, 11, 1, 8, 6, 1, 4

Step 1: Organize the data, or arrange the numbers from smallest to largest.

1, 1, 3, 4, 4, 6, 7, 8, 9, 11

Step 2: Since the number of data values is even, the median will be the mean value of the numbers found before and after the n + 1 / 2 position.

n+1 /2 = 10 + 1 /2 = 11/2 = 5.5

Step 3: The number found before the 5.5 position is 4 and the number found after the 5.5 position is 6. Now, you need to find the mean value.

1, 1, 3, 4, 4, 6, 7, 8, 9, 11

4 + 6 / 2 = 10/2 = 5

The median is 5.

The Mode

What is the Mode?

The mode of a set of data is simply the value that appears most frequently in the set.

If two or more values appear with the same frequency, each is a mode. The downside to using the mode as a measure of central tendency is that a set of data may have no mode, or it may have more than one mode. However, the same set of data will have only one mean and only one median.

The word **modal** is often used when referring to the mode of a data set.

If a data set has only one value that occurs most often, the set is called **unimodal**.

A data set that has two values that occur with the same greatest frequency is referred to as **bimodal**.

When a set of data has more than two values that occur with the same greatest frequency, the set is called **multimodal**.

When determining the mode of a data set, calculations are not required, but keen observation is a must. The mode is a measure of central tendency that is simple to locate, but it is not used much in practical applications.

Example

Find the mode of the following data:

76, 81, 79, 80, 78, 83, 77, 79, 82, 75

There is no need to organize the data, unless you think that it would be easier to locate the mode if the numbers were arranged from least to greatest. In the above data set, the number 79 appears twice, but all the other numbers appear only once. Since 79 appears with the greatest frequency, it is the mode of the data values.

The mode is 79.

Example

The ages of 12 randomly selected customers at a local Best Buy are listed below:

23, 21, 29, 24, 31, 21, 27, 23, 24, 32, 33, 19

What is the mode of the above ages?

The above data set has three values that each occur with a frequency of 2. These values are 21, 23, and 24. All other values occur only once. Therefore, this set of data has three modes.

The modes are 21, 23, and 24.

Remember that the mode can be determined for qualitative data as well as quantitative data, but the mean and the median can only be determined for quantitative data.

Example

You begin to observe to the color of clothing your employees wear. Your goal is to find out what color is worn most frequently so that you can offer company shirts to your employees.

Monday: Red, Blue, Black, Pink, Green, and Blue

Tuesday: Green, Blue, Pink, White, Blue, and Blue

Wednesday: Orange, White, White, Blue, Blue, and Red

Thursday: Brown, Black, Brown, Blue, White, and Blue

Friday: Blue, Black, Blue, Red, Red, and Pink

What is the mode of the colors above?

The color blue was worn 11 times during the week. All other colors were worn with much less frequency in comparison to the color blue.

The mode is blue.

ASSESSMENT

1. What is another name for mean
2. Find the mean of the following; 124, 125, 131, 150, 153, 166, 178
3. What is the median of a set of numbers?
4. What is the mode of a set of numbers?
5. Find the mode of the following data 7,6,8,1,7,9,8,7,8,8,3,7,7,7,9,8,2,7,5
6. What is the median of these data set; 12, 2, 16, 8, 14, 10, 6, 7, 5, 3, 4, 76, 12, 356?

WEEK 8

Topic: Collection, Tabulation and Presentation of Grouped Data

In some investigations you may collect an awful lot of information. How can you use this raw data and make it meaningful? This section will help you to collect, organise and interpret the data efficiently.

Explaining your results

Imagine that you are asked to carry out a survey to find the number of pets owned by pupils in your school. You decide to ask 50 people, and record your results as follows:

0 2 1 2 0 4 1 0 2 2 1 6 1 1 2 8 0 12

2 1 2 0 3 2 0 1 3 0 1 4 0 3 0 2 3 6

3 3 0 1 2 0 1 1 3 0 2 0 3 2

You now have the information you need, but is this the most efficient way to collect and display the data?

Tallying

Tallying is a method of counting using groups of five.

|= 1

||= 2

||| = 3

|||| = 4

𝍸= 5

𝍸| = 6

𝍸|| = 7

卌||| = 8

卌|||| = 9

卌卌 = 10

Because we have used groups of five, it is easy to find the total.

Question

Which numbers do the following tally marks represent?

a.) 卌 卌 |||

b.) 卌 卌 卌 卌 ////

Answer

卌 卌 ||| =13

卌 卌 卌 卌 |||| =24

Using the tally system to record results is much faster than writing out words or figures all the time. For example, if you had to investigate the most popular type of vehicle that passed the school gates, it would be easier to draw tally marks in one of three columns than write: car, car, lorry, bike, car, car, and so on.

By using a tally chart, the data is already collected into groups, and will not require further grouping at a later date.

Collecting data

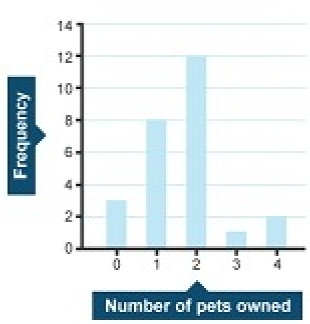
The easiest way to collect data is to use a tally chart.

When collecting data for the number of pets survey, it would have been useful to draw a table similar to this one.

As each person answers the question, we put a tally next to the appropriate number of pets. The frequency column is completed once all of the data has been collected. The table below shows the results of a new pets survey.

Number of Pets	Tally	Frequency
0	III	3
1	卌 III	8
2	卌 卌 II	12
3	I	1
4	II	2

These frequencies can be displayed in a bar chart, as shown.



Frequency means the 'number of times it occurs'.

In this example, three people had no pets, so the frequency of 0 pets was three.

Remember that the total frequency should be the same as the number of people in your survey. Always check that this is correct.

In this example, we know that 26 people were questioned in the survey. Check this by adding up the frequency totals: 3 + 8 + 12 + 1 + 2 = 26

Here is the same information but this time we have two tables, one for the number of pets owned by boys and one for the number of pets owned by girls.

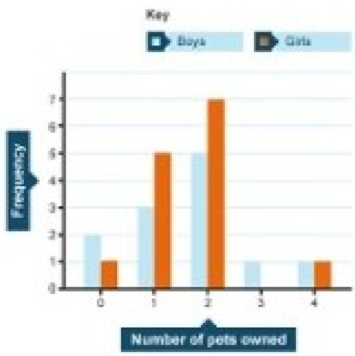
Number of pets owned by boys

Number of Pets	**Tally**	**Frequency**
0	II	2
1	III	3
2	卌	5
3	I	1
4	I	1

Number of pets owned by girls

Number of Pets	Tally	Frequency
0	|	1
1	卌	5
2	卌||	7
3	|	0
4	|	1

These frequencies can be displayed in a dual bar chart.



We can find more information from looking at this graph.

Question

How many pets were owned by the same number of boys as for girls?

Answer

Four pets.

We can see that the height of the bar is the same for both boys and girls for four pets.

Question

How many more girls than boys were there in the survey?

Answer

Two more girls.

Adding the bars for girls and for boys we find:

the total for girls is: 1 + 5 + 7 + 0 + 1 = 14 the total boys is: 2 + 3 + 5 + 1 + 1= 12

Grouping data

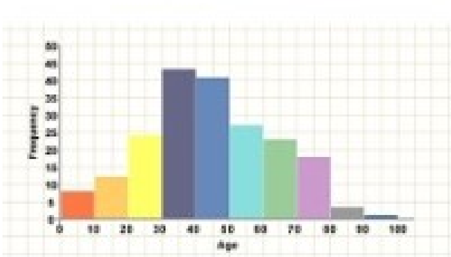
When a large amount of data has to be collected, use a **grouped** frequency distribution.

The following tally chart represents the ages of 200 people entering a park on a Saturday afternoon.

The ages have been grouped into the classes 0-9, 10-19, 20-29, and so on.

Age	**Tally**	**Frequency**
0-9	𝍸 𝍸 III	8
10-19	𝍸 𝍸 II	12
20-29	𝍸 𝍸 𝍸 𝍸 IIII	24
30-39	𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 III	43
40-49	𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 I	41
50-59	𝍸 𝍸 𝍸 𝍸 𝍸II	27
60-69	𝍸 𝍸 𝍸 𝍸 III	23
70-79	𝍸 𝍸 𝍸 III	18
80-89	III	3
90-99	I	1

These frequencies can also be shown in a histogram.



<u>ASSESSMENT</u>

1. What is tallying?
2. What is the easiest way to collect data?
3. What is frequency?

WEEK 9

Topic: STATISTICAL GRAPHS

Bar Chart

A bar graph is a way to visually represent qualitative data. Qualitative or categorical data occurs when the information concerns a trait or attribute and is not numerical. This kind of graph emphasizes the relative sizes of each of the categories being measured by using vertical or horizontal bars. Each trait corresponds to a different bar. The arrangement of the bars is by frequency. By looking at all of the bars, it is easy to tell at a glance which categories in a set of data dominate the others. The larger a category, the bigger that its bar will be.

Big Bars or Small Bars?

To construct a bar graph we must first list all the categories. Along with this we denote how many members of the data set are in each of the categories. Arrange the categories in order of frequency. We do this because the category with the highest frequency will end up being represented by the largest bar, and the category with the lowest frequency will be represented by the smallest bar.

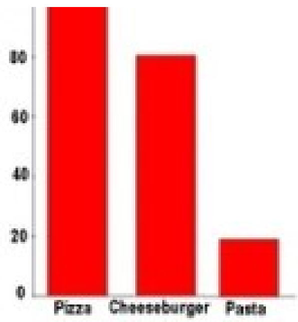
For a bar graph with vertical bars, draw a vertical line with a numbered scale. The numbers on the scale will correspond to the height of the bars. The greatest number that we need on the scale is the category with the highest frequency. The bottom of the scale is typically zero, however if the height of our bars would be too tall, then we can use a number greater than zero.

We draw this bar, and label the bottom of it with the title of the category. We then continue the above process for the next category, and conclude when bars for all categories have been included. The bars should have a gap separating each of them from one another.

An Example

To see an example of a bar graph, suppose that we gather some data by surveying students at a local elementary school. We ask every one of the students to tell us what his or her favorite food is. Of 200 students, we find that 100 like pizza the best, 80 like cheeseburgers, and 20 have a favorite food of pasta. This means that the highest bar (of height 100) goes to the category of pizza. The next highest bar is 80 units high, and corresponds to cheeseburgers. The third and final bar represents the students who like pasta the best, and is only 20 units high.

The resulting bar graph is depicted below. Notice that both the scale and categories are clearly marked and that all the bars are separated. At a glance we can see that although three foods were mentioned, pizza and cheeseburgers are clearly more popular than pasta.



Pie Chart

One of the most common ways to represent data graphically is called a pie chart. It gets its name by how it looks, just like a circular pie that has been cut into several slices. This kind of graph is helpful when graphing qualitative data, where the information describes a trait or attribute and is not numerical. Each trait corresponds to a different slice of the pie. By looking at all of the pie pieces, you can compare how much of the data fits in each category. The larger a category, the bigger that its pie piece will be.

Big or Small Slices?

How do we know how large to make a pie piece? First we need to calculate a percentage. Ask what percent of the data is represented by a given category. Divide the number of elements in this category by the total number. We then convert this decimal into a percentage.

A pie is a circle. Our pie piece, representing a given category, is a portion of the circle. Because a circle has 360 degrees all the way around, we need to multiply 360 by our percentage. This gives us the measure of the angle that our pie piece should have.

An Example

To illustrate the above, let’s think about the following example. In a cafeteria of 100 third graders, a teacher looks at the eye color of each student and records it. After all 100 students are examined, the results show that 60 students have brown eyes, 25 have blue eyes and 15 have hazel eyes.

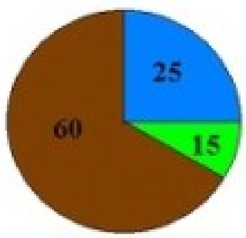
The slice of pie for brown eyes needs to be the largest. And it needs to be over twice as large as the slice of pie for blue eyes. To say exactly how large it should be, first find out what percent of the students have brown eyes. This is found by dividing the number of brown eyed students by the total number of students, and converting to a percent. The calculation is 60/100 x 100% = 60%.

Now we find 60% of 360 degrees, or 0.60 x 360 = 216 degrees. This reflex angle is what we need for our brown pie piece.

Next look at the slice of pie for blue eyes. Since there are a total of 25 students with blue eyes out of a total of 100, this means that this trait accounts for 25/100×100% = 25% of the students. One quarter, or 25% of 360 degrees is 90 degrees, a right angle.

The angle for the pie piece representing the hazel eyed students can be found in two ways. The first is to follow the same procedure as the last two pieces. The easier way is to notice that there are only three categories of data, and we have accounted for two already. The remainder of the pie is corresponds to the students with hazel eyes.

The resulting pie chart is pictured below. Note that number of students in each category is written on each pie piece.



Limitations of Pie Charts

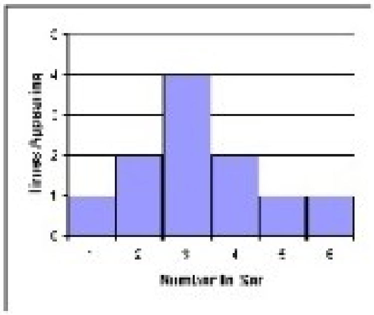
Pie charts are to be used with qualitative data, however there are some limitations in using them. If there are too many categories, then there will be a multitude of pie pieces. Some of these are likely to be very skinny, and can be difficult to compare to one another.

If we want to compare different categories that are close in size, a pie chart does not always help us to do this. If one slice has central angle of 30 degrees, and another has a central angle of 29 degrees, then it would be very hard to tell at a glance which pie piece is larger than the other.

Histogram

A *histogram* is "a representation of a frequency distribution by means of rectangles whose widths represent class intervals and whose areas are proportional to the corresponding frequencies."

Sounds complicated . . . but the concept really is pretty simple. We graph groups of numbers according to how often they appear. Thus if we have the set {1,2,2,3,3,3,3,4,4,5,6}, we can graph them like this:



This graph is pretty easy to make and gives us some useful data about the set. For example, the graph peaks at 3, which is also the median and the mode of the set. The mean of the set is 3.27—also not far from the peak. The shape of the graph gives us an idea of how the numbers in the set are distributed about the mean; the distribution of this graph is wide compared to size of the peak, indicating that values in the set are only loosely bunched around the mean.

Cumulative Frequency

Cumulative frequency is defined as a running total of frequencies. The frequency of an element in a set refers to how many of that element there are in the set. Cumulative frequency can also defined as the sum of all previous frequencies up to the current point.

The cumulative frequency is important when analyzing data, where the value of the cumulative frequency indicates the number of elements in the data set that lie below the current value. The cumulative frequency is also useful when representing data using diagrams like histograms.

Cumulative Frequency Table

The cumulative frequency is usually observed by constructing a cumulative frequency table. The cumulative frequency table takes the form as in the example below.

Example 1

The set of data below shows the ages of participants in a certain summer camp. Draw a cumulative frequency table for the data.

Age (years)	**Frequency**
10	3
11	18
12	13
13	12
14	7

15	27

Solution:

The cumulative frequency at a certain point is found by adding the frequency at the present point to the cumulative frequency of the previous point.

The cumulative frequency for the first data point is the same as its frequency since there is no cumulative frequency before it.

Age (years)	Frequency	Cumulative Frequency
10	3	3
11	18	3+18 = 21
12	13	21+13 = 34
13	12	34+12 = 46
14	7	46+7 = 53
15	27	53+27 = 80

Cumulative Frequency Graph (Ogive)

A cumulative frequency graph, also known as an Ogive, is a curve showing the cumulative frequency for a given set of data. The cumulative frequency is plotted on the y-axis against the data which is on the x-axis for un-grouped data. When dealing with grouped data, the Ogive is formed by plotting the cumulative frequency against the upper boundary of the class. An Ogive is used to study the growth rate of data as it shows the accumulation of frequency and hence its growth rate.

Example 2

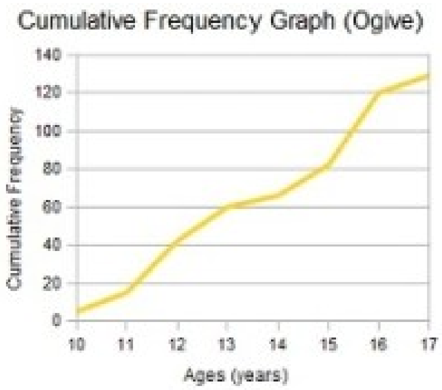
Plot the cumulative frequency curve for the data set below

Age (years)	Frequency
10	5
11	10
12	27
13	18
14	6
15	16
16	38
17	9

Solution

Age (years)	Frequency	Cumulative Frequency

10	5	5
11	10	5+10 = 15
12	27	15+27 = 42
13	18	42+18 = 60
14	6	60+6 = 66
15	16	66+16 = 82
16	38	82+38 = 120
17	9	120+9 = 129



ASSESSMENT

1. What is a cumulative frequency?
2. List the statistical graphs

WEEK 10

Topic: CALCULATION OF MEAN, MEDIAN, MODE OF GROUPED DATA

Let's start off with some raw data (**not a grouped frequency**) .

Example: Alex did a survey of how many games each of 20 friends owned, and got this:

9, 15, 11, 12, 3, 5, 10, 20, 14, 6, 8, 8, 12, 12, 18, 15, 6, 9, 18, 11

To find the Mean, add up all the numbers, then divide by how many numbers there are:

Mean = 9+15+11+12+3+5+10+20+14+6+8+8+12+12+18+15+6+9+18+11/20 = 11.1

To find the Median, place the numbers in value order and find the middle number (or the mean of the middle two numbers). In this case the mean of the 10^{th} and 11^{th} values:

3, 5, 6, 6, 8, 8, 9, 9, 10, 11, 11, 12, 12, 12, 14, 15, 15, 18, 18, 20:

12 appears three times, more often than the other values, so **Mode = 12**

Grouped Frequency Table

Now, let's make a Grouped Frequency Table of Alex's data:

Number of games	**Frequency**
1-5	2
6-10	7
11-15	8
16-15	3

(It says that 2 of Alex's friends own somewhere between 1 and 5 games, 7 own between 6 and 10 games, etc)

Estimating the Mean from Grouped Data

So all we have left is:

Number of games	**Frequency**
1-5	2
6-10	7
11-15	8
16-15	3

The groups (1-5, 6-10, etc) also called **class intervals**, are of **width** 5

The numbers 1, 6, 11 and 16 are the **lower class boundaries**

The numbers 5, 10, 15 and 20 are the **upper class boundaries**

The **midpoints** are halfway between the lower and upper class boundaries

So the **midpoints** are 3, 8, 13 and 18

We can estimate the **Mean** by using the **midpoints**.

So, how does this work?

Think about Alex's 7 friends who are in the group **6 – 10**: all we know is that they each have between 6 and 10 games:

Maybe all seven of them have 6 games,

Maybe all seven of them have 10 games,

But it is more likely that there is a spread of numbers: some have 6, some have 7, and so on

So we take an average: we **assume** that all seven of them have 8 games (8 is the average of 6 and 10), which is the **midpoint** of the group.

So, we could make the table in a different way:

Midpoint **Frequency**

3	2
8	7
13	8
18	3

Now we think “2 people have 3 games, 7 people have 8 games, 8 people have 13 games and 3 people have 18 games”, so we **imagine** the data looks like this:

3, 3, 8, 8, 8, 8, 8, 8, 8, 13, 13, 13, 13, 13, 13, 13, 13, 18, 18, 18

Now we can add them all up and divide by 20. This is the quick way to do it:

Midpoint	**Frequency**	
x	**f**	**fx**
3	2	6
8	7	56
13	8	104
18	3	54
Totals	20	220

So an **estimate** of the mean number of games is:

Estimated Mean = 220/20 = 11

Estimating the Median from Grouped Data

To **estimate** the Median, let’s look at our data again:

Number of games	**Frequency**
1-5	2
6-10	7
11-15	8

16-20	3

The median is the mean of the middle two numbers (the 10th and 11th values) ...

... and they are both in the 11 – 15 group:

We can say "the **median group** is 11 – 15"

But if we need to estimate a single **Median value** we can use this formula:

Estimated Median = L = (n/2) – cf_b x w / f_m

where:

L is the lower class boundary of the group containing the median, **n** is the total number of data **cf_b** is the cumulative frequency of the groups before the median group , **f_m** is the frequency of the median group, **w** is the group width

For our example:

L = 11, **n** = 20, **cf_b** = 2 + 7 = 9, **f_m** = 8, **w** = 5

Estimated Median = 11 + (20/2) – 9/8 x 5 = 11 + (1/8) x 5 = 11.625

Estimating the Mode from Grouped Data

Again, looking at our data:

Number of games	**Frequency**
1-5	2
6-10	7
11-15	8
16-20	3

We can easily identify the modal group (the group with the highest frequency), which is **11 – 15**

We can say "the **modal group** is 11 – 15"

But the actual **Mode** may not even be in that group! Or there may be more than one mode. Without the raw data we don't really know.

But, we can **estimate** the Mode using the following formula:

Estimated Mode = L + $f_m - f_{m-1}$ / $(f_m - f_{m-1}) + (f_m - f_{m+1})$ x w

where:

L is the lower class boundary of the modal group, f_{m-1} is the frequency of the group before the modal group, f_m is the frequency of the modal group, f_{m+1} is the frequency of the group after the modal group, w is the group width

In this example:

L = 11, f_{m-1} = 7, f_m = 8, f_{m+1} = 3, w = 5

Estimated Mode = 8 − 7 / (8 − 7) + (8 − 3) x 5 = 11 + (1/6) x 5 = **11.833**

Our final result is:

Estimated Mean: 11

Estimated Median: 11.625

Estimated Mode: 11.833...

(Compare that with the true Mean, Median and Mode of **11.1, 11 and 12** that we got at the very start.)

And that is how it is done.

Now let us look at two more special examples, and get some more practice along the way!

WEEK 11

Topic: MEAN DEVIATION FOR GROUPED DATA

Range and quartile deviations are positional measures of dispersion, wherein all the observations are not taken into account in the calculation. Now we consider a measure of dispersion called mean deviation based on all observations.

Consider the observation 3, 5, 6, 7, 9

A.M. = `(3+5+6+7+9)/(5)` = 6

The sum of the deviations of the items from the mean is zero. Consider the A.M. of theabsolute deviations of those observations from their mean.

i.e., = `(3+1+0+1+3)/(5)` = 8/5 = 1.6

This is called the mean deviation about the mean. This tells that on the average the observations are deviated away from the mean by 1.6 on either side.

Definition of Mean Deviation for Grouped Data:

Let x_1, x_2, x_3,...x_n be n values. The mean deviation about the mean of these values is given by

M.D. = `(sum I X i – barxI)/(n)`

In a frequency distribution with frequencies f_1,f_2,f_3,...f_n against the values x_1, x_2, x_3,...x_n,

M.D. about the mean = `(sum fi I X i – barx I)/(N)`

where N is the total frequency

Relative measure:

Co efficient of mean deviation about the mean = `(M.D.)/(Mean)`

Co efficient of mean deviation about the median = `(M.D.)/(Median)`

Finding Mean Deviation for Grouped Data:

Classfrequency0-535-10510-151215-20620-254Total30

Find the mean deviation for the grouped data.

Solution:

Class	Mid X	f	d	fd	X – `barx`	f IX – `barx` I
0-5	2.5	3	–2	–6	10.5	31.5
5-10	7.5	5	–1	–5	5.5	27.5
10-15	12.5	12	0	0	0.5	6.0
15-20	17.5	6	1	6	4.5	27.0
20-25	22.5	4	2	8	9.5	38.0
		30		3		130.0

A = 12.5 d = `(x – 7.5)/(5)`

`barx` = A + `(sum fd )/(N)` x c

= 12.5 + `(3)/(30)` = 13

Mean deviation about the mean = `(sum f I X – barx I)/(N)` x c

= `(130 X 5)/(30)` = 21.67

Standard Deviation of Grouped Data

The **standard deviation** measures the spread of the data about the **mean value**. It is useful in comparing sets of data which may have the same mean but a different range. For example, the mean of the following two is the same: 15, 15, 15, 14, 16 and 2, 7, 14, 22, 30. However, the second is clearly more spread out. If a set has a low standard deviation, the values are not spread out too much.

Just like when working out the mean, the method is different if the data is given to you in groups.

Grouped Data

When dealing with grouped data, such as the following:

x	**f**
4	9
5	14
6	22

7	11
8	17

the formula for standard deviation becomes:

$\sigma = \sqrt{\sum f(X - \bar{X})^2 / \sum f}$

Try working out the standard deviation of the above data. You should get an answer of 1.32
.

You may be given the data in the form of groups, such as:

Number	**Frequency**
3.5 – 4.5	9
4.5 – 5.5	14
5.5 – 6.5	22
6.5 – 7.5	11
7.5 – 8.5	17

In such a circumstance, x is the midpoint of groups.

To illustrate the variability of a group of scores, in statistics, we use “variance” or “standard deviation”. We define the deviation of a single score as its distance from the mean: ***Variance*** is symbolized by s^2. ***Standard Deviation*** is s. N is the number of scores.

Deviation = $X - \bar{X}$

$\Sigma^2 = \sqrt{\sum (X - \bar{X})^2 / N}$

$\sigma = \sqrt{\sigma^2} = \sqrt{\sum (X - \bar{X})^2 / N}$

Summary of the calculation procedures:

Subtract the mean from each score

Square each result

Sum all the squares

Divide the sum of square by N. Now you getvariance

Problem 1:

Bridget surveyed the price of petrol at petrol stations in Cape Town and Durban. The data, in rands per litre, are given below.

Cape Town	3,96	3,76	4,00	3,91	3,69	3,72
Durban	3,97	3,81	3,52	4,08	3,88	3,68

Find the mean price in each city and then state which city has the lower mean.

Find the standard deviation of each city's prices.

Which city has the more consistently priced petrol? Give reasons for your answer.

Answer 1:

Cape Town: 3,84. Durban: 3,82. Durban has the lower mean.

Standard deviation:

$\sigma = \sqrt{\sum(X - \bar{X})^2 / n}$

For Cape Town:

$\sigma = \sqrt{\sum(X-(3{,}84))^2 / 6}$

$= \sqrt{0{,}0882 / 6}$

$= \sqrt{0{,}0147} \approx 0{,}121$

For Durban:

$\sigma = \sqrt{\sum(X-(3{,}82\dot{3}))^2 / 6}$

$= \sqrt{0{,}20\dot{3} / 6}$

$= \sqrt{0{,}033\dot{8}} \approx 0{,}184$

The standard deviation of Cape Town's prices is lower than that of Durban's. That means that Cape Town has more consistent (less variable) prices than Durban.

Exercise

1. Compute the mean 150 ; 300 ; 250 ; 270 ; 130 ; 80 ; 700 ; 500 ; 200 ; 220 ; 110 ; 320 ; 420 ; 140

A. 370,7 B. 470,2 C 270,7 D. 170,7

2. Compute variance of the following set of values 150 ; 300 ; 250 ; 270 ; 130 ; 80 ; 700 ; 500 ; 200 ; 220 ; 110 ; 320 ; 420 ; 140

A. 27 435,2 B. 29 435,2 C. 37 435,2 D. 27 625,2

3. Consider the following three data sets A, B and C.

A = {9,10,11,7,13}

B = {10,10,10,10,10} Find

C = {1,1,10,19,19}

Calculate the mean of each data set.

A. 9, 10, 15   B. 10, 10, 15 C. 10, 10, 10 D. 9, 15, 10

4. Calculate the standard deviation of data set A, B and C.
A. 2, 1, 8 B. 2, 0, 8.05 C. 3, 2, 9.05 D. 2, 2, 8.09
5. Which set has the largest standard deviation amongst the above?

A. Data set A B. Data set B C. Data set C D. All equal

Answers

1. C 2. A 3. C 4. B 5. C

Week 12

Tabulation of Grouped Data

The process of placing classified data into tabular form is known as tabulation. A table is a symmetric arrangement of statistical data in rows and columns. Rows are horizontal arrangements whereas columns are vertical arrangements. It may be simple, double or complex depending upon the type of classification.

Types of Tabulation:

(1) Simple Tabulation or One-way Tabulation:

When the data are tabulated to one characteristic, it is said to be simple tabulation or one-way tabulation.

For Example: Tabulation of data on population of world classified by one characteristic like Religion is example of simple tabulation.

(2) Double Tabulation or Two-way Tabulation:

When the data are tabulated according to two characteristics at a time. It is said to be double tabulation or two-way tabulation.

For Example: Tabulation of data on population of world classified by two characteristics like Religion and Sex is example of double tabulation.

(3) Complex Tabulation:

When the data are tabulated according to many characteristics, it is said to be complex tabulation.

Below is an Example of grouped data

An estimate, , of the mean of the population from which the data are drawn can be calculated from the grouped data as:

In this formula, x refers to the midpoint of the class intervals, and f is the class frequency. Note that the result of this will be different from the sample mean of the ungrouped data. The mean for the grouped data in the above example, can be calculated as follows:

Class Intervals	Frequency (f)	Midpoint (x)	fx
5 and above, below 10	1	7.5	7.5
10 ≤ t < 15	4	12.5	50
15 ≤ t < 20	6	17.5	105
20 ≤ t < 25	4	22.5	90
25 ≤ t < 30	2	27.5	55
30 ≤ t < 35	3	32.5	97.5
TOTAL	**20**		**405**

Thus, the mean of the grouped data is

{

ASSESSMENT

1. The process of placing classified data into tabular form is known as (a) tabulation

   (b) classification

   (c) sorting

   (d) statistics

2. When the data are tabulated to one characteristic, it is said to be (a) simple tabulation

   (b)complex tabulation

   (b) double tabulation

   (d) two-way tabulation

3. Second step in constructing frequency distribution is to (a) determine class limits

   (b) determine midpoints of classes

   (c) select appropriate class intervals

   (d) determine width of class intervals

4. Complex type of table in which variables to be studied are subdivided with interrelated characteristics is called (a) two way table

   (b) one way table

   (c)subparts of table

   (d) order level table

5. Largest numerical value is 45 and smallest numerical value is 25 and classes desired are 4 then width of class interval is (a) 45

   (b) 65

   (c) 5

   (d) 17.5

ANSWERS

1. a
2. a
3. d
4. a
5. c

Week 13

The Processes for Calculating the Range, Median and Mode of Grouped Frequencies is hereby explained with Examples

This starts with some raw data which is **not yet a grouped frequency-**

Ade timed twenty one people in the sprint race, to the nearest second:

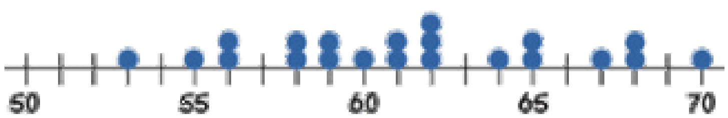
59, 65, 61, 62, 53, 55, 60, 70, 64, 56, 58, 58, 62, 62, 68, 65, 56, 59, 68, 61, 67

To find the Mean, Ade adds up all the numbers, then divides by how many numbers:

$$\textbf{Mean} = \frac{59+65+61+62+53+55+60+70+64+56+58+58+62+62+68+65+56+59+68+61+67}{21}$$

$$= \quad \textbf{61.38095...}$$

To find the Median, Ade places the numbers in value order and finds the middle number.



In this case the median is the 11^{th} number:

53, 55, 56, 56, 58, 58, 59, 59, 60, 61, 61, 62, 62, 62, 64, 65, 65, 67, 68, 68, 70

Median = 61

To find the Mode, or modal value, Ade places the numbers in value order then counts how many of each number. The Mode is the number which appears most often (there can be more than one mode):

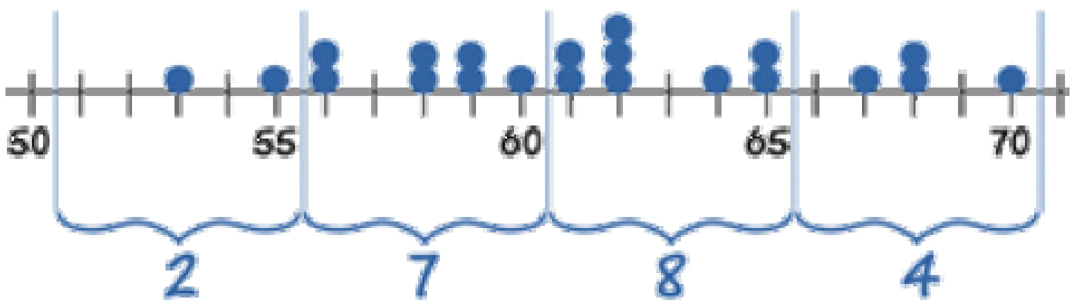
53, 55, 56, 56, 58, 58, 59, 59, 60, 61, 61, 62, 62, 62, 64, 65, 65, 67, 68, 68, 70

62 appears three times, more often than the other values, so **Mode = 62**

Grouped Frequency Table

Ade then makes a Grouped Frequency Table:

Seconds	Frequency
51 – 55	2
56 – 60	7
61 – 65	8
66 – 70	4



So two runners took between 51 and 55 seconds, 7 took between 56 and 60 seconds, etc

Suddenly all the original data gets lost (naughty pup!)

Only the Grouped Frequency Table survived …

Can we help Ade calculate the Mean, Median and Mode from just that table?

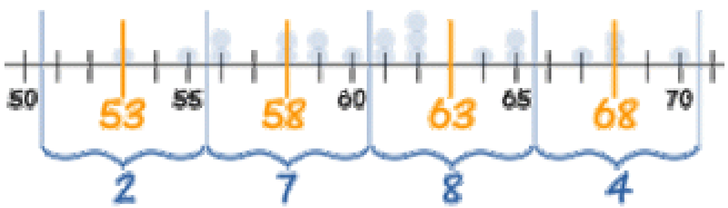
The answer is … no we can't. Not accurately anyway. But, we can make estimates.

<u>Estimating the Mean from Grouped Data</u>

So all we have left is:

Seconds	Frequency
51 – 55	2
56 - 60	7
61 – 65	8
66 – 70	4

- The groups (51-55, 56-60, etc), also called class intervals, are of width 5
- The midpoints are in the middle of each class: 53, 58, 63 and 68



We can estimate the Mean by using the **midpoints**.

So, how does this work?

Think about the 7 runners in the group **56 – 60**: all we know is that they ran somewhere between 56 and 60 seconds:

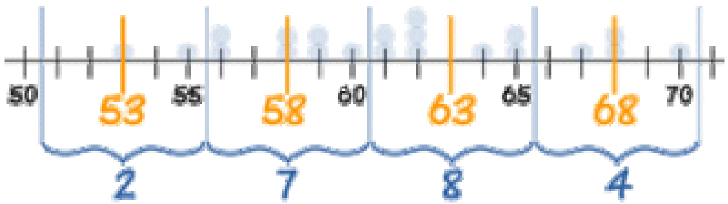
- Maybe all seven of them did 56 seconds,
- Maybe all seven of them did 60 seconds,
- But it is more likely that there is a spread of numbers: some at 56, some at 57, etc

So we take an average and **assume** that all seven of them took 58 seconds.

Let's now make the table using midpoints:

Midpoint	Frequency
53	2

58	7
63	8
68	4



Our thinking is: “2 people took 53 sec, 7 people took 58 sec, 8 people took 63 sec and 3 took 68 sec”. In other words we **imagine** the data looks like this:

53, 53, 58, 58, 58, 58, 58, 58, 58, 63, 63, 63, 63, 63, 63, 63, 63, 68, 68, 68, 68

Then we add them all up and divide by 21. The quick way to do it is to multiply each midpoint by each frequency:

Midpoint x	Frequency f	Midpoint × Frequency fx
53	2	106
58	7	406
63	8	504
68	4	272
Totals:	**21**	**1288**

And then our **estimate** of the mean time to complete the race is:

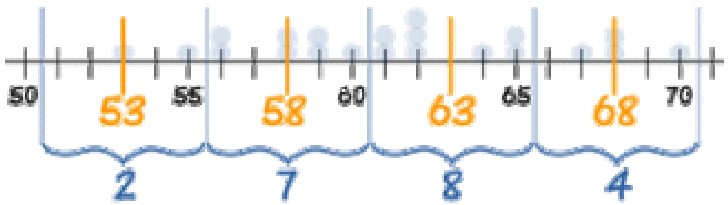
$$\textbf{Estimated Mean} = \frac{1288}{21} = \textbf{61.333...}$$

Very close to the exact answer we got earlier.

Estimating the Median from Grouped Data

Let's look at our data again:

Seconds	Frequency
51 – 55	2
56 – 60	7
61 – 65	8
66 – 70	4



The median is the middle value, which in our case is the 11th one, which is in the 61 – 65 group:

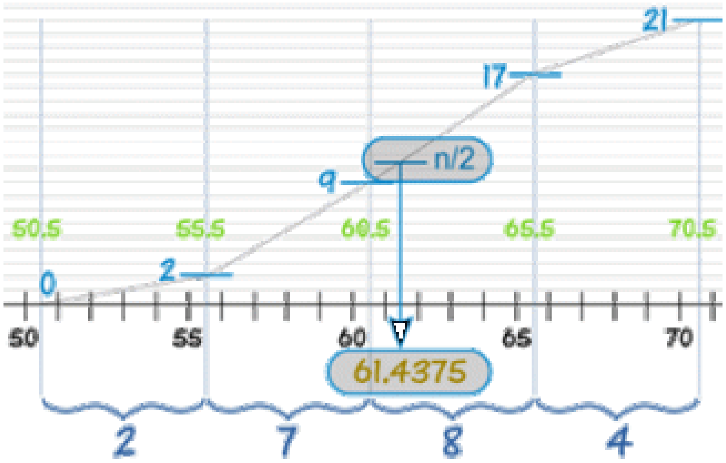
We can say "the **median group** is 61 – 65"

But if we want an estimated **Median value** we need to look more closely at the 61 – 65 group.

We call it "61 – 65", but it really includes values from 60.5 up to (but not including) 65.5.

Why? Well, the values are in whole seconds, so a real time of 60.5 is measured as 61. Likewise 65.4 is measured as 65.

At 60.5 we already have **9** runners, and by the next boundary at 65.5 we have **17** runners. By drawing a straight line in between we can pick out where the median frequency of **n/2** runners is:



And this handy formula does the calculation:

$$\textbf{Estimated Median} = L + \frac{(n/2) - B}{G} \times w$$

where:

- **L** is the lower class boundary of the group containing the median
- **n** is the total number of values
- **B** is the cumulative frequency of the groups before the median group
- **G** is the frequency of the median group
- **w** is the group width

For our example:

- **L** = 60.5
- **n** = 21
- **B** = 2 + 7 = 9
- **G** = 8
- **w** = 5

$$\textbf{Estimated Median} = 60.5 + \frac{(21/2) - 9}{8} \times 5$$

$$= \mathbf{60.5 + 0.9375}$$

$$= \mathbf{61.4375}$$

Estimating the Mode from Grouped Data

Again, looking at our data:

Seconds	Frequency
51 – 55	2
56 – 60	7
61 – 65	8
66 – 70	4

We can easily find the modal group (the group with the highest frequency), which is **61 – 65**

We can say "the **modal group** is 61 – 65"

But the actual Mode may not even be in that group! Or there may be more than one mode. Without the raw data we don't really know.

But, we can estimate the Mode using the following formula:

$$\textbf{Estimated Mode} = L + \frac{f_m - f_{m-1}}{(f_m - f_{m-1}) + (f_m - f_{m+1})} \times w$$

where:

- L is the lower class boundary of the modal group

- f_{m-1} is the frequency of the group before the modal group
- f_m is the frequency of the modal group
- f_{m+1} is the frequency of the group after the modal group
- w is the group width

In this example:

- L = 60.5
- $f_{m-1} = 7$
- $f_m = 8$
- $f_{m+1} = 4$
- w = 5

$$\textbf{Estimated Mode} = 60.5 + \frac{8 - 7}{(8 - 7) + (8 - 4)} \times 5$$

$$= 60.5 + (1/5) \times 5$$

$$= \mathbf{61.5}$$

Our final result is:

- Estimated Mean: **61.333...**
- Estimated Median: **61.4375**
- Estimated Mode: **61.5**

(Compare that with the true Mean, Median and Mode of **61.38..., 61 and 62** that we got at the very start.)

And that is how it is done.

Now let us look at two more examples, and get some more practice along the way!

ASSESSMENT

1. If mean is 11 and median is 13 then value of mode is

(a) 15

(b) 13

(c) 11

(d) 17

2. If value of mode is 14 and value of arithmetic mean is 5 then value of median is

(a) 12

(b) 18

(c) 8

(d) 14

3. Median of 7, 6, 4, 8, 2, 5, 11 is

(a) 6

(b) 12

(c) 11

(d) 4

4. Number which occurs most frequently in a set of numbers is

(a) mean

(b) median

(c) mode

(d) None of above

5. Mode of 12, 17, 16, 14, 13, 16, 11, 14 is

(a) 13

(b) 11

(c) 14

(d) 14 and 16

ANSWERS

1. D
2. C
3. A
4. C
5. D