MATHEMATICS

For

Senior Secondary School

2

EDUBASE

SS 2
FIRST TERM NOTES
ON
MATHEMATICS

TABLE OF CONTENT

FIRST TERM

WEEK 1

Introduction to the Logarithm of Number Less than One

The characteristics of the logs of all numbers equal to or greater than 1 are equal to or greater than 0, i.e. are all positive or zero, if m is the characteristics of all $\log_{10}$ N where N is greater than or equal to 1 then m is greater than or equal to 0. We talk of negative characteristics when we consider logarithms of numbers which lie between 0 and 1 i.e. all decimals numbers less than 1, eg 0.0314. Here again we bring in the standard form of the number, where the power of 10 gives us the characteristics, thus $0.0314 = 3.14 \times 10^{-2}$,and so -2 is the characteristics of the log of 0.0314.

Alternatively, we obtain the characteristics by doing these two things

a. add 1 to the number of zeros between the decimal point and the first significant figure in the given number and,

b. make this value obtained negative.

Example

There is only one zero between the decimal point and 3, the first significant figure in the given number 0.0314 1 + 1 = 2 then (b) this two is made negative, i.e. -2 So the characteristics is -2. But we put the minus sign on top of the characteristics only, thus ‘2 pronounced “bar” 2 not “minus” 2 even though they have the same value to show that the minus only refers to the characteristics and not to both the characteristics and mantissa part of the log. If the minus was in front like -2.(...) it will seem as if both the characteristics and mantissa are negative. This negative characteristics is more obvious when the number is expressed in its standard form $a \times 10^{-n}$ in which case the characteristics is (-n) i.e. `n.

Example

Find the characteristics (negative) and hence the logarithm of the following numbers

1. 0.051 2. 0.0084 3. 0.0000765

	Number	No of zeros between decimal pt. and 1st signf figure +1	Standardform of number	Charact-eristics	Answers
a	0.051	One + 1→ 2 neg	5.1 x 10^{-2}	2	Log of 0.051 =`2.7076
b	0.0084	Two + 1→ 3 neg	8.4 x 10^{-3}	3	Log of 0.084 = `3.9243
c	0.0000765	Four + 1→ 5 neg	7.65 x 10^{-5}	5	Log of 0.0000765 = `5.8837

Note in seeking for the logarithm of any number, first decide what the characteristics is. This can be either positive, zero or negative as shown in the examples. Then after finding the characteristic, look up the mantissa part of the log on the log table. The mantissa is always positive. Using the logarithm tables, makes calculation especially with very little or small numbers, easier and more time saving. Great care should be taken in applying the laws of logarithm. The antilogarithms are always positive.
Note: Since logarithm, the laws of indices are the same as the laws of logarithm.

Use of Logarithm in Solving Problems
Squares and Square Roots

Definitions Square of 2 means 2 x 2 = 4 and written as 2^2. The square root of 4 means the number which when multiplied by itself gives 4 which is = 2 or -2. Square root of 4 is written as √4. Again 3^3, the square of 3 = 9 and √9 = 3 or -3. Hence the square root of a number m is that number n which when squared equals the original number m, i.e. if the square root of m=n, then n^2 = m. So squares and square roots are so connected as described above, the square root being usually the smaller number. Note that each number has two possible square roots, the positive root and the negative root. This is because – x – = +, + x + = +.

Example

√4 = +2 or -2 because both (+2) and (-2) = 4 and (-2) x(-2) = 4. Hence all squares are positive numbers. The squares and square roots of numbers can be found by several calculative methods such as

a. using logarithm table

b. using squares and square root tables

c. using factor methods

d. using long division method (for square root),

e. using long multiplication (for squares)

Example Find (i) the square of 76, (ii) the square root of 400.

(i) Using long Multiplication Method

$(76)^2$ = 76 x 76 =

76

X 76

456

532

5776

(ii) Using logs

Log of 76 = 1.8808

Log of $(76)^2$ = 1.8808

x 2 = 3.7616

Antilog = 5776

$(76)^2$ = 5776

Using Square Tables

Look up for 76 under 0, we have 5776.

Reciprocal

A reciprocal, or *multiplicative inverse,* is simply one of a pair of numbers that, when multiplied together, equal 1. If you can reduce the number to a fraction, finding the reciprocal is simply a matter of transposing the numerator and the denominator. To find the reciprocal of a whole number, just turn it into a fraction in which the original number is the denominator and the numerator is 1.

For example, the reciprocal of 2/3 is 3/2 (or 1-1/2) , because 2/3 x 3/2 = 1. The reciprocal of 7 is 1/7 because 7 x 1/7 = 1.

Decimal numbers, too, have reciprocals. To find the reciprocal of a decimal number, divide 1 by that number. For instance, to find the reciprocal of 1.25, divide 1 by 1.25:

1 ÷ 1.25 = 0.8

The multiplicative inverse of 1.25, therefore, is 0.8.

Understanding reciprocals can simplify many math problems when you understand that dividing by a number is the same as multiplying by the reciprocal of that number. For example

5 ÷ 1/4

is the same as

5 x 4/1 (which is simply 5 x 4, which of course equals 20)

10 ÷ 1/10

3 ÷ 3/8

Questions

1. Without using table write down the square roots of 0.0009

A. 0.003   B. 0.6   C. 0.03   D. 0.06

2. What is the reciprocal of this expression 8 ÷ 1/5?

A. 30 B. 30.625 C. 40 D. 1.6

3. Find the reciprocal of this expression 10 ÷ 1/10.

A. 1 B. 100 C. 1000 D. 0.01

4. Find reciprocal of -5

A. 1/5 B. 5/1 C. -1/5 D. -5/1

5. The characteristics of the logs of all numbers equal to or greater than 1are equal to or greater than

A. 0 B. 1 C. 10   D. 11

Answers

1. C 2. C 3. B 4. C 5.

WEEK 2

Introduction to Approximation and Percentage Error

Sometimes we do not need to measure of calculate things exactly. We may only wish to have a rough idea, or to calculate only to a certain degree of accuracy. It is in such cases we talk of decimal places, significant figures or rounding off to the nearest unit, tens, etc. When this happens, we say that we are approximating.

When we approximate we use such terms as

a. rounding off to the nearest unit, or ten, or hundred, or .....

b. decimal places

c. significant places etc.

Rounding Off

In rounding off we recall the place-value of numbers

e.g. 743,285 means 7 hundreds of thousands

plus 4 tens of thousands

plus 3 units of thousands

plus 2 hundreds

plus 5 tens

and 8 units

which we can write in the abacus form as

H of Th	**T of Th**	**Th**	**H**	**T**	**U**
7	4	3	2	5	8

Now to round off this number to the nearest ten, for example, we look at the digit to the right of 5 tens. If this digit is equal to , or greater than 5 (^{3} 5) then we add 1 to the digit in the tens place. Like here the digit to the right of 5 tens is 8: and 8 > 5 then 5 tens becomes 6 tens i.e. to the nearest ten, 743,285 = 743,260.

But if the digit to the right of 5 tens is less than 5 (< 5) then we do not add 1 to the tens value

i.e. To the nearest ten, 743,253 = 743,250

Example

488 = 490 to the nearest ten

488 = 500 to the nearest hundred

119,347 = 119,300 to the nearest hundred

= 120,000 to the nearest tens of thousands

= 100,000 to the nearest hundreds of thousands

To round off a number to the nearest **Unit** usually involves numbers which are written in decimal form or mixed number form. If the fractional part is greater than ½ we add 1 to the unit digit of the number.

In the decimal form, if the digit after the decimal point is 5 or greater than 5, we add 1 to the unit digit of the number.

Decimal Places

In order to calculate to a required number of decimal place, we usually calculate to one place more than the required. If that last digit is less than 5, it will be discarded but if it is 5 or more than 5, then 1 is added to the digit just before it.

Examples

1. 45.475 to two decimal places= 45.48 since the last digit which is 8 is more than 5 we add 1 to 7
2. 122.184 to two decimal places = 122.18. We discard 4 which is less than 5

Significant Figures

The first non-zero digit in any number is its first significant figure. A zero between two significant figures is counted as a significant figure. We calculate to a required number of significant places by the same method a s we do for decimal places. Numbers can be rounded off to any required number of the significant figures.

Example

(i) To 3 significant figures, 0.02364 = 0.0236, Note we do not count the first digit, 0.

(ii) To 2 significant figures, 5286 = 5300, since 8 is more than 5, 1 is added to 2
(iii) Ti 1 significant figure, 9.103 = 9. (Since 1 is less than 5)

Percentage Error
When we make estimate or approximation we do not have the exact value of the result, but an approximation to it, i.e. an idea of the value.
Now the difference between the actual result and the estimated or approximated result can be calculated in percentage. This is known a s percentage error

Note The nearer the percentage error is to zero, the more accurate the result.

Example
The exact distance a boy walk to school every morning is 4km, when asked he said 4 ½ km while his parents said he walks 3 km to school, Find their percentage error.

Solution
Both the boy and the parents made errors. The boy **over estimated** by ½ km and the parents **under estimated** by 1 km.
½ km and 1 km are called the **absolute error**s.
The percentage error of Boy = ½ /4 x 100% = 1/8 x 100 = 12 ½ %
The Percentage error of the Parents = ¼ x 100% = 25%
So the boy is more accurate than the parents.
The absolute error is difference between the estimate and the exact values. This percentage error is usually referred to as the **Relative Error** i.e. Relative error is often given as a percentage.
Percentage Error = absolute error/exact value x 100%

Approximation and Estimations in Everyday Life
a. A mother who wants to buy, say 3 loaves of bread, 5kg of garri. 1 packet of sugar and 1 bottle of oil will ensure that she has enough money for those

items before leaving her house for the market. She may not know the exact costs of the items, but by estimation she will give some prices to the items, add the costs up, and then go shopping with approximately the amount needed for those items.

b. Cooks have to estimate the quantity of food that will satisfy a customer in a restaurant (or even at home), and multiply that quantity by the number of people to feed, then round off the quantities to take care of wastages etc. Life is full of activities involving estimations and approximations.

Questions

1. Round off 1,478 to the 1000
A. 1,480 B. 1,479 C. 2000 D, 1,400

2. Round off 479 to the nearest 100
A. 480 B. 500 C. 789 D. 501

Find both the absolute errors and the Percentage errors of the following
3. Exact Value: ₦400, Approximate Measurement
A. 27 and 6.25% B. 25 and 6.25% C. 25 and 7.35% D. 24 and 6.25%

4. Exact value : 6.25 m, Approximate Measurement: 5.5 m
A. 0.65 and 12 B. 0.75 and 15 C. 0.75 and 12 D. 1.02 and 12
5. What is the first significant figure of 46.057?
A. 46.10 B. 46.06 C. 46.5 D. 47

Answers

1. C 2. B 3. B 4. C 5. B

WEEK 3

Arithmetic Progression (A.P.)

If you can recall some of the formulae you must have studied earlier, you'd be able to remember that a sequence $a_1, a_2, a_3, \ldots, a_n, \ldots$ is called arithmetic sequence or arithmetic progression if $a_{n+1} = a_n + d,\ n \in N$, where a_1 is called the first term and the constant term d is called the

Let us consider an A.P. (in its standard form) with first term a and common difference d, i.e., a, a + d, a + 2d, …

Then the n^{th} term (general term) of the A.P. is $a_n = a + (n - 1) d$.

We can verify the following simple properties of an A.P. :

(i) If a constant is added to each term of an A.P., the resulting sequence is also an A.P.

(ii) If a constant is subtracted from each term of an A.P., the resulting sequence is also an A.P.

(iii) If each term of an A.P. is multiplied by a constant, then the resulting sequence is also an A.P.

(iv) If each term of an A.P. is divided by a non-zero constant then the resulting sequence is also an A.P.

Here, we shall use the following notations for an arithmetic progression:

a = the first term, l = the last term, d = common difference,

n = the number of terms.

S_n= the sum to n terms of A.P.

Let a, a + d, a + 2d, …, a + (n − 1) d be an A.P. Then

$l = a + (n - 1) d$

$S_n = n/2[2a + (n-1)d]$

We can also write, $S_n = n/2[a + l]$

Let us consider some examples.

Example In an A.P. if m^{th} term is n and the n^{th} term is m, where $m \neq n$, find the pth term.

Solution We have $a_m = a + (m - 1) d = n$, … (1)
And $a_n = a + (n - 1) d = m$ … (2)
Solving (1) and (2), we get
$(m - n) d = n - m$, or $d = -1$, … (3)
and $a = n + m - 1$ … (4)
Therefore $a_p = a + (p - 1)d$
$= n + m - 1 + (p - 1)(-1) = n + m - p$
Hence, the p^{th} term is $n + m - p$.

Example If the sum of n terms of an A.P. is $nP + ½ n(n - 1)Q$, where P and Q are constants, find the common difference.
Solution Let $a_1, a_2, … a_n$ be the given A.P. Then
$Sn = a_1 + a_2 + a_3 +…+ a_{n-1} + a_n = nP + ½ n (n - 1) Q$
Therefore $S_1 = a_1 = P$, $S_2 = a_1 + a_2 = 2P + Q$
So that $a_2 = S_2 - S_1 = P + Q$
Hence, the common difference is given by $d = a_2 - a_1 = (P + Q) - P = Q$.

Example The income of a person is Rs. 3,00,000, in the first year and he receives an increase of Rs.10,000 to his income per year for the next 19 years. Find the total amount, he received in 20 years.
Solution Here, we have an A.P. with a = 3,00,000, d = 10,000, and n = 20.

Using the sum formula, we get,
S_{20} = 20/2 [600000 + 19 x 10000] = 10 (790000) = 79,00,000.
Hence, the person received Rs. 79,00,000 as the total amount at the end of 20 years.

Arithmetic mean
Given two numbers a and b. We can insert a number A between them so that a, A, b is an A.P. Such a number A is called the arithmetic mean (A.M.) of the numbers a and b. Note that, in this case, we have $A - a = b - A$, i.e., $A = (a + b) / 2$

We may also interpret the A.M. between two numbers a and b as their average (a + b) / 2. For example, the A.M. of two numbers 4 and 16 is 10. We have, thus constructed an A.P. 4, 10, 16 by inserting a number 10 between 4 and 16. The natural question now arises : Can we insert two or more numbers between given two numbers so that the resulting sequence comes out to be an A.P. ? Observe that two numbers 8 and 12 can be inserted between 4 and 16 so that the resulting sequence 4, 8, 12, 16 becomes an A.P. More generally, given any two numbers a and b, we can insert as many numbers as we like between them such that the resulting sequence is an A.P.

Let $A_1, A_2, A_3, \ldots, A_n$ be n numbers between a and b such that a, $A_1, A_2, A_3, \ldots, A_n$, b is an A.P.

Here, b is the $(n + 2)^{th}$ term, i.e., $b = a + [(n + 2) - 1]d = a + (n + 1) d$. This gives $d = (b - a)/(n + 1)$.

Thus, n numbers between a and b are as follows:

Example Insert 6 numbers between 3 and 24 such that the resulting sequence is an A.P.

Solution Let A_1, A_2, A_3, A_4, A_5 and A_6 be six numbers between 3 and 24 such that

3, $A_1, A_2, A_3, A_4, A_5, A_6$, 24 are in A.P. Here, a = 3, b = 24, n = 8.

Therefore, $24 = 3 + (8 - 1) d$, so that d = 3.

Thus $A_1 = a + d = 3 + 3 = 6$; $A_2 = a + 2d = 3 + 2 \times 3 = 9$;

$A_3 = a + 3d = 3 + 3 \times 3 = 12$; $A_4 = a + 4d = 3 + 4 \times 3 = 15$;

$A_5 = a + 5d = 3 + 5 \times 3 = 18$; $A_6 = a + 6d = 3 + 6 \times 3 = 21$.

Hence, six numbers between 3 and 24 are 6, 9, 12, 15, 18 and 21.

Geometric Progression

In mathematics, a **geometric progression**, also known as a **geometric sequence**, is a sequence of numbers where each term after the first is found by multiplying the previous one by a fixed, non-zero number called the *common ratio*. For example, the sequence 2, 6, 18, 54, ... is a geometric

progression with common ratio 3. Similarly 10, 5, 2.5, 1.25, ... is a geometric sequence with common ratio 1/2.
Examples of a geometric sequence are powers r^k of a fixed number r, such as 2^k and 3^k. The general form of a geometric sequence is

where $r \neq 0$ is the common ratio and a is a scale factor, equal to the sequence's start value.
A **geometric sequence** is a sequence such that any element after the first is obtained by multiplying the preceding element by a constant called the **common ratio** which is denoted by r. The common ratio (r) is obtained by dividing any term by the preceding term, i.e.,

$$r = \frac{a_2}{a_1} = \frac{a_3}{a_2} = \ldots = \frac{a_n}{a_{n-1}}$$

where	r	common ratio
	a_1	first term
	a_2	second term
	a_3	third term
	a_{n-1}	the term before the nth term
	a_n	the nth term

The **geometric sequence** is sometimes called the **geometric progression** or **GP**, for short.

For example, the sequence 1, 3, 9, 27, 81 is a geometric sequence. Note that after the first term, the next term is obtained by multiplying the preceding element by 3.
The geometric sequence has its sequence formation: $a_1, a_1r, a_1r^2, \ldots, a_1r^{n-1}, a_1r^n$
To find the *nth* term of a geometric sequence we use the formula:

$$a_n = a_1 r^{n-1}$$

where	r	common ratio
	a_1	first term
	a_{n-1}	the term before the nth term

n number of terms

Sum of Terms in a Geometric Progression

Finding the sum of terms in a geometric progression is easily obtained by applying the formulas:

nth partial sum of a geometric sequence

$$S_n = \frac{a_1(1-r^n)}{1-r}, \qquad r \neq 1$$

sum to infinity

$$S_\infty = \sum_{n=1}^{\infty} ar^{n-1} = \frac{a_1}{1-r}, -1 < r < 1$$

where S_n sum of GP with n terms

S_∞ sum of GP with infinitely many terms

a_1 the first term

r common ratio

n number of terms

Examples of Common Problems to Solve

Write down a specific term in a Geometric Progression

Question

Write down the 8th term in the Geometric Progression 1, 3, 9, ...

Answer

$a_1 = 1;\ a_2 = 3;\ a_3 = 9; n = 8$	write down key terms
$r = \frac{a_2}{a_1} = \frac{3}{1} = 3$	find the common ratio r using $r = \frac{a_2}{a_1}$
$a_8 = a_1 r^{8-1}$	substitute $n = 8$ to $a_n = a_1 r^{n-1}$
$= (1)(3)^7$	substitute $a_1 = 1$ and $r = 3$
$= (3)^7$	multiply (1) and $(3)^7$
$a_8 = 2187$	simplify (3)7 = 2187

Finding the number of terms in a Geometric Progression

Question

Find the number of terms in the geometric progression 6, 12, 24, ..., 1536

Answer

$a_1 = 6;\ a_2 = 12;\ a_3 = 24; a_n = 1536$	write down key terms
$r = \frac{a_2}{a_1} = \frac{12}{6} = 2$	find r using $r = \frac{a_2}{a_1}$
$1536 = (6)(2)^{n-1}$	substitute the values of a_1, a_n and r to $a_n = a_1 r^{n-1}$ to find n
$256 = (2)^{n-1}$	divide both sides by 6
$2^8 = 2^{n-1}$	change 256 to its exponential form form whose base = r
$8 = n - 1$	equate the indices since they both have the same base
$8 + 1 = n$	add 1 to both sides
$9 = n$	add 8 and 1

Hence, 1536 is the **9th** term.

Finding the sum of a Geometric Series

Question

Find the sum of each of the geometric series $-2, \frac{1}{2}, -\frac{1}{8}, \ldots, -\frac{1}{37268}$

Answer

$a_1 = -2;\ a_2 = \frac{1}{2};\ a_3 = -\frac{1}{8}; a_n = -\frac{1}{37268}$	write down key terms
$r = \frac{a_2}{a_1} = \frac{\frac{1}{2}}{-2} = -\frac{1}{4}$	find r using $r = \frac{a_2}{a_1}$
$-\frac{1}{37268} = (-2)\left(-\frac{1}{4}\right)^{n-1}$	substitute the values of a_1, a_n and r to $a_n = a_1 r^{n-1}$ to find n
$\frac{1}{65536} = \left(-\frac{1}{4}\right)^{n-1}$	divide both sides by -2
$\left(-\frac{1}{4}\right)^8 = \left(-\frac{1}{4}\right)^{n-1}$	change $\frac{1}{65536}$ to its exponential form form whose base=r
$8 = n - 1$	equate the indices since they both have the same base
$8 + 1 = n$	add 1 to both sides
$9 = n$	add 8 and 1
$S_9 = \frac{-2\left(1-\left(-\frac{1}{4}\right)^9\right)}{1-\left(-\frac{1}{4}\right)}$	substitute the values of a_1, r and n to $S_n = \frac{a_1(1-r^n)}{1-r}$
$= \frac{1\left(1-\left(-\frac{1}{262144}\right)\right)}{\frac{5}{4}}$	evaluate $\left(-\frac{1}{4}\right)^9$ then subtract $-\frac{1}{4}$ from *1*
$= \frac{\frac{262145}{262144}}{\frac{5}{4}}$	evaluate $\left(1 - \left(-\frac{1}{262144}\right)\right)$
$S_9 = \frac{52429}{60536}$	divide $\frac{262145}{262144}$ by $\frac{5}{4}$

Finding the sum of a Geometric Series to Infinity

Question

Work out the sum $\sum_{r=1}^{\infty}\left(\frac{1}{3}\right)^r$

Answer

$\sum_{r=1}^{\infty}\left(\frac{1}{3}\right)^r = \left(\frac{1}{3}\right)^1 + \left(\frac{1}{3}\right)^2 + \left(\frac{1}{3}\right)^3 + ... + \left(\frac{1}{3}\right)^n + ...$ expand the given sum

$= \frac{1}{3} + \frac{1}{9} + \frac{1}{27} + ... + \left(\frac{1}{3}\right)^n + ...$

$r = \frac{a_2}{a_1} = \frac{\frac{1}{9}}{\frac{1}{3}} = \frac{1}{3}$ solve for r using $r = \frac{a_2}{a_1}$

$S_\infty = \frac{\frac{1}{3}}{1-\frac{1}{3}}$ substitute $a_1 = \frac{1}{3}$ and $r = \frac{1}{3}$ to $S_\infty = \frac{a_1}{1-r}$

$= \frac{\frac{1}{3}}{\frac{2}{3}}$ subtract $\frac{1}{3}$ from *1*

$S_\infty = \frac{1}{2}$ divide $\frac{1}{3}$ by $\frac{2}{3}$

Converting a Recurring Decimal to a Fraction

Decimals that occurs in *repetition infinitely* or are *repeated in period* are called **recurring decimals**.

For example, 0.22222222… is a recurring decimal because the number 2 is repeated infinitely.

The recurring decimal 0.22222222… can be written as $0.\dot{2}$.

Another example is 0.234523452345… is a recurring decimal because the number 2345 is repeated periodically.

Thus, it can be written as $0.\dot{2}\dot{3}\dot{4}\dot{5}$ or it can also be expressed in fractions.

Question

Express $0.\dot{7}\dot{2}$ as a fraction in their lowest terms.

Answer

$0.\dot{7}\dot{2} = 0.727272.$	equivalent of the recurring decimal
Let $x = 0.727272$ [1]	
$100x = 72.7272$ [2]	multiply x and 0.727272 by 100
$100x - x = 72.7272 - 0.727272$	Subtract the equation [1] from [2]
$99x = 72$	evaluate both sides of the equation
$= \frac{72}{99}$	divide both sides by 99
$x = \frac{8}{11}$	simplify $\frac{72}{99}$

Go to the next page to start putting what you have learnt into practice.

ASSESSMENT

1. Number of observations are 30 and value of arithmetic mean is 15 then sum of all values is
   (a) 15
   (b) 450
   (c) 200
   (d) 45
2. In arithmetic mean, sum of deviations of all recorded observations must always be
   (a) two
   (b) minus one
   (c) one
   (d) zero
3. Arithmetic mean is 25 and all sum of observations is 350 then number of observations are
   (a) 25
   (b) 70
   (c) 14
   (d) 75

4. Arithmetic mean is 12 and number of observations are 20 then sum of all values is
   (a) 8
   (b) 32
   (c) 240
   (d) 1.667
5. Arithmetic mean is multiplied to coefficient of mean absolute deviation to calculate the
   (a) absolute mean deviation
   (b) absolute median deviation
   (c) relative mean deviation
   (d) relative median deviation

ANSWERS

1. b
2. d
3. c
4. c
5. a

WEEK 4

Definition of Common Ratio

Let's define a few basic terms before jumping into the subject of this lesson. A **sequence** is a group of numbers. It can be a group that is in a particular order, or it can be just a random set. A **geometric sequence** is a group of numbers that is ordered with a specific pattern. The pattern is determined by a certain number that is multiplied to each number in the sequence. This determines the next number in the sequence. The number multiplied must be the same for each term in the sequence and is called a **common ratio**.

Determining the Common Ratio

The common ratio is the amount between each number in a geometric sequence. It is called the common ratio because it is the same to each number, or common, and it also is the ratio between two consecutive numbers in the sequence.

To determine the common ratio, you can just divide each number from the number preceding it in the sequence. For example, what is the common ratio in the following sequence of numbers?

{2, 4, 8, 16}

Starting with the number at the end of the sequence, divide by the number immediately preceding it

16 / 8 = 2

Continue to divide to ensure that the pattern is the same for each number in the series.

8 / 4 = 2

4 / 2 = 2

Since the ratio is the same for each set, you can say that the common ratio is 2.

Therefore, you can say that the formula to find the common ratio of a geometric sequence is:

$d = a(n) / a(n - 1)$

Where $a(n)$ is the last term in the sequence and $a(n - 1)$ is the previous term in the sequence.
If you divide and find that the ratio between each number in the sequence is not the same, then there is no common ratio, and the sequence is not geometric.
Examples
Let's take a look at a few examples.
1.) What is the common ratio in the following sequence?
{3, 9, 27, 81}
81 / 27 = 3
27 / 9 = 3
9 / 3 = 3
The ratio between each of the numbers in the sequence is 3, therefore the common ratio is 3.
2.) What is the common ratio in the following sequence?
{5, 10, 15, 20}
20/15=1.3
15/10=1.5
10/5=2

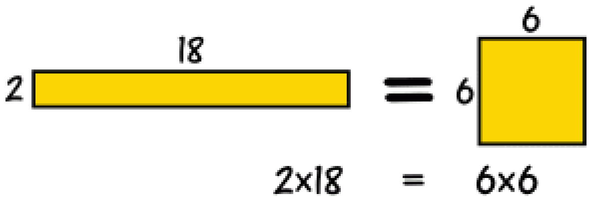
Geometric Mean
The Geometric Mean is a special type of average where we multiply the numbers together and then take a square root (for two numbers), cube root (for three numbers) etc.

Example: What is the Geometric Mean of 2 and 18?

- First we multiply them: 2 × 18 = 36
- Then (as there are two numbers) take the square root: √36 = **6**

In one line:

Geometric Mean of 2 and 18 = √(2 × 18) = 6
It is like the area is the same!



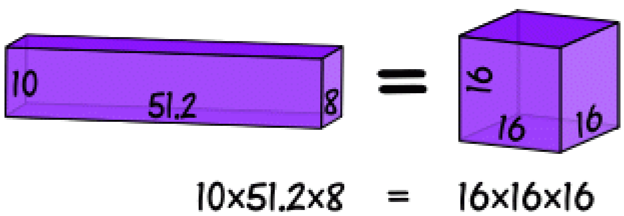
Example: What is the Geometric Mean of 10, 51.2 and 8?

- First we multiply them: 10 × 51.2 × 8 = 4096
- Then (as there are three numbers) take the cube root: $\sqrt[3]{4096}$ = **16**

In one line:

Geometric Mean = $\sqrt[3]{(10 \times 51.2 \times 8)}$ = 16

It is like the volume is the same:



Example: What is the Geometric Mean of 1, 3, 9, 27 and 81?

- First we multiply them: 1 × 3 × 9 × 27 × 81 = 59049
- Then (as there are 5 numbers) take the 5th root: $\sqrt[5]{59049}$ = **9**

In one line:

Geometric Mean = $\sqrt[5]{(1 \times 3 \times 9 \times 27 \times 81)}$ = 9

I can't show you a nice picture of this, but it is still true that:

1 × 3 × 9 × 27 × 81 = 9 × 9 × 9 × 9 × 9

Example: What is the Geometric Mean of a Molecule and a Mountain

Using scientific notation:

- A molecule of water (for example) is 0.275×10^{-9} m

- Mount Everest (for example) is 8.8×10^3 m

$$\textbf{Geometric Mean} = \sqrt{(0.275 \times 10^{-9} \times 8.8 \times 10^3)}$$
$$= \sqrt{(2.42 \times 10^{-6})}$$
$$\approx 0.0016 \text{ m}$$

Which is **1.6 millimeters**, or about the thickness of a coin.
We could say, in a rough kind of way,
"a millimeter is half-way between a molecule and a mountain!"
So the geometric mean gives us a way of finding a value in between widely different values.

ASSESSMENT

1. Sum of an infinite geometric series exist only if condition on common ratio r is
   (a) $-1 < r < 1$
   (b) $-1 \le r \le 1$
   (c) $r < -1, r > 1$
   (d) $r \le -1, r \ge 1$
2. G1,G2,....,Gn are said to be n geometric means between a and b if a,G1,...Gn,b is
   (a) a sequence
   (b) not a sequence
   (c) G.P
   (d) A.P
3. No terms of a geometric sequence be
   (a) 3
   (b) 1
   (c) 2
   (d) 0
4. If n is total number of geometric mean between a and b than nth geometric mean between a and b is
   (a) $a(b/a)^{n/n+}$
   (b)$a(b/a)^{m/n+1}$

(c) $b(a/b)^{m/n+1}$
(d) ab

5. A number G is said to be geometric mean between two numbers a and b if a,G,b is
(a) a sequence
(b) A.P
(c) not a sequence
(d) G.P

ANSWERS

1. a
2. c
3. d
4. b
5. c

WEEK 5

Factorization of Perfect Square

In factorization of perfect square we will learn how to factor different types of algebraic expressions using the following identities.

(i) $a^2 + 2ab + b^2 = (a + b)^2 = (a + b)(a + b)$

(ii) $a^2 - 2ab + b^2 = (a - b)^2 = (a - b)(a - b)$

Solved examples on factorization of perfect square:

1. Factorize the perfect square completely:

(i) $4x^2 + 9y^2 + 12xy$

Solution:

First we arrange the given expression $4x^2 + 9y^2 + 12xy$ in the form of $a^2 + 2ab + b^2$.

$4x^2 + 12xy + 9y^2$

$= (2x)^2 + 2(2x)(3y) + (3y)^2$

Now applying the formula of $a^2 + 2ab + b^2 = (a + b)^2$ then we get,

$= (2x + 3y)^2$

$= (2x + 3y)(2x + 3y)$

(ii) $25x^2 - 10xz + z^2$

Solution:

We can express the given expression $25x^2 - 10xz + z^2$ as $a^2 - 2ab + b^2$

$= (5x)^2 - 2(5x)(z) + (z)^2$

Now we will apply the formula of $a^2 - 2ab + b^2 = (a - b)^2$ then we get,

$= (5x - z)^2$

$= (5x - z)(5x - z)$

(iii) $x^2 + 6x + 8$

Solution:

We can that the given expression is not a perfect square. To get the expression as a perfect square we need to add 1 at the same time subtract 1 to keep the expression unchanged.

$= x^2 + 6x + 8 + 1 - 1$

$= x^2 + 6x + 9 - 1$
$= [(x)^2 + 2 (x) (3) + (3)^2] - (1)^2$
$= (x + 3)^2 - (1)^2$
$= (x + 3 + 1)(x + 3 - 1)$
$= (x + 4)(x + 2)$

2. Factor using the identity:
(i) $4m^4 + 1$

Solution:
$4m^4 + 1$
To get the above expression in the form of $a^2 + 2ab + b^2$ we need to add $4m^2$and to keep the expression same we also need to subtract $4m^2$ at the same time so that the expression remain same.
$= 4m^4 + 1 + 4m^2 - 4m^2$
$= 4m^4 + 4m^2 + 1 - 4m^2$, rearranged the terms
$= (2m^2)^2 + 2 (2m^2) (1) + (1)^2 - 4m^2$

Now we apply the formula of $a^2 + 2ab + b^2 = (a + b)^2$
$= (2m^2 + 1)^2 - 4m^2$
$= (2m^2 + 1)^2 - (2m)^2$
$= (2m^2 + 1 + 2m) (2m^2 + 1 - 2m)$
$= (2m^2 + 2m + 1) (2m^2 - 2m + 1)$

(ii) $(x + 2y)^2 + 2(x + 2y) (3y - x) + (3y - x)^2$
Solution:
We see that the given expression $(x + 2y)^2 + 2(x + 2y) (3y - x) + (3y - x)^2$ is in the form of $a^2 + 2ab + b^2$.
Here, $a = x + 2y$ and $b = 3y - x$

Now we will apply the formula of $a^2 + 2ab + b^2 = (a + b)^2$ then we get,
$[(x + 2y) + (3y - x)]^2$
$= [x + 2y + 3y - x]^2$
$= [5y]^2$

$= 25y^2$

Revision: Completing the Square

"**Completing the Square**" is where we ...

... take a Quadratic Equation like this: $ax^2 + bx + c = 0$

and turn it into this: $a(x+d)^2 + e = 0$

For those of you in a hurry, I can tell you that: d = b**2a**

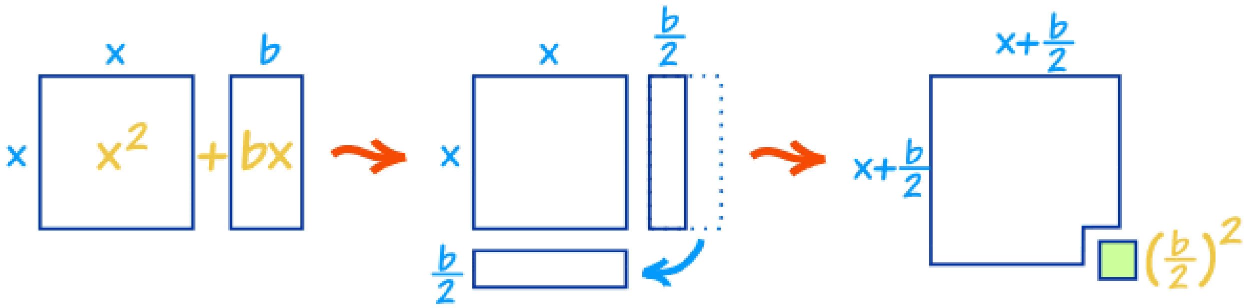
and: e = c − b^2**4a**

But if you have time, let me show you how to "**Complete the Square**" yourself.

Completing the Square

Say we have a simple expression like $x^2 + bx$. Having x twice in the same expression can make life hard. What can we do?

Well, with a little inspiration from Geometry we can convert it, like this:



As you can see $x^2 + bx$ can be rearranged ***nearly*** into a square ...

... and we can **complete the square** with $(b/2)^2$

In Algebra it looks like this:

$$x^2 + bx \quad + (b/2)^2 \quad = \quad (x+b/2)^2$$

"Complete the Square"

So, by adding $(b/2)^2$ we can complete the square.

And $(x+b/2)^2$ has x only **once**, which is easier to use.

Keeping the Balance

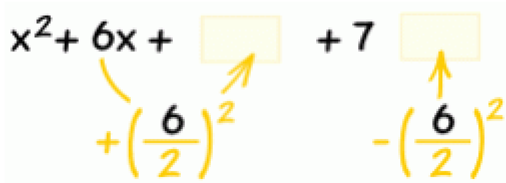
Now ... we can't just ***add*** $(b/2)^2$ without also ***subtracting*** it too! Otherwise the whole value changes.

So let's see how to do it properly with an example:

Start with: $x^2 + 6x + 7$

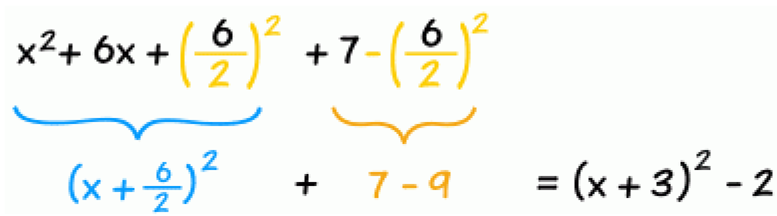
("b" is 6 in this case)

Complete the Square:



Also **subtract** the new term

Simplify it and we are done.



The result:

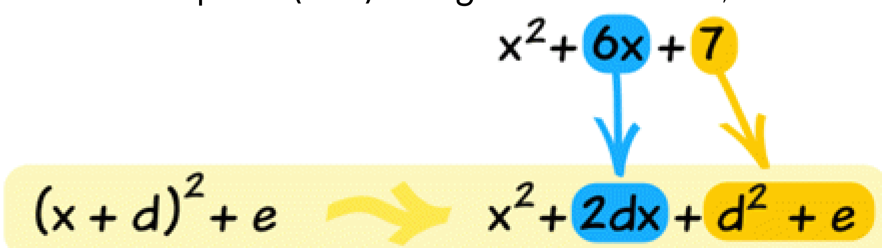
$x^2 + 6x + 7 \;=\; (x+3)^2 - 2$

And now x only appears once, and our job is done!

A Shortcut Approach

Let us look at the result we want: $(x+d)^2 + e$

When we expand $(x+d)^2$ we get $x^2 + 2dx + d^2$, so:



Now we can "force" an answer:

- We know that 6x must end up as 2dx, so **d must be 3**
- Next we see that 7 must become $d^2 + e = 9 + e$, so **e must be −2**

And we get the same result $(x+3)^2 - 2$ as above!

Now, let us look at a useful application: solving Quadratic Equations ...

Solving General Quadratic Equations by Completing the Square

We can complete the square to **solve** a Quadratic Equation (find where it is equal to zero).

But a general Quadratic Equation can have a coefficient of a in front of x^2:

$ax^2 + bx + c = 0$

But that is easy to deal with … just divide the whole equation by “a” first, then carry on:

$x^2 + (b/a)x + c/a = 0$

Steps

Now we can **solve** a Quadratic Equation in 5 steps:

- **Step 1** Divide all terms by **a** (the coefficient of $\mathbf{x^2}$).
- **Step 2** Move the number term (**c/a**) to the right side of the equation.
- **Step 3** Complete the square on the left side of the equation and balance this by adding the same value to the right side of the equation.

We now have something that looks like $(x + p)^2 = q$, which can be solved rather easily:

- **Step 4** Take the square root on both sides of the equation.

- **Step 5** Subtract the number that remains on the left side of the equation to find **x**.

Examples

Here are two examples:

Example 1: Solve $x^2 + 4x + 1 = 0$

Step 1 can be skipped in this example since the coefficient of x^2 is 1

Step 2 Move the number term to the right side of the equation:

$x^2 + 4x = -1$

Step 3 Complete the square on the left side of the equation and balance this by adding the same number to the right side of the equation.

$(b/2)^2 = (4/2)^2 = 2^2 = 4$

$x^2 + 4x + 4 = -1 + 4$

$(x + 2)^2 = 3$

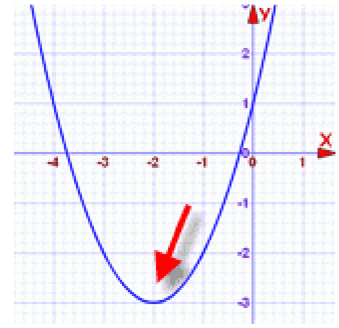
Step 4 Take the square root on both sides of the equation:

$x + 2 = \pm\sqrt{3} = \pm 1.73$ (to 2 decimals)

Step 5 Subtract 2 from both sides:

$x = \pm 1.73 - 2 = -3.73$ or -0.27

And here is an interesting and useful thing.
At the end of step 3 we had the equation:
$(x + 2)^2 = 3$
It gives us the **vertex** (turning point) of $x^2 + 4x + 1$: **(-2, -3)**



Example 2: Solve $5x^2 - 4x - 2 = 0$
Step 1 Divide all terms by 5
$x^2 - 0.8x - 0.4 = 0$
Step 2 Move the number term to the right side of the equation:
$x^2 - 0.8x = 0.4$
Step 3 Complete the square on the left side of the equation and balance this by adding the same number to the right side of the equation:
$(b/2)^2 = (0.8/2)^2 = 0.4^2 = 0.16$
$x^2 - 0.8x + 0.16 = 0.4 + 0.16$
$(x - 0.4)^2 = 0.56$
Step 4 Take the square root on both sides of the equation:
$x - 0.4 = \pm\sqrt{0.56} = \pm 0.748$ (to 3 decimals)
Step 5 Subtract (-0.4) from both sides (in other words, add 0.4):
$x = \pm 0.748 + 0.4 = -0.348$ or 1.148
Why "Complete the Square"?
Why complete the square when we can just use the Quadratic Formula to solve a Quadratic Equation?
Well, one reason is given above, where the new form not only shows us the vertex, but makes it easier to solve.
There are also times when the form **ax^2 + bx + c** may be part of a **larger** question and rearranging it as **a(x+*d*)^2 + *e*** makes the solution easier, because **x** only appears once.
For example "x" may itself be a function (like *cos(z)*) and rearranging it may open up a path to a better solution.
Also Completing the Square is the first step in the Derivation of the Quadratic Formula
Just think of it as another tool in your mathematics toolbox.

Question 1 Question 2 Question 3 Question 4 Question 5Question 6 Ques tion 7 Question 8 Question 9 Question 10

Footnote: Values of “d” and “e”
How did I get the values of **d** and **e** from the top of the page?

Start with	$ax^2 + bx + c = 0$
Divide the equation by **a**	$x^2 + \frac{b}{a}x + \frac{c}{a} = 0$
Put **c/a** on other side	$x^2 + \frac{b}{a}x = -\frac{c}{a}$
Add **(b/2a)²** to both sides	$x^2 + \frac{b}{a}x + \left(\frac{b}{2a}\right)^2 = -\frac{c}{a} + \left(\frac{b}{2a}\right)^2$
“Complete the Square”	$\left(x + \frac{b}{2a}\right)^2 = -\frac{c}{a} + \left(\frac{b}{2a}\right)^2$
Now bring everything back...	
... to the left side	$\left(x + \frac{b}{2a}\right)^2 + \frac{c}{a} - \left(\frac{b}{2a}\right)^2 = 0$
... to the original multiple **a** of x^2	$a\left(x + \frac{b}{2a}\right)^2 + c - \frac{b^2}{4a} = 0$
And you will notice that we have got:	$a(x+d)^2 + e = 0$
Where:	d = *b***2a**
and:	e = c − b^2**4a**

Just like at the top of the page!

ASSESSMENT

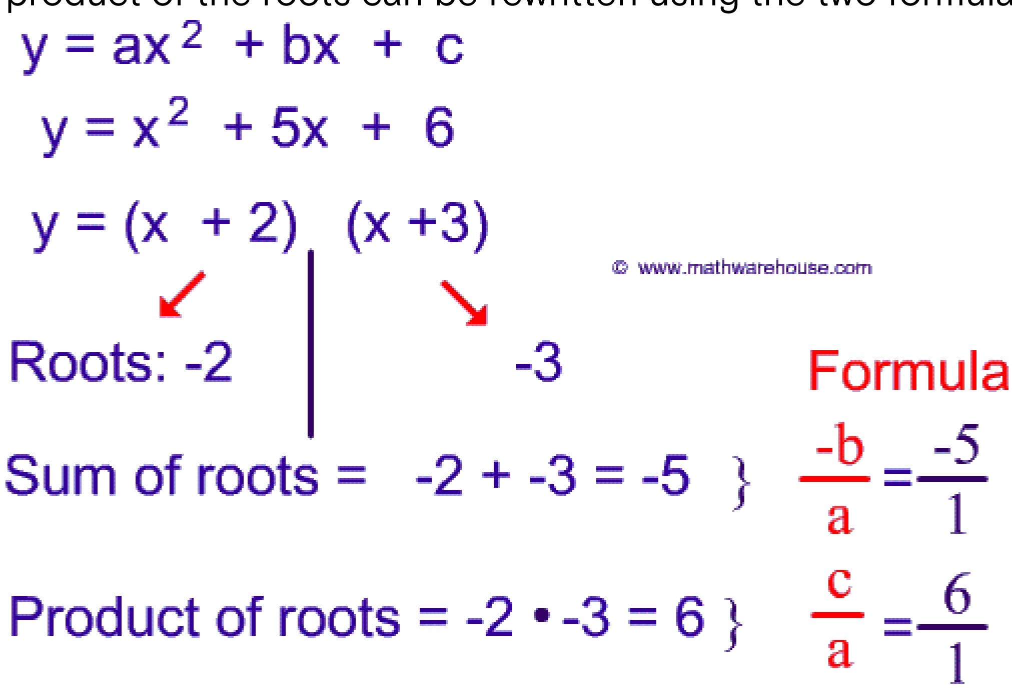
Solve the following quadratic equations;

1. $x^2 + 7x + 11 = -1$
2. $x^2 - 5x + 6 = 0$
3. $x^2 + 4x + 4 = -1$

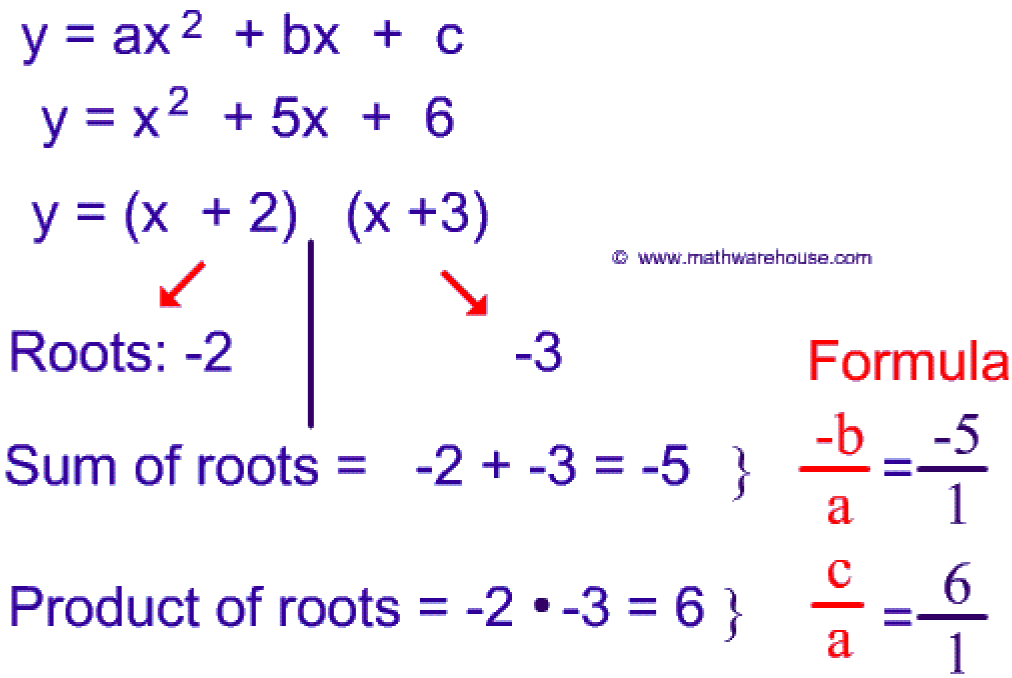
WEEK 6

How to Construct Quadratic Equation from Sum and Product Roots

The formulas
sum of roots: −ba−ba
product of roots: caca
As you can see from the work below, when you are trying to solve a quadratic equations in the form of ax2+bx+cax2+bx+c. The sum and product of the roots can be rewritten using the two formulas above.

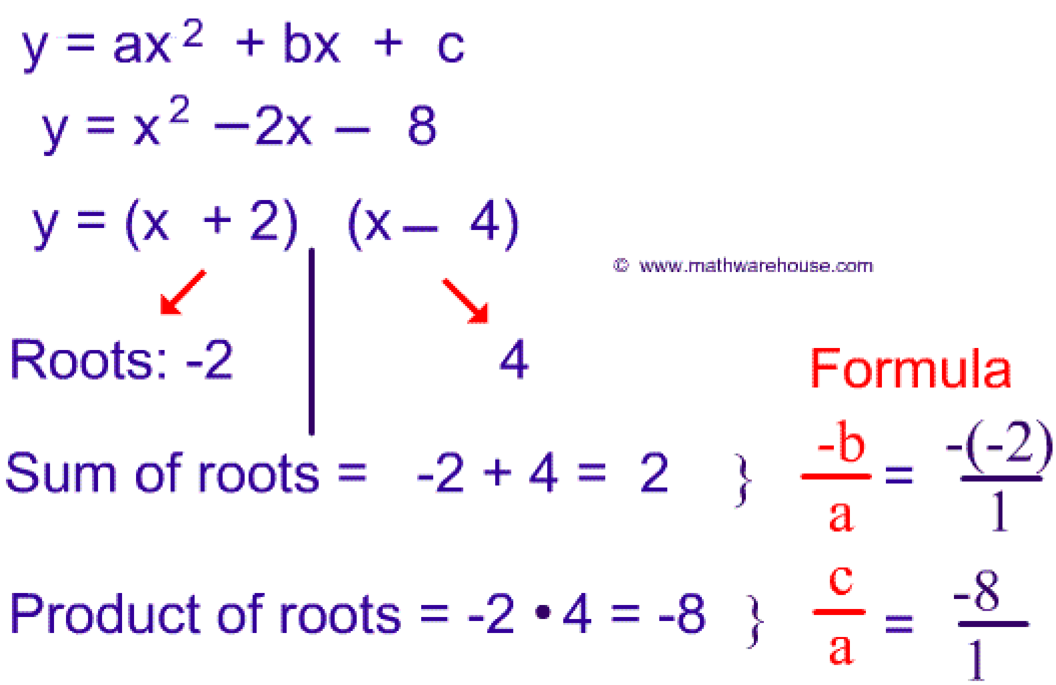


Example 1
The example below illustrates how this formula applies to the quadratic equation x2+5x+6x2+5x+6. As you, can see the sum of the roots is indeed −ba−ba and the product of the roots is caca.



Example 2

The example below illustrates how this formula applies to the quadratic equation $x^2 - 2x - 8$. Again, both formulas – for the sum and the product boil down to -b/a and c/a, respectively.



Practice Problems

Without solving, find the sum and product of the roots of the equation: $2x^2 - 3x - 2 = 0$

Word Problem Involving Quadratic Equations

Write a “let” statement and equation for each example. Explain the difference.

1. Find two consecutive integers whose sum is 13

Let x = first integer

x + 1 = second integer

$x + x + 1 = 13$

Find two consecutive two consecutive integers whose products is 42

Let x = first integer

X + 1 = second integer

$x(x + 1) = 42$

Write “let” statement (if consecutive integers, use x; x + 1; x + 2 if consecutive even or odd, use x; x+2;x+4)

Write equation using keywords from statement: sum (add), product (multiply, more than (add), less than (subtract), etc.

Solve equation:

Check in original problem statement

1. Let x = first integer, x+2= second integer

$x(x + 2) = 143$

$x^2 + 2x = 143$

$x^2 + 2x - 143 = 0$

$(x + 13)(x - 11) = 0$

Integers: -13 and -11 i.e. 11 and 13

Find two numbers whose sum is 9 and whose product is 20.

Solution

Let one of the numbers be x

The other number is 9-x

Their product is 20 : x(9-x) = 20

$x(9 - x) = 20$

i.e. $9x - x^2 - 20 = 0$

Þ $x^2 + 9x + 20 = 0$ (the resulting quadratic equation)

Factorizing: $(x - 4)(x - 5) = 0$

\x = 4 or 5 √

\ The numbers are 4 and 5

If check sum of numbers: 4 + 5 = 9

The product of numbers:4 x 5 = 20

Example

The difference of two numbers is 2 and their product is 224. Find the numbers.

Let x and y be the numbers. Their difference is 2, so I can write

$x - y = 2$

Their product is 224, so

$xy = 224$

From $x - y = 2$, I get $x = y + 2$. Plug this into $xy = 224$ and solve for y:

$(y + 2)y = 224$

$y^2 + 2y = 224$

$y^2 + 2y - 224 = 0$

$(y + 16)(y - 14) = 224$

If y = -16, then x = − 16 + 2 = − 14.

If y = 14, then x = 14 + 2 = 16.

So two pairs work: -14 and -16, and 14 and 16

Problem

Motorboat moving upstream and downstream on a river. A motorboat makes a round trip on a river 56 miles upstream and 56 miles downstream, maintaining the constant speed 15 miles per hour relative to the water.

The entire trip up and back takes 7.5 hours.
What is the speed of the current?

Solution

Denote the unknown current speed of the river as x miles/hour.
When motorboat moves upstream, its speed relative to the bank of the river is 15 – x miles/hour, and the time spent moving upstream is 56/15 – x hours.
When motorboat moves downstream, its speed relative to the bank of the river is 15 + x miles/hour, and the time spent moving downstream is 56/15 + x hours.
So, the total time up and back is 56/15 – x + 56/15 + x, and it is equal to 7.5 hours, according to the problem input.
This gives an equation 56/15 – x + 56/15 + x = 7.5.
To simplify the equation, multiply both sides by (15 – x)(15 + x) and collect common terms. Step by step, you get 56(15 + x) + 56(15 – x) = 7.5(15 – x)(15 + x),
,1680 = $7.5(15^2 - x^2)$
$1680/7.5 = 225 - x^2$
$224 = 225 - x^2$
$x^2 = 1$.

Problem

Andrew and Bill, working together, can cover the roof of a house in 6 days. Andrew, working alone, can complete this job in 5 days less than Bill. How long will it take Bill to make this job?

Solution

Denote x number of days for Bill to cover the roof, working himself, if Andrew works alone, he can complete this job in x – 5 days.
Thus, in one single day Andrew covers 1/x -5 part of the roof area, while Bill covers 1/x part of the roof area.
Working together, Andrew and Bill make 1/x – 5 + 1/x of the whole work in each single day.

Since they can cover the entire roof in 6 days working together, the equation for the unknown value x is as follows: $6/x - 5 + 6/x = 1$.
To simplify this equation, multiply both sides by $(x - 5)x$, then transfer all terms from the right side to the left with the opposite signs, then collect common terms and adjust the signs. In this way you get $6x + 6(x - 5) = x(x-5)$,
$6x - 6x - 30 = x^2 - 5x$,
$-x^2 + 6x + 6x + 5x - 30 = 0$,
$- x^2 - 17x - 30 = 0$,
You get the quadratic equation. Apply the quadratic formula to solve this equation. You get
$x = 17 \pm Ö17^2 - 4.30/2 = 17 \pm Ö289 - 120/2 = 17 \pm Ö169/2$

Note: square root represent Ö
The equation has two roots: $x_1 = 17 + 13/2$ and $x_2 = 17 - 13/2 = 2$ and. The second root $x_2 = 2$ does not fit the given conditions (if Bill covers the roof in two days, then Andrew has 2 – 5= -3 days, what has no sense).
So, the potentially correct solution is $x_1 = 15$: Bill covers the roof in 15 days.
Let us check it. If Bill gets the job done in 15 days, then Andrew makes it in 10 days, working separately.
Since $6/10 + 6/15 = 1$, this solution is correct.
Answer: Bill covers the roof in 15 days

EXERCISES

Lets see how much you've learnt, attach the following answers to the comment below

1. One leg of a right triangle exceeds the other leg by four inches. The hypotenuse is 20 inches. Find the length of the shorter leg of the right triangle. Hint: Pythagorean Theorem. A. 12 B. 14 16 D. 182.
2. The product of two consecutive integers is 56. Find the integers. Hint: Pythagorean Theorem A. -8, 7 -8, -7 C. 8, -7 D. -8, -8

3. The area of a rectangle is $80cm^2$. If the length is 2cm more than the width, find the width. A. 9cm B 10cm C. 6cm D. 8cm
4. Solve this quadratic equation $x^2 - 7x + 12.25 = 0$, using Quadratic formula.
5. Solve these equations by factoring x: $y = x^2 - 5x + 7$ A. 1 and 7 B. 2 and 6 C. 1 and 6 D. 2 and 7

WEEK 7

Graphical Solution of Linear and Quadratic Equation

The basic idea behind solving by graphing is that since the "solutions" to "$ax^2 + bx + c = 0$" are the x-intercepts of "$y = ax^2 + bx + c$", you can look at the x-intercepts of the graph to find the solutions to the equation. There are difficulties with "solving" this way, though....

When you graph a straight line like "$y = 2x + 3$", you can find the x-intercept (to a certain degree of accuracy) by drawing a really neat axis system, plotting a couple points, grabbing your ruler and drawing a nice straight line, and reading the (approximate) answer from the graph with a fair degree of confidence.

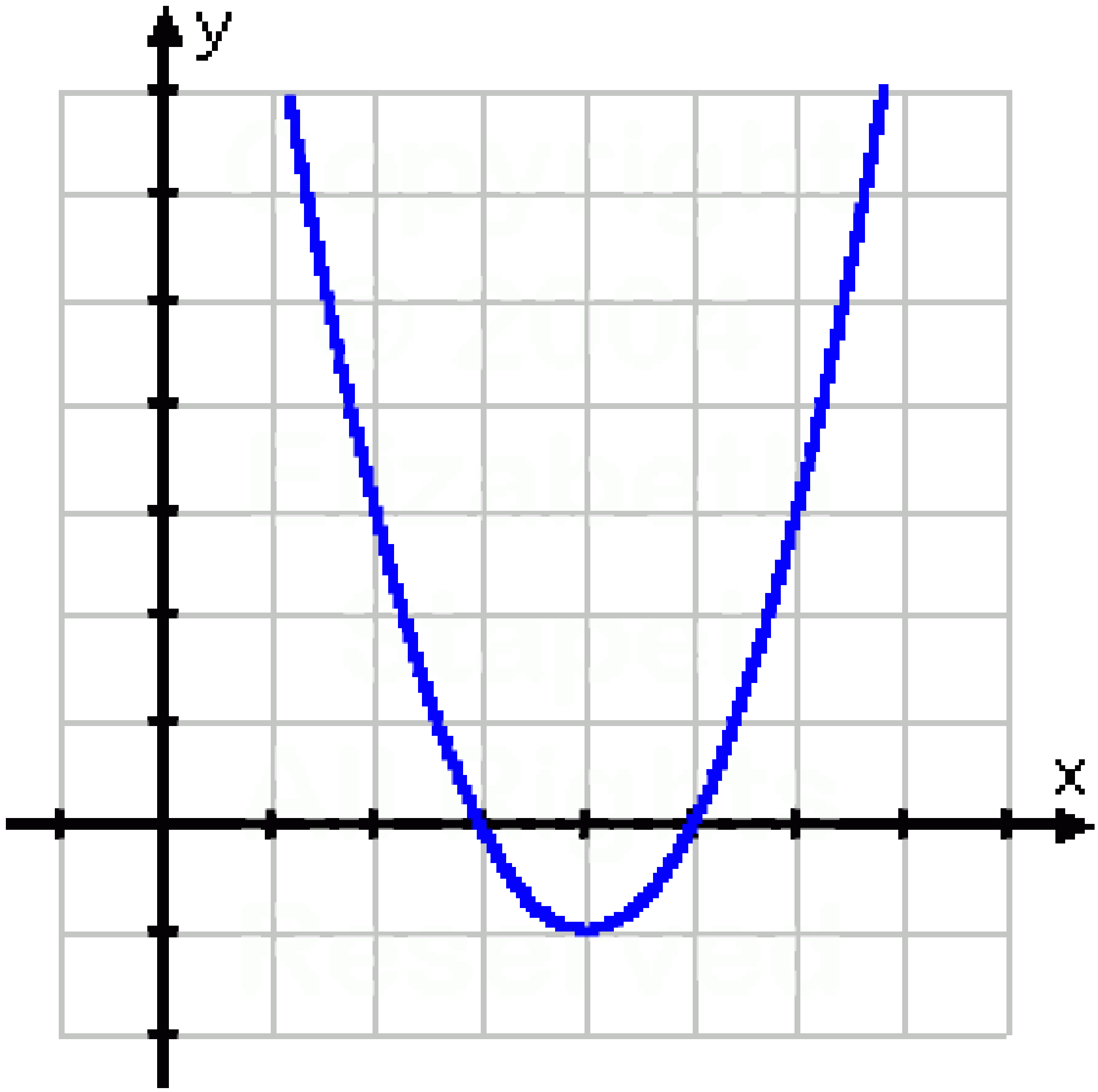
On the other hand, a quadratic graphs as a wiggly parabola. If you plot a few non-x-intercept points and then draw a curvy line through them, how do you know if you got the x-intercepts even close to being correct? You don't. The only way you can be sure of your x-intercepts is to set the quadratic equal to zero and solve. But the whole point of this topic is that they don't want you to do the (exact) algebraic solving; they want you to guess from the pretty pictures.

So "solving by graphing" tends to be neither "solving" nor "graphing". That is, you don't actually graph anything, and you don't actually do any of the "solving". Instead, you are told to punch some buttons on your graphing calculator and look at the pretty picture, and then you're told which other buttons to hit so the software can compute the intercepts (or you're told to guess from the pretty picture in the book, hoping that the printer lined up the different print runs for the different ink colors exactly right). I think the educators are trying to "help" you "discover" the connection between x-intercepts and solutions, but the concept tends to get lost in all the button-pushing. Okay, enough of my ranting...

Solve $x^2 – 8x + 15 = 0$ by using the following graph.

To "solve" by graphing, the book may give you a very neat graph, probably with at least a few points labelled; the book will ask you to state the points on the graph that represent solutions. Otherwise, it will give you a quadratic, and you will be using your graphing calculator to find the answer. Since different calculator models have different key-sequences, I cannot give instruction on how to "use technology" to find the answers, so I will only give a couple examples of how to solve from a picture that is given to you.

- **Solve $x^2 - 8x + 15 = 0$ by using the following graph.**

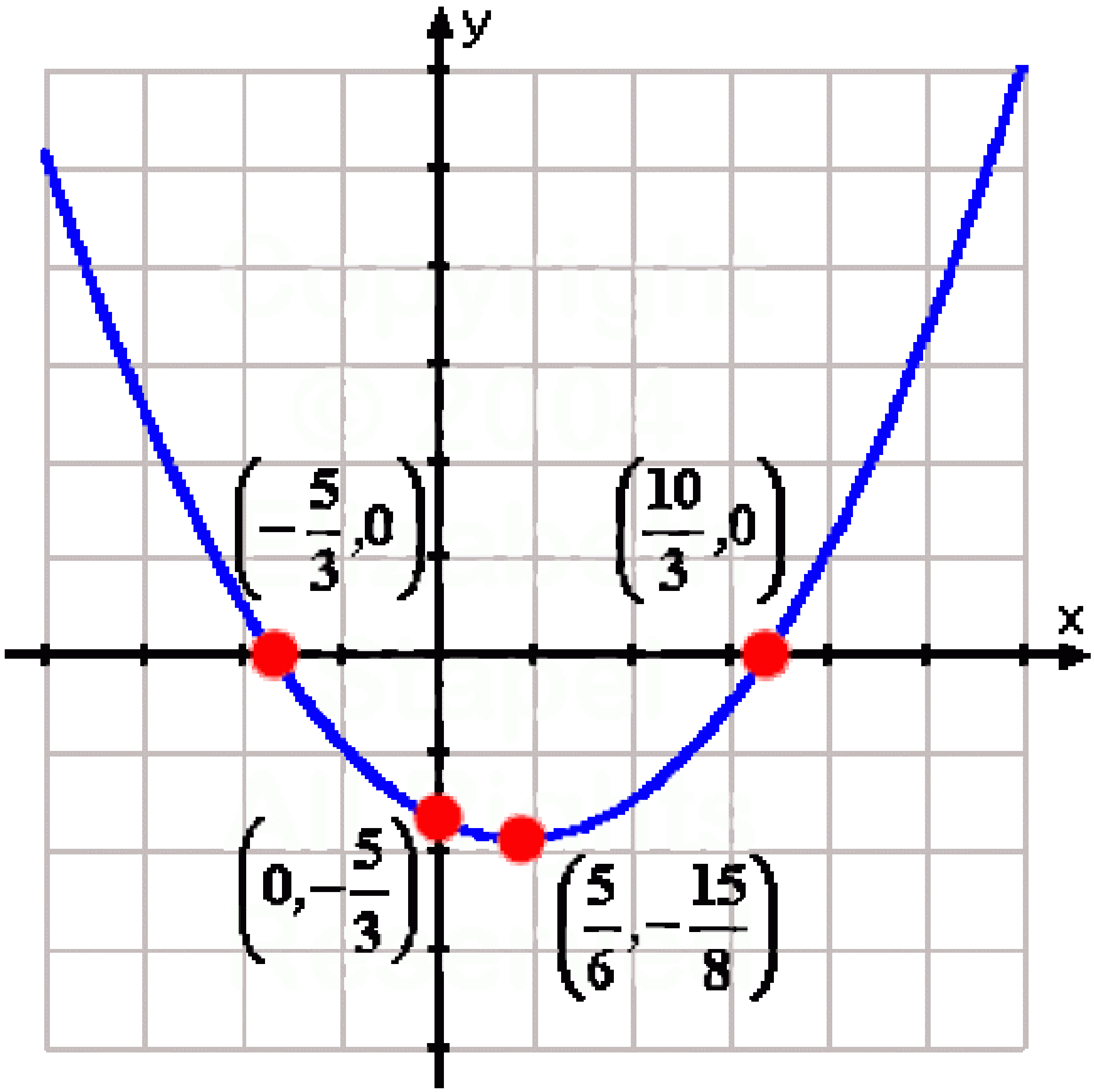


The graph is of the related quadratic, $y = x^2 - 8x + 15$, with the x-intercepts being where $y = 0$. The point here is to look at the picture (hoping that the points really do cross at whole numbers, as it appears), and read the x-intercepts (and hence the solutions) from the picture.

The solution is $x = 3, 5$

Since $x^2 - 8x + 15$ factors as $(x - 3)(x - 5)$, we know that our answer is correct.

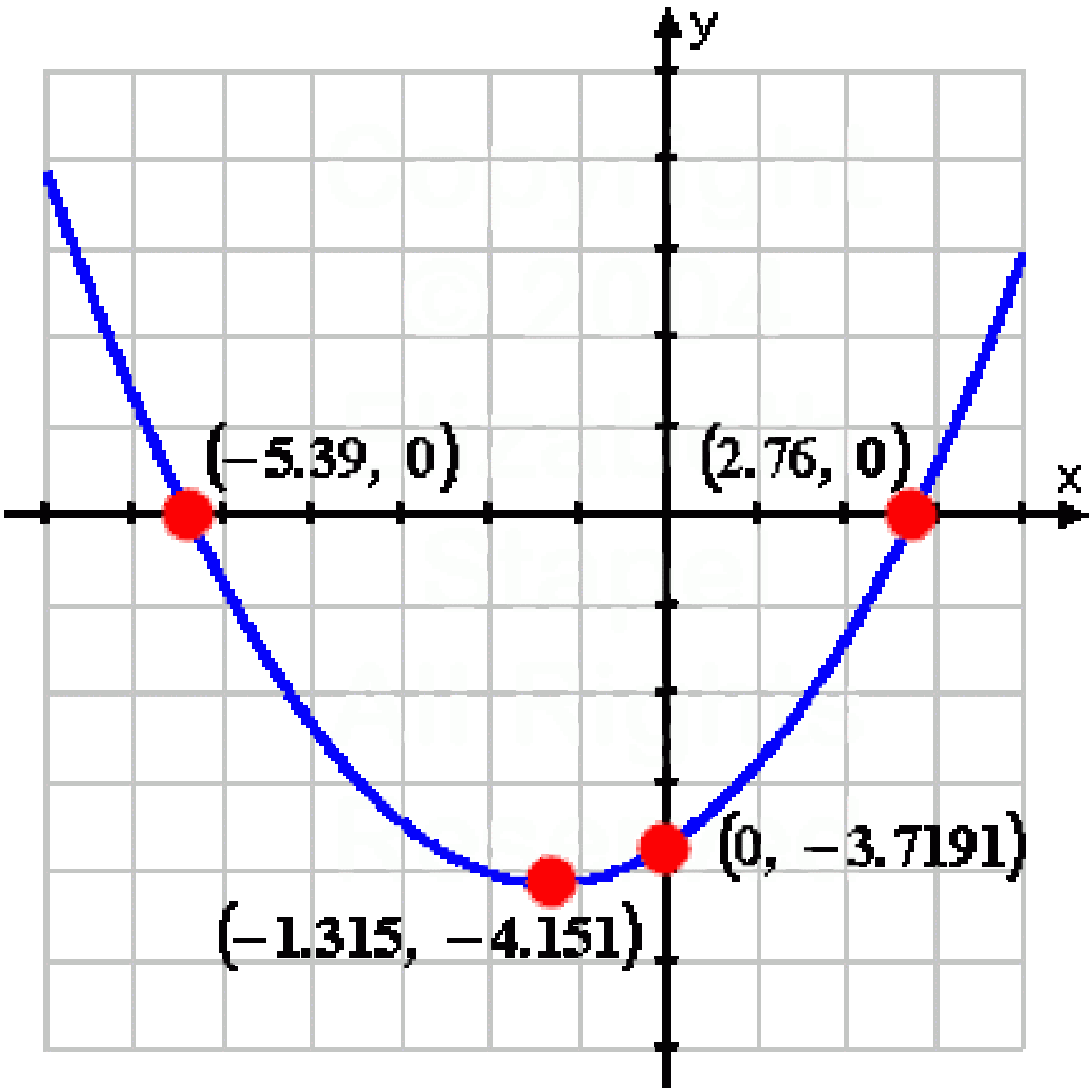
-
  - **Solve $0.3x^2 - 0.5x - {}^5/_3 = 0$ by using the following graph.**



For this picture, they labelled a bunch of points. Partly, this was to be helpful, because the *x*-intercepts are messy (so I could not have guessed their values without the labels), but mostly this was in hopes of confusing me, in case I had forgotten that only the *x*-intercepts, not the vertices or *y*-intercepts, correspond to “solutions”.

The *x*-values of the two points where the graph crosses the *x*-axis are the solutions to the equation.
The solution is $x = {}^{-5}/_3, {}^{10}/_3$

-
  - **Find the solutions to the following quadratic:**



They haven't given me the quadratic equation, so I can't check my work algebraically. (And, technically, they haven't even given me a quadratic to solve; they have only given me the picture of a parabola from which I am supposed to approximate the *x*-intercepts, which really is a different question....)

I ignore the vertex and the *y*-intercept, and pay attention only to the *x*-intercepts. The "solutions" are the *x*-values of the points where the pictured line crosses the *x*-axis:

The solution is $x = -5.39, 2.76$

"Solving" quadratics by graphing is silly in "real life", and requires that the solutions be the simple factoring-type solutions such as "$x = 3$", rather than something like "$x = -4 + \text{sqrt}(7)$". In other words, they either have to "give" you the answers (by labelling the graph), or they have to ask you for solutions that you could have found easily by factoring. About the only thing you can gain from this topic is reinforcing your understanding of the connection between solutions and *x*-intercepts: the solutions to "(some polynomial) equals (zero)" correspond to the *x*-intercepts of "*y* equals (that same polynomial)". If you come away with an understanding of that concept, then you will know when best to use your graphing calculator or other graphing software to help you solve general polynomials; namely, when they aren't factorable.

Graphical Solution of Linear and Simultaneous Equation

Recall in the coordinate geometry that the graph of a linear equation or linear function is always a straight line. To solve simultaneous linear equations graphically, we are required to draw the graphs of the two equations on the same graph sheet. Then the point (x, y) of intersection of the straight line gives the solution of the equations.It is important to note that when plotting the graph of a linear equation, two points are sufficient to determine the line: However it is advisable to plot more than two points, as a check.

Also known that if the two lines drawn do not intersect one another, the simultaneous equations have no solution.

Example 3

Draw the graph for $y = 3x^2$

x	-2	-1	0	1	2
x^2	$(-2)^2$	$(-1)^2$	0	$(1)^2$	$(2)^2$
$3x^2$	3 4	31	0	31	34
y	12	3	0	3	12

Plot the points (-2,12), (-1,3), (0,0), (1,3), (2,12) and join them to form a smooth curve.
This curve is also a parabola, which is symmetric about the y-axis and lies above the x-axis.

Example 4

Draw the following curves and make a comparison.

1. $f(y) = y^2$
2. $f(y) = 2y^2$
3. $f(y) = -1 / 3\ y^2$
4. $f(x) = -y^2$

1. **Let $x = f(y) = y^2$**

We can find the points to be plotted as follows.

y	-3	-2	-1	0	1	2	3
x^2	$(-3)^2$	$(-2)^2$	$(-1)^2$	0	12	22	32
x	9	4	1	0	1	4	9

However, when we plot the points, we take them in the form (x,y).Plot the points (9,-3), (4,-2), (1,-1), (0,0), (1,1), (4,2), (9,3) and join them to form a smooth curve.

1. **Let $x = f(y) = 2y^2$**

y	-3	-2	-1	0	1	2	3
y^2	$(-3)^2$	$(-2)^2$	$(-1)^2$	0	12	22	32
$2y^2$	2*9	2*4	2*1	0	2*1	2*4	2*9
x	18	8	2	0	2	8	18

Plot the points (18,-3), (8,-2), (2,-1), (0,0), (2,1), (8,2), (18,3) and join them to form a smooth curve.

1. **Let x = -1/3 y^2**

y	-6	-3	0	3	6
y^2	$(-6)^2$	$(-3)^2$	0	32	62
$-1/3y^2$	-1/3 * 36	-1/3 * 9	0	-1/3 * 9	-1/3 * 36
x	-12	-3	0	-3	-12

Plot the points (-12,-6), (-3,-3), (0,0), (-3,3), (-12,6) and join them to form a smooth curve.

1. **Let $x=-y^2$**

y	-3	-2	-1	0	1	2	3
$-y^2$	$-(-3)^2$	$-(-2)^2$	$-(-1)^2$	0	$-(-1)^2$	$-(-2)^2$	$-(-3)^2$
x	-9	-4	-1	0	-1	-4	-9

Plot the points (-9,-3), (-4,-2), (-1,-1), (0,0), (-1,1), (-4,2), (-9,3) and join them to form a smooth curve.From the graphs, we find that the curves $x = y^2$, $x = 2y^2$are parabolas symmetric about the x-axis and lie toward the right of the y-axis. They also lie in the first and fourth quadrants.
The curves $x = -1/3\ y^2$, $x = -y^2$are parabolas symmetric about the x-axis but lie to the left of the y-axis. They lie in the second and third quadrants.

Example 5

Draw the graph for $f(x)=x^2-x-12$Let $y=x^2-x-12$
To plot the points, we find the value of y as follows.$f(-3)=(-3)^2-(-3)-12$
=9+3-12
=0f(2) $=(2)^2-(2)-12$
=4-2-12
=4-14
=-10In general, we use the following format.

y	-4	-3	-2	-1	0	1	2	3	4	5

x^2	16	9	4	1	0	1	4	9	16	25
-x	+4	3	2	+1	0	-1	-2	-3	-4	-5
-12	-12	-12	-12	-12	-12	-12	-12	-12	-12	-12
y	8	0	-6	-10	-12	-12	-10	-6	0	8

We plot the points (-4,8), (-3,0), (-2,6), (-1,-10), (0,-12), (1,-12), (2,-10), (3,6), (4,0), (5,8), and join them to form a smooth curve.Observe that when y=0, x= -3 or 4 or y = 0 $\Rightarrow x^2-x-12=0$

∴ the roots or solutions of $x^2-x-12=0$ are x=-3, 4

Further, the curve cuts the x-axis at the points (-3,0) and (4,0). So the roots of the equation are the x-coordinates of the points, where the curve cuts the x-axis.

By looking at the graph, we can solve $x^2-x-12=0$ by taking the points where the curve cuts the x-axis, i.e., (-3.0), (4,0).

We will now consider the graphs for equations of the form $y=ax^2+bx+c$ and see what happens when we change the coefficients of the x^2 and x terms, and the constant term.

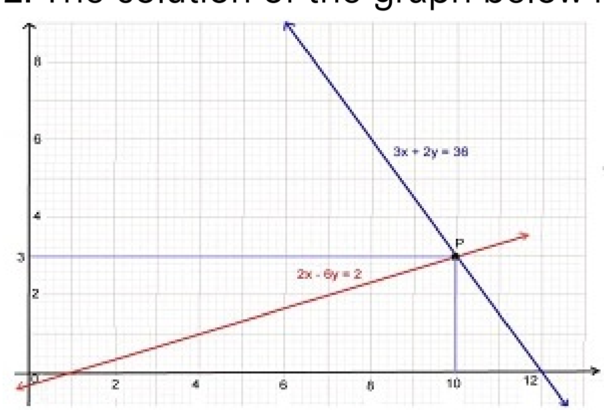
First, we consider the quadratic equations of the form

$y = ax^2+b+c$ where the coefficient of x^2, which is a, changes.

Questions1. When plotted graphs are parallel to each other, the equation is said to have

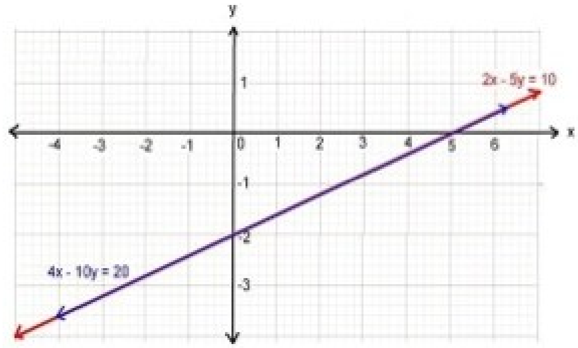
A. Single number of solution B. Infinite number of solution C. Double number of solution D. No solution

2. The solution of the graph below is



A. 9, 12 B. 1, 10 C. 10, 3 D. 3, 12

3. The graphs below is said to have



A. No solution B. Infinite number of solution C. Single solution D. Double solution

4.What is the solution of a system of equations?

A. Where the equations meet or intersect B. Where the equations end C. Where the equations start D. The middle point of the equations.

5. Supply the missing word. An equation of the form $ax^2+bx+c=0, a\neq 0$, is called a.................... equation

A. Linear equation B. Quadratic equation C. Simultaneous equation D. Guassian equation

Answer

1. D. 2. C 3. B 4. A 5. B

WEEK 8

HERE ARE SOME EXAMPLES of problems that lead to simultaneous equations.

Example 1. Andre has more money than Bob. If Andre gave Bob \$20, they would have the same amount. While if Bob gave Andre \$22, Andre would then have twice as much as Bob. How much does each one actually have?

Solution. Let x be the *amount of money that Andre has.* Let y be the *amount that Bob has*.

Always let x and y answer the question — and be perfectly clear about what they represent!

Now there are two unknowns. Therefore there must be two equations. (In general, the number of equations must equal the number of unknowns.) How can we get two equations out of the given information? We must translate each verbal sentence into the language of algebra.

Here is the first sentence:

"If Andre gave Bob \$20, they would have the same amount."

Algebraically:

1) $x - 20 = y + 20$.

(Andre — x — has the same amount as Bob, after he gives him \$20.)

Here is the second sentence:

"While if Bob gave Andre \$22, Andre would then have twice as much as Bob."

Algebraically:

2) $x + 22 = 2(y - 22)$.

(Andre has twice as much as Bob — after Bob gives him \$22.)

To solve any system of two equations, we must reduce it to one equation in one of the unknowns. In this example, we can solveequation 1) for x—

$x - 20 \quad = y + 20$

implies $x = y + 40$

— and substitute it into equation 2):

$y + 40 + 22 = 2(y - 22).$

That is,

$y + 62 = 2y - 44,$

$y - 2y = -44 - 62,$

according to the techniques of Lesson 9,

$-y = -106$

$y = 106.$

Bob has $106. Therefore, according to the exression for *x*, Andre has 106 + 40 = $146.

Example 2. 1000 tickets were sold. Adult tickets cost $8.50, children's cost $4.50, and a total of $7300 was collected. How many tickets of each kind were sold?

Solution. Let *x* be the *number of adult tickets*. Let *y* be the *number of children's tickets*.

Again, we have let *x* and *y* answer the question. And again we must get two equations out of the given information. Here they are:

1) Total number of tickets: $x + y = 1000$

2) Total money collected: $8.5x + 4.5y = 7300$

In equation 2), we will make the coefficients into whole numbers by multiplying both sides of the equation by 10:

1) $x + y = 1000$

2′) $85x + 45y = 73{,}000$

We call the second equation 2′ ("2 prime") to show that we obtained it from equation 2).

These simultaneous equations are solved in the usual way.

The solutions are: $x = 700$, $y = 300$.

To see the answer, pass your mouse over the colored area.

To cover the answer again, click "Refresh" ("Reload").

Do the problem yourself first!

Example 3. Mrs. B. invested \$30,000; part at 5%, and part at 8%. The total interest on the investment was \$2,100. How much did she invest at each rate?

Solution.

1) Total investment: $x + y = 30{,}000$

2) Total interest $.05x + .08y = 2{,}100$

(To change a percent to a decimal, see Skill in Arithmetic, Lesson 4.)

Again, in equation 2) let us make the coefficients whole numbers by multiplying both sides of the equation by 100:

1) $x + y = 30{,}000$

2′) $5x + 8y = 210{,}000$

These are the simultaneous equations to solve.

The solutions are: x = \$10,000, y = \$20,000.

Problem. Samantha has 30 coins, consisting of quarters and dimes, which total \$5.70. How many of each does she have?

To see the answer, pass your mouse from left to right
over the colored area.

To cover the answer again, click “Refresh” (“Reload”).

Do the problem yourself first!

Let x be the number of quarters. Let y be the number of dimes.

The equations are:

1) Total number of coins: $x + y = 30$

2) Total value: $.25x + .10y = 5.70$

To eliminate y:

Multiply equation 1) by −10 and equation 2) by 100:

1′) $-10x - 10y = -300$

2′) $25x + 10y = 570$

Add:

$$15x = 270$$

$$x = \frac{270}{15}$$

$$= \frac{300 - 30}{15}$$

$$= 20 - 2 \quad \text{(Lesson 11 of Arithmetic)}$$

$$x = 18.$$

Therefore, $y = 30 - 18 = 12$.

Example 4. Mixture problem 1. First:
"36 gallons of a 25% alcohol solution"
means: 25%, or one quarter, of the solution is pure alcohol.
One quarter of 36 is 9. That solution contains 9 gallons of pure alcohol.
Here is the problem:
How many gallons of 30% alcohol solution and how many of 60% alcohol solution must be mixed to produce 18 gallons of 50% solution?
"18 gallons of 50% solution" means: 50%, or half, is pure alcohol. The final solution, then, will have 9 gallons of pure alcohol.
Let x be the *number of gallons* of 30% solution. Let y be the *number of gallons* of 60% solution.

1) Total number of gallons $x + y = 18$

2) Gallons of pure alcohol $.3x + .6y = 9$

2′) $3x + 6y = 90$

Equations 1) and 2′) are the two equations in the two unknowns.
The solutions are: $x = 6$ gallons, $y = 12$ gallons.

Example 5. Mixture problem 2. A saline solution is 20% salt. How much water must you add to how much saline solution, in order to dilute it to 8 gallons of 15% solution?
(This is more an arithmetic problem than an algebra problem.)

Solution. Let s be the number of gallons of saline solution. Now all the salt will come from those s gallons. So the question is, What is s so that 20% of s — the salt — will be 15% of 8 gallons?
$.2s = .15 \times 8 = 1.2$
That is,

$2s = 12.$
$s = 6.$
Therefore, to 6 gallons of saline solution you must add 2 gallons of water.

Example 6. Upstream/Downstream problem. It takes 3 hours for a boat to travel 27 miles upstream. The same boat can travel 30 miles downstream in 2 hours. Find the speeds of the boat and the current.

Solution. Let x be the *speed* of the boat (without a current). Let y be the speed of the current.
The student might review the meanings of “upstream” and “downstream,” Lesson 25. We saw there that speed, or velocity, is *distance* divided by *time*:

$$v = \frac{d}{t}$$

Therefore, according to the problem:

$$\text{Upstream speed} = \frac{\text{Upstream distance}}{\text{Upstream time}} = \frac{27}{3} = 9$$

$$\text{Downstream speed} = \frac{\text{Downstream distance}}{\text{Downstream time}} = \frac{30}{2} = 15$$

Here are the equations:
1) Upstream speed: $x - y = 9$
2) Downstream speed: $x + y = 15$
Enjoy!
(The solutions are: $x = 12$ mph, $y = 3$ mph.)

ASSESSMENT

1. The sum of two number is 14 and their difference is 2. Find the numbers.
   (a) 4 and 8
   (b) 2 and 5

(c) 3 and 6
(d) 6 and 8

2. The sum of the digits of a two digit number is 7. When the digits are reversed, then number is decreased by 9. Find the number
   (a) 32
   (b) 34
   (c) 33
   (d) 35
3. The sum of two numbers is 12. When three times the first number is added to 5 times the second number, the resultant number is 44. Find the two numbers.
   (a) 8 and 4
   (b) 6 and 3
   (c) 2 and 4
   (d) 8 and 6
4. James bought 5 apples and 10 oranges for $4. Donald bought 3 apples and 9 oranges for $3. The shop keeper strictly told that there will not be any discounts. What is the cost of an apple and an orange?
   (a) Cost of an apple = $0.40, Cost of an orange = $0.20
   (b) Cost of an apple = $0.50, Cost of an orange = $0.30
   (c) Cost of an apple = $0.60, Cost of an orange = $0.40
   (d) Cost of an apple = $0.70, Cost of an orange = $0.50
5. In a two digit number. The units digit is thrice the tens digit. If 36 is added to the number, the digits interchange their place. Find the number.
   (a) 24
   (b) 25
   (c) 26
   (d) 27

ANSWERS

1. d
2. b
3. a
4. a
5. c

SS 2
SECOND TERM NOTES ON MATHEMATICS

TABLE OF CONTENT

SECOND TERM

WEEK 1

Introduction to Straight-Line Graphs

Linear Graphs

A **graph** is a picture that represents numerical data. Most of the graphs that you have been taught are **straight-line** or **linear graphs**. This topic shows how to use linear graphs to represent various real-life situations.

If the rule for a relation between two variables is given, then the graph of the relation can be drawn by constructing a table of values.

To plot a **straight line graph** we need to find the coordinates of *at least two points* that fit the rule.

Example

Plot the graph of $y = 3x + 2$.

Solution

Construct a table and choose simple x values.

X	-2	-1	O	1	2
Y					

In order to find the y values for the table, substitute each x value into the rule y = 3x + 2

When x = −2, y = 3(−2) + 2

= −6 + 2 = = −4

When x = −1, y = 3(−1) + 2

= −3 + 2 = 1

When x = O, y = 3 X O + 2

= O + 2 = 2

When x = 1, y = 3 X 1 + 2

= 3 + 2 = 5

When x = 2, y = 3 X 2 + 2

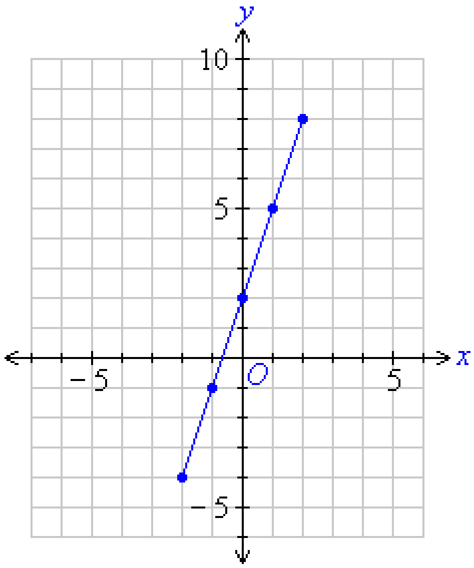
= 6 + 2 = 8

The table of values obtained after entering the values of *y* is as follows:

X	-2	-1	O	1	2

Y	-4	-1	2	5	8

Draw a Cartesian plane and plot the points. Then join the points with a ruler to obtain a straight line graph.



Setting out:

Often, we set out the solution as follows.

Y = 3x + 2

When x = -2, y = 3(-2) + 2

= -6 + 2 = -4

When x = -1, y = 3(-1) + 2

= -3 + 2 = = -1

When x = 0, y = 3 X 0 + 2

= 0 + 2 = 2

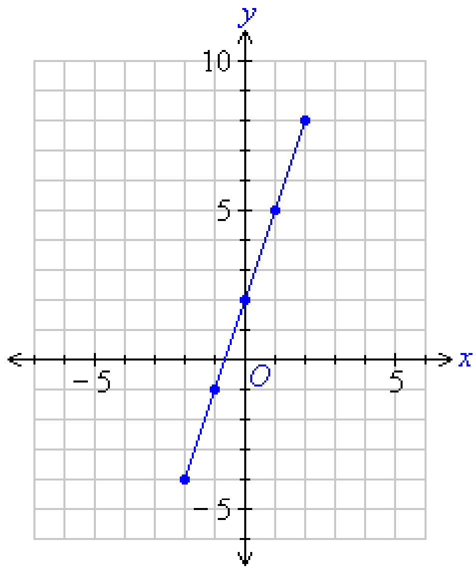
When x = 1, y = 3 X 1 + 2

= 3 + 2 = 5

When x = 2, y = 3 X 2 + 2

= 6 + 2 = 8

X	-2	-1	0	1	2
Y	-4	-1	2	5	8



Example

Plot the graph of $y = -2x + 4$.

Solution

Y = -2x + 4

When x = -2, y = -2(-2) + 4

= 4 + 4 = 8

When x = -1, y = -2(-1) + 4

= 2 + 4 = 6

When x = 0, y = -2 X 0 + 4

= 0 + 4 = 4

When x = 1, y = -2(1) + 4

= -2 + 4 = 2

When x = 2, y = -2(2) + 4

= -4 + 4 = 0

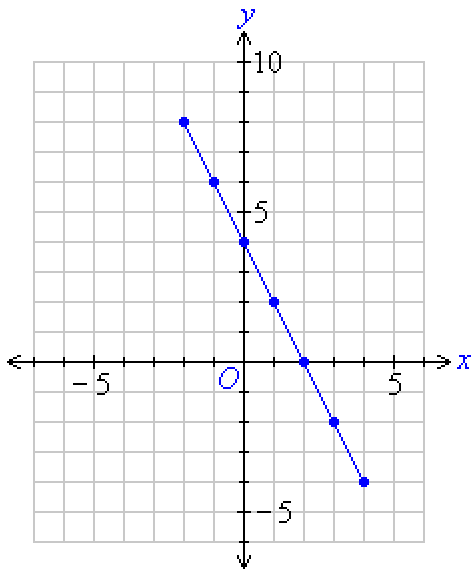
When x = 3, y = -2(3) + 4

= -6 + 4 = -2

When x = 4, y = -2(4) + 4

= -8 + 4 = -4

x	-2	-1	0	1	2	3	4
y	8	6	4	2	0	-2	-4

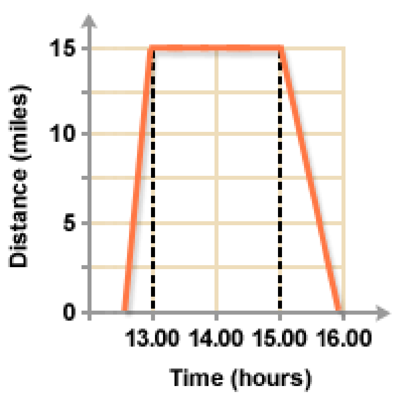


Distance-time graphs

We use distance-and-time graphs to show journeys. It is always very important that you read **all** the information shown on these type of graphs.

A graph showing one vehicle's journey

If we look at the graph shown below, you can see that the time in hours is along the horizontal, and the distance in miles is on the vertical axis. This graph represents a journey that Jan took, in travelling to Glasgow and back, from Aberdeen.



Important points to note are:

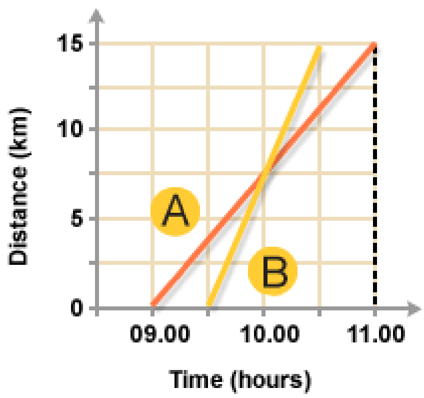
It took half an hour to travel a distance of miles

Between 1pm and 3pm there was no distance travelled. This means that the car had stopped.

The journey back, after 3pm, took one hour.

A graph showing two different journeys in the same direction

The next graph shows two different journeys. You can see that there is a difference with the steepness of the lines drawn. Remember that, the steeper the line, the faster the average speed. We can calculate the average speeds, by reading distances from the graph, and dividing by the time taken.



Line A: How long does journey A last, and what distance is travelled?
The journey takes 2 hours, and the distance travelled is 15km.

Line B : How long does journey B last, and what distance is travelled?
The journey takes 1 hour, and the distance travelled is also 15 km.
This means that the average speeds are:

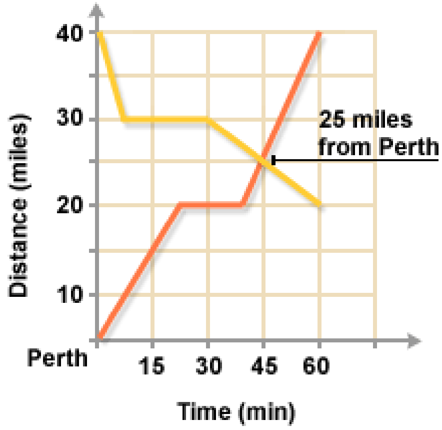
A: 15/2 = 7.5 km per hour

B: 15/1 km per hour

You will also notice from the graph that the two lines cross. This means that, if the two vehicles were travelling along the same route, they would have met at that point, which was just after 10am. The vehicle on journey B overtook vehicle

A graph showing two journeys in the opposite direction

A different pair of journeys is shown below. It is important to note that one journey begins at a distance of , and the other at a distance of miles, from Perth. In fact, what is happening is that one journey is travelling away from, and the other is travelling towards Perth. Again, the two journeys meet. This time it is miles from Perth.



You will also see that the two journeys contain stops.
If we were to calculate the average speeds for each total journey we would have to include this time as well.
A graph showing a journey (or journeys) should have time on the horizontal axis, and distance from somewhere on the vertical axis.
A line moving up, as it goes from left to right, shows a journey moving away from a place, and a line moving down, as it goes from left to right, represents a journey towards a place.
A horizontal line is a break or rest.
Two lines, sloping the same way, cut: then an overtaking has taken place.
Two lines, sloping opposite ways, cut: a meeting has taken place.

Speed-time graphs

A **speed-time graph,** velocity-time graph, shows how the speed of an object varies with time during a journey.
There are two very important things to remember about velocity – time graphs.
The distance travelled is the area under the graph.
The gradient or slope of the graph is equal to the acceleration. If the gradient is negative, then there is a deceleration. We may use the equations(1) or some rearrangement of this equation.

Example. A car starts on a journey. It accelerates for 10 seconds at It then travels at a constant speed for 50 seconds before coming to rest in a further 4 seconds.

a. Sketch a velocity – time graph.

b. Find the total distance travelled.

c. Find the deceleration when the car is coming to a stop at the end.

d. Find the average speed.

a. We may rearrange (1) to obtain v = u + at = 0 + 3 X 10 = 30m/s. Hence we may draw a straight line from (0,0) to (3,30). During the second part the car is travelling at a constant speed of 30m/3. Hence we can draw a straight line to (3,30)+(50,0)=(53,30). During the last part, which takes a further 4 seconds the car comes to a rest, and it's final velocity will be zero. Hence we can draw a straight line to (57,0). We can now draw the velocity time graph.

b. Distance travelled = Area under the graph. The graph is a trapezium so use the formula for the area of a trapezium: ½(a + b) X h = 1/2 (57 + 50) X 30 = 1535m

c. During the final part of the journey the velocity decreases from 30 to 0 in 4 seconds so a = v – u/t = (0 – 30)/4 = -7.5m/s^2 → deceleration = 7.5m/s^2

d. Average speed = Total Distance/Total Time = 1535/57 = 26.93m/s^2.

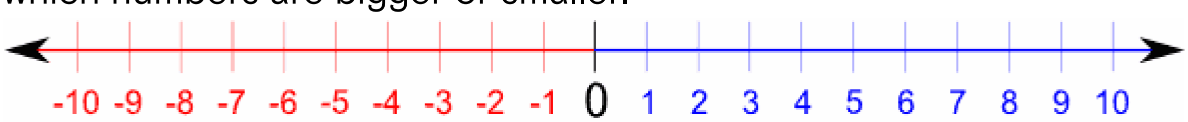
GRAPHS – CARTESIAN PLANE AND COORDINATES

The position of points

A **graph** is a picture of numerical data. We used graphs in statistics in class 1, where they represented number patterns. Here we extend graphs to identifying and drawing the position of points.

Points on a line

Writing numbers down on a Number Line makes it easy to tell which numbers are bigger or smaller.

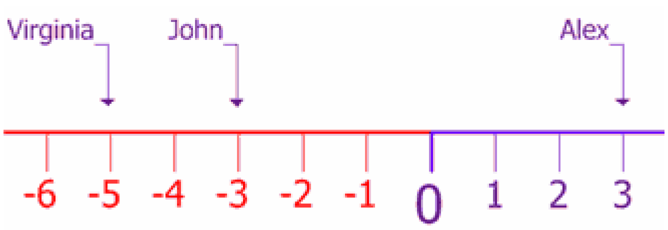


Numbers on the left are stronger than numbers on the right.

Example

5 is smaller than 8
-1 is smaller than 1
-8 is smaller than -5

Example
Example: John owes $3, Virginia owes $5 but Alex doesn't owe anything, in fact he has $3 in his pocket. Place these people on the number line to find who is poorest and who is richest.
Having money in your pocket is positive, owing money is negative.
So John has "−3", Virginia "−5" and Alex "+3"



Now it is easy to see that Virginia is poorer than John (−5 is less than −3) and John is poorer than Alex (−3 is smaller than 3), and Alex is, of course, the richest!

Plotting Points on a Cartesian Plane
A Cartesian plane (named after French mathematician Rene Descartes, who formalized its use in mathematics) is defined by two perpendicular number lines: the ***x*-axis**, which is horizontal, and the ***y*-axis**, which is vertical. Using these axes, we can describe any point in the plane using an ordered pair of numbers.
The Cartesian plane extends infinitely in all directions. To show this, math textbooks usually put arrows at the ends of the axes in their drawings.
The location of a point in the plane is given by its coordinates, a pair of numbers enclosed in parentheses: (x, y). The first number x gives the point's horizontal position and the second number y gives its vertical position. All positions are measured relative to a "central" point called the origin, whose coordinates are (0, 0). For example, the point (5, 2) is 5 units to the right of the origin and 2 units up, as shown in the figure. Negative

coordinate numbers tell us to go left or down. See the other points in the figure for examples.

The Cartesian plane is divided into four quadrants. These are numbered from I – IV, starting with the upper right and going around counterclockwise. (For some reason everybody uses roman numerals for this).

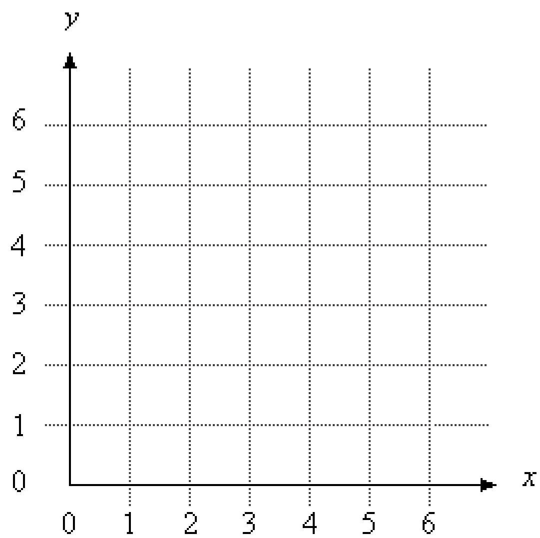
In Quadrant I, both the x– and y–coordinates are positive; in Quadrant II, the x–coordinate is negative, but the y–coordinate is positive; in Quadrant III both are negative; and in Quadrant IV x is positive but y is negative.

Points which lie on an axis (i.e., which have at least one coordinate equal to 0) are said not to be in any quadrant. Coordinates of the form $(x, 0)$ lie on the horizontal x–axis, and coordinates of the form $(0, y)$ lie on the vertical y–axis.

Coordinate Graphing

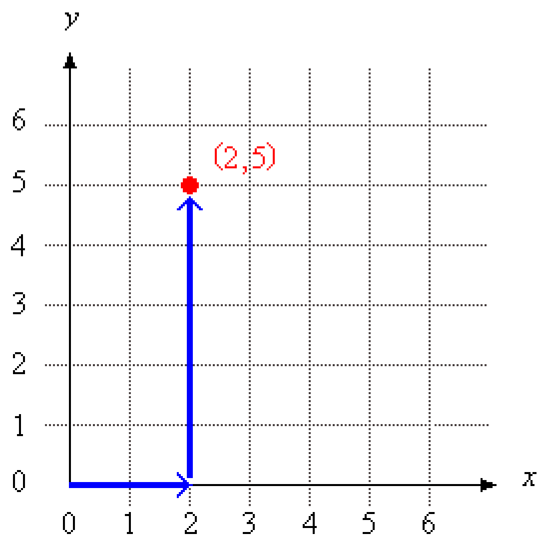
Coordinate graphing sounds very dramatic but it is actually just a visual method for showing relationships between numbers. The relationships are shown on a **coordinate grid**. A coordinate grid has two perpendicular lines, or **axes**, labeled like number lines. The **horizontal axis** is called the ***x*–axis**. The **vertical axis** is called the ***y*–axis**. The point where the x–axis and y–axis intersect is called the **origin**.

Coordinate Grid

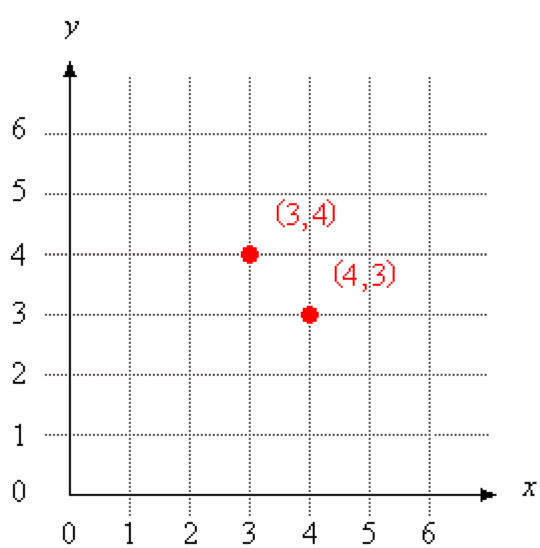


The numbers on a coordinate grid are used to locate points. Each point can be identified by an **ordered pair** of numbers; that is, a number on the *x*-axis called an ***x*-coordinate**, and a number on the *y*-axis called a **y-coordinate**. Ordered pairs are written in parentheses (*x*-coordinate, *y*-coordinate). The origin is located at (0,0). Note that there is no space after the comma.

The location of (2,5) is shown on the coordinate grid below. The *x*-coordinate is 2. The *y*-coordinate is 5. To locate (2,5), move 2 units to the right on the *x*-axis and 5 units up on the *y*-axis.



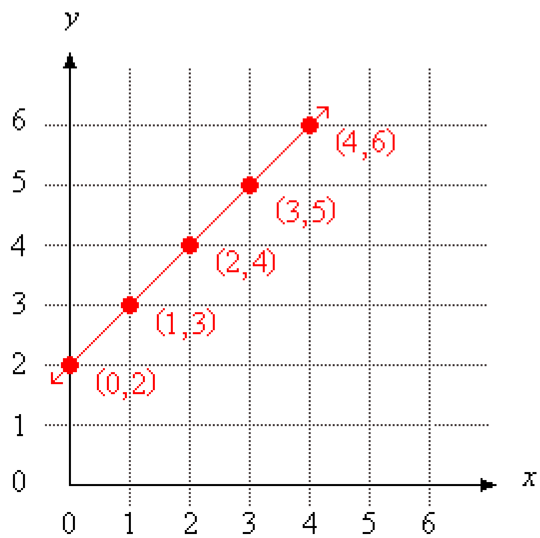
The order in which you write *x*– and *y*–coordinates in an ordered pair is very important. The *x*–coordinate always comes first, followed by the *y*–coordinate. As you can see in the coordinate grid below, the ordered pairs (3,4) and (4,3) refer to two different points!



The function table below shows the *x*– and *y*–coordinates for five ordered pairs. You can describe the relationship between the *x*– and *y*–coordinates for each of these ordered pairs with this rule: the x-coordinate plus two equals the y-coordinate. You can also describe this relationship with the algebraic equation $x + 2 = y$.

***x*–coordinate**	***x* + 2 = *y***	***y*–coordinate**	**ordered pair**
0	**0** + 2 =**2**	2	(0,2)
1	**1** + 2 =**3**	3	(1,3)
2	**2** + 2 =**4**	4	(2,4)
3	**3** + 2 =**5**	5	(3,5)
4	**4** + 2 =**6**	6	(4,6)

To graph the equation $x + 2 = y$, each ordered pair is located on a coordinate grid, then the points are connected. Notice that the graph forms a straight line. The arrows indicate that the line goes on in both directions. The graph for any simple addition, subtraction, multiplication, or division equation forms a straight line.



Plotting Points

To plot a point means to raw its position on a Cartesian plane. The easiest way to plot a point is as follows:

1. Start at the origin.
2. Move along the x-axis by an amount and in a direction given by the x-coordinate of the point.
3. Move up or down parallel to the y-axis by an amount and in a direction given by the y-coordinate.

WEEK 2

Introduction to Inequalities

Linear Inequalities in one variable

When working with linear equations involving one variable whose highest degree (or order) is one, you are looking for the one value of the variable that will make the equation true. But if you consider an inequality such as x + 2 < 7, then values of x can be 0, 1, 2, 3, any negative number, or any fraction in between. In other words, there are many solutions for this inequality. Fortunately, solving an inequality involves the same strategies as solving a one variable equation. So even though there are an infinite number of answers to an inequality, you do not have to work any harder to find the answer. To review how to solve one variable equations,

However, there is one major difference that you must keep in mind when working with any inequality. If you multiply or divide by a negative number, you must change the direction of the inequality sign. You'll see why this is the case soon.

Let's go back and look at x + 2 < 7. If this were an equation, you would only need to subtract 2 from both sides to have x by itself.

x + 2 < 7

– 2   – 2

....

x < 5

Keep in mind that the new rule for inequalities only applies to multiplying or dividing by a negative number. You can still add or subtract without having to worry about the sign of the inequality.

But what would happen if you had -2x ≥ 10? Before solving, If you let x = -5 or -6 or any other value that is less than -5, then the inequality will be true. So you would write your solution as x ≤ – 5. In the process of solving this inequality using algebraic methods, you would have something that looks like the following:

-2/-2 10/-2

x   -5

Let's Practice

2x + 3 > -11

Begin by getting the variable on one side by itself by subtracting 3 from both sides. Then divide both sides by 2. Since you are dividing by a positive 2, there is no need to worry about changing the sign of the inequality.

2x + 3 > -11

2x > -14

x > -7

4 – 3x 20

The solution to this problem begins with subtracting 4 from both sides and then dividing by -3. As soon as you divide by -3, you must change the sign of the inequality.

4 – 3x 20

-3x 16

x   -16/3

5x – 7 > 3x + 9

This solution will require a little more manipulation than the previous examples. You have to gather the terms with the variables on one side and the terms without the variables on the other side.

5x – 7 > 3x + 9

-3x   -3x

2x – 7 > 9

+7   +7

2x > 16

x > 8

There is another type of inequality called a double inequality. This is when the variable appears in the middle of two inequality signs. This is simply a shortcut way of writing two separate inequalities into one and using a shorter process for finding the solution.

Basic Rules of Inequalities

Rule 1

If a > b then b < a, i.e. if a is greater than b then b is less than a, If a < b then b > a and if a is less than b then b is greater than a

Rule 2

If a > b and b > e then a > e, e. g. if 6 > 4 and 4 > 2 then 6 > 2, If a < band b < e then a < e, e. g. if 3 < 7 and 7 < 10 then 3 < 10

Rule 3

If a > b then a + e > b + c or a − c > b − c, If a < b then a + e < b + c or a − c < b − c

i.e. we can add to or subtract from both sides of an inequality the same quantity without changing the sense (or sign) of the inequality.

Rule 4

If a > b and c is a positive number, i.e. c > 0 then ac > bc and a/c > b/c, If a < b and c > 0 then ac < bc and a/c < b/c

i.e. both sides of an inequality can be multiplied or divided by the same positive number without changing the sense of the inequality.

Rule 5

If a> b and c is negative i.e. c < 0 then ac < bc and a/c <b/c, If a < b and c < 0 then ac > bc and a/c > b/c

Note: Both sides of an inequality can be multiplied or divided by a negative number, but the sense of the inequality is reversed.

The sense of an inequality is changed if both sides are multiplied or divided by the same negative number.

Rule 6

If a > b and c > d then adding the inequalities a + c > b + d, If a < b and c < d then a + c < b + d

i.e. Inequalities having the same sense can be added side by side to each other without changing the sense of the inequalities.

Rule 7

If a > b and c > d then either a – c > b – d or c – a > d – b is true but not the two of them are true at the time.

Similarly if a < b and c < d then either a – c < b – d or c –a < d – b is true but not the two of them.

Rule 8

If a > b > 0 and c > d > 0 or a < b < 0 and c < d < 0 then ac > bd

Rule 9

If a > b and n > 0 then $a^n > b^n$

e.g. 5 > 3 and 2 > 0

$5^2 > 3^2$ i.e.25 > 9

If a > b and n < 0 then $a^n < b^n$

If a< b and n > 0 then $a^n < b^n$

If a < b and n < 0 then $a^n > b^n$

e.g. 4 < 6 and -2 < 0

$4^{-2} > 6^{-2}$

i.e. 1/16 > 1/36

Example

-2 6x – 1 10

The strategy for solving this inequality is not that much different than the other examples. Except in this case, you are trying to isolate the variable in the middle rather than on one side or the other. But the process for getting the x by itself in the middle, you should add 1 to all three parts of the inequality and then divide by 6.

-2 6x – 1 10

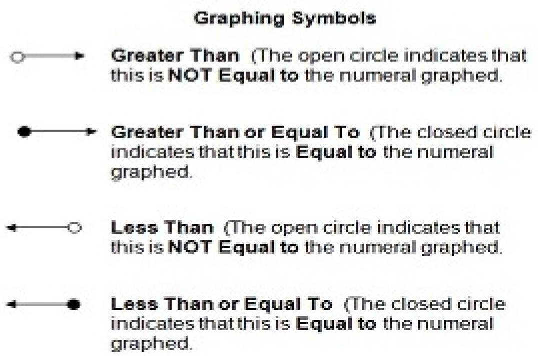
-1 6x 11

-1/6 x 11/6

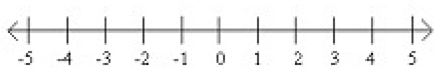
Graphing One-Variable Inequalities

Before graphing linear inequalities, we summarized below the different forms of inequalities, with its corresponding interval form and graph:
Let's take a look at the inequality symbols and their meanings again.

>	**Greater Than**
≥	**Greater Than or Equal To** (The line underneath the greater than sign indicates also equal to)
<	**Less Than** (Tip: To remember this sign, if you open the sign up a little more, it would look like a capital L for less than)
≤	**Less Than or Equal To** (The line underneath the less than sign indicates also equal to)



There are just a few important concepts that you must know in order to graph an inequality. Let's review a number line.



The negative numbers are on the left of the zero and the positive numbers are on the right.

Example

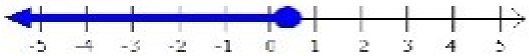
$r > -5$

This is read as "r is greater than -5." This means it includes all numbers greater than, or to the right, of -5 but does not include -5 itself. We will have to show this by using an open circle and having the arrow shoot out to the right.

Example 2

$x \leq 0.4$

This is read as “x is less than or equal to 0.4.” This time we include the 0.4 by using a closed circle and the arrow will shoot out to the left. The number 0.4 is in between the 0 and the 1 on a number line.



Here is a summary of the important details in graphing inequalities.

Make sure you read the inequality starting with the variable!

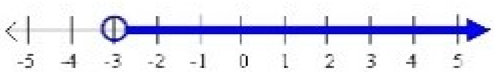
“greater than” or “greater than or equal to” – arrow shoots out to the right
“less than” or “less than or equal to” – arrow shoots out to the left will have open circles
≤ and ≥ will have closed circles.

ASSESSMENT

1. How is this read ?

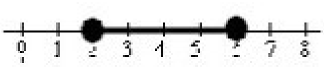
A. Less than or equal to B. greater than or equal to C. Less than D. Greater than

2. What inequality value can be read from the graph below?



A. −3 < *p* B. −3 > p C. −3 p D. −3 p

3. What inequality value can be read from the graph below?



A. 2 < *x*< 6 B. 2> x > 6 C. 6 > x > 8 D. $2^{3} x^{3} 8$

5. Find the solution of the inequality 2x – 4 > 8

A. x > 8 B. x < 8 C. x > 6 D. x < 6

5. 3x -8 10 + 5x

A. x 9 B. x -9 C. x 7 D. x > 9

ANSWERS

1. B 2. A 3. A 4. C 5. B

<u>WEEK 3</u>

Introduction Linear Inequalities in two Variables

We use **inequalities** when there is a range of possible answers for a situation. “I have to be there in less than 5 minutes,” “This team needs to score at least a goal to have a chance of winning,” and “To get into the city and back home again, I need at least $6.50 for train fare” are all examples of situations where a limit is specified, but a range of possibilities exist beyond that limit. That’s what we are interested in when we study inequalities—possibilities.

We can explore the possibilities of an inequality using a number line. This is sufficient in simple situations, such as inequalities with just one variable. But in more complicated circumstances, like those with two variables, it’s more useful to add another dimension, and use a **coordinate plane**. In these cases, we use **linear inequalities**—inequalities that can be written in the form of a linear equation.

The solution of a linear inequality in two variables like $Ax + By > C$ is an ordered pair (x, y) that produces a true statement when the values of x and y are substituted into the inequality.

Example

Is (1, 2) a solution to the inequality

$2x+3y>12x+3y>1$

$2\cdot1+3\cdot2>?12\cdot1+3\cdot2>?1$

$2+5>?12+5>?1$

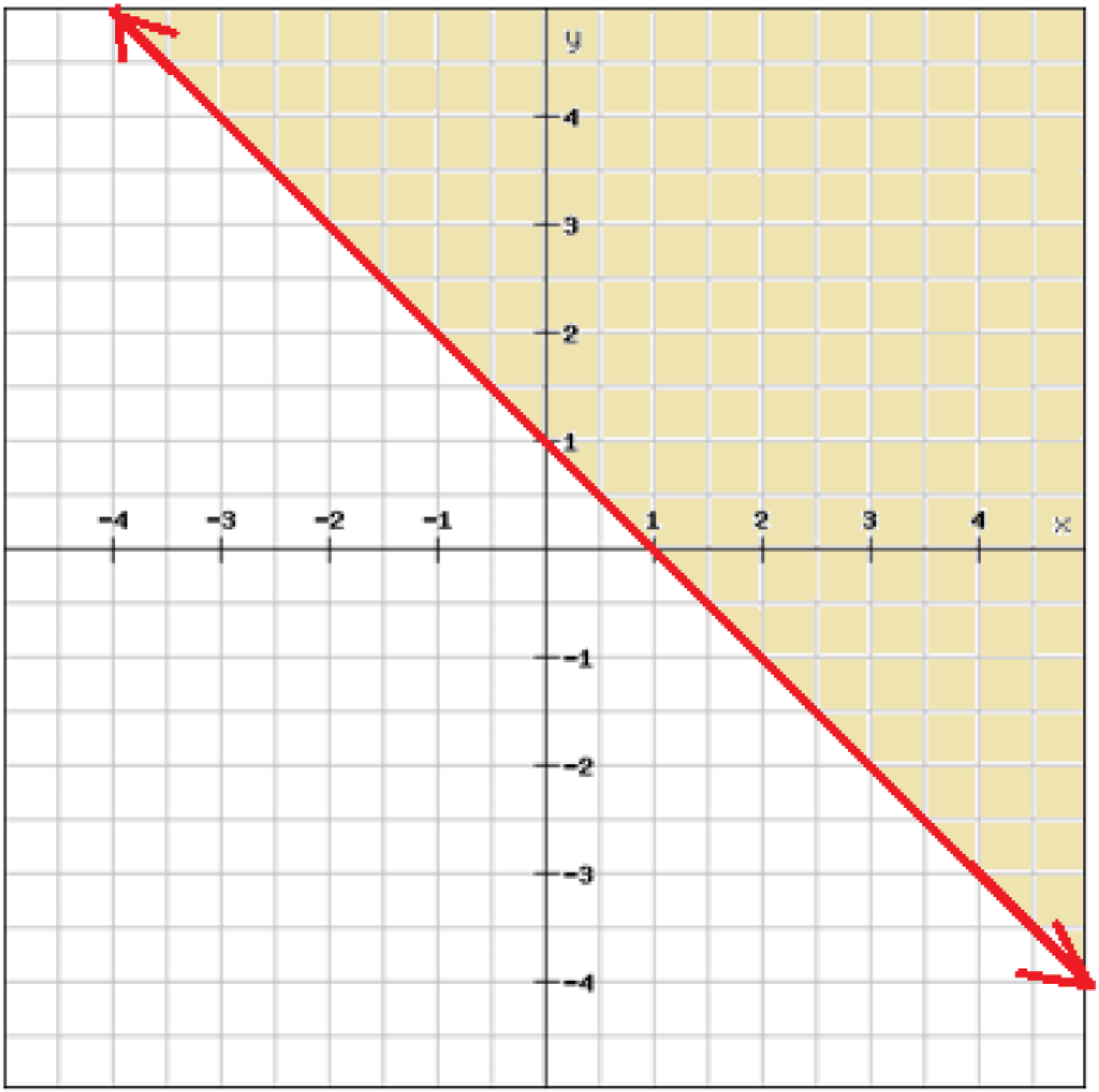
$7>17>1$

The graph of an inequality in two variables is the set of points that represents all solutions to the inequality. A linear inequality divides the coordinate plane into two halves by a boundary line where one half represents the solutions of the inequality. The boundary line is dashed for $>$ and $<$ and solid for $\leq$ and $\geq$. The half-plane that is a solution to the inequality is usually shaded.

Example

Graph the inequality

y≥−x+1y≥−x+1



One Variable Inequalities

Inequalities with one variable can be plotted on a number line, as in the case of the inequality $x \geq -2$: Here is another representation of the same inequality $x \geq -2$, this time plotted on a coordinate plane:

On this graph, we first plotted the line $x = -2$, and then shaded in the entire region to the right of the line. The shaded area is called the **bounded**

region, and any point within this region satisfies the inequality $x \geq -2$. Notice also that the line representing the region's boundary is a solid line; this means that values along the line $x = -2$ are included in the solution set for this inequality.

By way of contrast, look at the graph below, which shows $y < 3$:

In this inequality, the **boundary line** is plotted as a dashed line. This means that the values on the line $y = 3$ are not included in the solution set of the inequality.

Notice that the two examples above used the variables *x* and *y*. It is standard practice to use these variables when you are graphing an inequality on a (x, y) coordinate grid.

Two Variable Inequalities

There's nothing too compelling about the plots of $x \geq -2$ and $y < 3$, shown above. We could have represented both of these relationships on a number line, and depending on the problem we were trying to solve, it may have been easier to do so.

Things get a little more interesting, though, when we plot linear inequalities with two variables. Let's start with a basic two-variable inequality: $x > y$.

The boundary line is represented by a dotted line along $x = y$. All of the points under the line are shaded; this is the range of points where the inequality $x > y$ is true. Take a look at the three points that have been identified on the graph. Do you see that the points in the boundary region have *x* values greater than the *y* values, while the point outside this region do not?

Plotting other inequalities in standard $y = mx + b$ form is fairly straightforward as well. Once we graph the boundary line, we can find out which region to shade by testing some ordered pairs within each region or, in many cases, just by looking at the inequality.

The graph of the inequality $y > 4x - 5.5$ is shown below. The boundary line is the line $y = 4x - 5.5$, and it is dashed because our *y* term is "greater than," not "greater than or equal to."

To identify the bounded region, the region where the inequality is true, we can test a couple of coordinate pairs, one on each side of the boundary line.

If we substitute (-1, 3) into $y > 4x - 5.5$, we find $3 > 4(-1) - 5.5$, or $3 > -9.5$. This is a true statement. It looks like we need to shade the area to the left side of the line.

On the other hand, if we plug (2, -2) into $y > 4x - 5.5$, we find $-2 > 4(2) - 5.5$, or $-2 > 2.5$. This is not a true statement, so the point (2, -2) must not be within the solution set. Yes, the bounded region is to the left of the boundary line.

Inequalities in Context

Making sense of the importance of the shaded region in an inequality can be a bit difficult without assigning any context to it. The following problem shows one instance where the shaded region helps us understand a range of possibilities.

Jumoke and Ajoke want to donate some money to an orphanage. To raise funds, they are selling necklaces and earrings that they have made themselves. Necklaces cost N8 and earrings cost N5. What is the range of possible sales they could make in order to donate at least N100?

The first step here is to create the inequality. Once we have it, we can solve it and then create a graph of it to better understand the importance of the bounded region. Let's begin by assigning the variable x to the number of necklaces sold and y to the number of earrings sold. (Remember—since this will be mapped on a coordinate plane, we should use the variables x and y.)

amount of money earned from selling necklaces	**+**	**amount of money earned from selling earrings**	**≥**	**N100**
$8x$	+	$5y$	≥	100

We can rearrange this inequality so that it solves for y. That's the **slope-intercept form**, and it will make the boundary line easier to graph.

Example

Problem	**8*x***	**+**	**5*y***	**≥**	**100**
	8*x*− 8*x*	+	5*y*	≥	100 − 8*x*
			5*y*	≥	100 − 8*x*
				≥	
			y	≥	20 −
Answer			y	≥	− + 20

So the slope intercept form of the inequality is . Now let's graph it:
The shaded region represents all the possible combinations of necklaces and earrings that Celia and Juniper could sell in order to make at least $100 for the food pantry. It's quite a wide range!
We can look at the two ordered pairs for confirmation that we have shaded the correct region. If we substitute (10, 15) into the inequality, we find 8(10) + 5(15) ≥ 100, which is a true statement. However, using (5, 5) creates a false statement: 8(5) + 5(5) is only 65, and is thus less than 100.
Note that while all points will satisfy the inequality, not all points will make sense in this context. Take (21.25, 10.5), for example. While it does fall within the shaded region, it's hard to expect them to sell 21.25 necklaces and 10.5 earrings! The women can look for whole number combinations in the bounded region to plan how much jewelry to produce.

ASSESSMENT

1. Any new equation obtained by raising both member of an equation to same power may have solutions is called
   (a) extraneous solutions
   (b) absolute value
   (c) radical signs
   (d) simultaneous equation
2. If x = 0 then | x | is equal to
   (a) x

(b) 0
(c) 1
(d) −x

3. Symbols <, >, ≤, ≥ are called
   (a) equality symbols
   (b) inequalities symbols
   (c) operational signs
   (d) radical signs
4. Solution of 4x-2 > 6 is
   (a) x > 2
   (b) x < 6
   (c) x = 4
   (d) x > 4
5. | 5 | should be equal to
   (a) 5
   (b) −5
   (c) 0
   (d) 1/5;

ANSWERS

1. a
2. b
3. b
4. a
5. a

WEEK 4

Algebraic Fractions

Algebraic fractions are simply fractions with algebraic expressions on the top and/or bottom. When adding or subtracting algebraic fractions, the first thing to do is to put them onto a common denominator (by cross multiplying).

When adding or subtracting algebraic fractions, the first thing to do is to put them onto a common denominator (by cross multiplying).

e.g. 1 + 4

(x + 1) (x + 6)

= 1(x + 6) + 4(x + 1)

(x + 1)(x + 6)

= x + 6 + 4x + 4

(x + 1)(x + 6)

= 5x + 10

(x + 1)(x + 6)

Solving equations

When solving equations containing algebraic fractions, first multiply both sides by a number/expression which removes the fractions.

Example

Solve 10 −2 = 1

(x + 3) x

multiply both sides by x(x + 3):

∴ 10x(x + 3) − 2x(x + 3) = x(x + 3)

(x + 3) x

∴ 10x − 2(x + 3) = x^2 + 3x [after cancelling]

∴ 10x − 2x − 6 = x^2 + 3x

∴ x^2 − 5x + 6 = 0

∴ (x − 3)(x − 2) = 0

∴ either x = 3 or x = 2

Adding Fractions

To add fractions there is a simple rule:

$$\frac{a}{b} + \frac{c}{d} = \frac{ad + bc}{bd}$$

(See why this works on the Common Denominator page).

Example:

$\frac{x}{2} + \frac{y}{5} = \frac{(x)(5) + (2)(y)}{(2)(5)}$

$= \frac{5x+2y}{10}$

Example:

$\frac{x + 4}{3} + \frac{x - 3}{4} = \frac{(x+4)(4) + (3)(x-3)}{(3)(4)}$

$= \frac{4x+16 + 3x-9}{12}$

$= \frac{7x+7}{12}$

Subtracting Fractions

Subtracting fractions is very similar to adding, except that the + is now −

$$\frac{a}{b} - \frac{c}{d} = \frac{ad - bc}{bd}$$

Example:

$\frac{x + 2}{x} - \frac{x}{x - 2} = \frac{(x+2)(x-2) - (x)(x)}{x(x-2)}$

$= \frac{(x^2 - 2^2) - x^2}{x^2 - 2x}$

$= \frac{-4}{x^2 - 2x}$

Multiplying Fractions

Multiplying fractions is the easiest one of all, just multiply the tops together, and the bottoms together:

$$\frac{a}{b} \times \frac{c}{d} = \frac{ac}{bd}$$

Example:

$\frac{3x}{x-2} \times \frac{x}{3} = \frac{(3x)(x)}{3(x-2)}$

$= 3x^2$**3(x−2)**
$= x^2$**x−2**
Dividing Fractions
To divide fractions, first “flip” the fraction we want to divide by, then use the same method as for multiplying:

$$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \frac{d}{c} = \frac{ad}{bc}$$

Example:
$3y^2$**x+1** ÷ y**2** = $3y^2$**x+1** × 2**y**
$= (3y^2)(2)$**(x+1)(y)**
$= 6y^2$**(x+1)(y)**
$= 6y$**x+1**

ASSESSMENT

1. Solving following expression 3/(2 – a/6)
   (a) 18/(12 – a)
   (b) 12a/18
   (c) 6a/4
   (d) none of above
2. Single denominator expression for (1/4)c/(c + 1/3)
   (a) 3c/(4(3c + 1))
   (b) 4c + 1/3c
   (c) 3c/4
   (d) 4/3c
3. Simplifying given expression 7/(2x – 5) + 4/(x – 3) gives us
   (a) 23x – 31
   (b) (15x – 41)/(2x – 5)(x – 3)
   (c) (x – 3)(2x – 5)/23x
   (d) none of above
4. Solving expression (2a + 3c)/3b + (a – c)/b gives us
   (a) 4a/3b
   (b) 5a/3b

(c) 3a/2b
(d) 3b/5a

5. If we simplify $m^2 /(m^2 - mp)$ we get
   (a) m/(m – p)
   (b) p/m
   (c) (m – 1)/p
   (d) p/(m – 1)

ANSWERS

1. a
2. a
3. b
4. b
5. b

WEEK 5

Introduction to Linear Programming

Linear programming is the process of taking various linear inequalities relating to some situation, and finding the “best” value obtainable under those conditions. A typical example would be taking the limitations of materials and labor, and then determining the “best” production levels for maximal profits under those conditions.

In “real life”, linear programming is part of a very important area of mathematics called “optimization techniques”. This field of study (or at least the applied results of it) are used every day in the organization and allocation of resources. These “real life” systems can have dozens or hundreds of variables, or more. In algebra, though, you’ll only work with the simple (and graphable) two-variable linear case.

The general process for solving linear-programming exercises is to graph the inequalities (called the “constraints”) to form a walled-off area on the x,y-plane (called the “feasibility region”). Then you figure out the coordinates of the corners of this feasibility region (that is, you find the intersection points of the various pairs of lines), and test these corner points in the formula (called the “optimization equation”) for which you’re trying to find the highest or lowest value.

Find the maximal and minimal value of $z = 3x + 4y$ subject to the following constraints:

= 0, x – y <= 2″ style=”border: 0px;”>

The three inequalities in the curly braces are the constraints. The area of the plane that they mark off will be the feasibility region. The formula “$z = 3x + 4y$” is the optimization equation. I need to find the (x, y) corner points of the feasibility region that return the largest and smallest values of z.

My first step is to solve each inequality for the more-easily graphed equivalent forms:

= x – 2″ style=”border: 0px;”>

It’s easy to graph the system:

To find the corner points — which aren't always clear from the graph — I'll pair the lines (thus forming a system of linear equations) and solve:

$\boldsymbol{y = -(\,^{1}/_{2}\,)x + 7}$
$\boldsymbol{y = 3x}$
$-(\,^{1}/_{2}\,)x + 7 = 3x$
$-x + 14 = 6x$
$14 = 7x$
$2 = xy = 3(2) = 6$
corner point at (2, 6)

$\boldsymbol{y = -(\,^{1}/_{2}\,)x + 7}$
$\boldsymbol{y = x - 2}$
$-(\,^{1}/_{2}\,)x + 7 = x - 2$
$-x + 14 = 2x - 4$
$18 = 3x$
$6 = xy = (6) - 2 = 4$
corner point at (6, 4)

$\boldsymbol{y = 3x}$
$\boldsymbol{y = x - 2}$
$3x = x - 2$
$2x = -2$
$x = -1y = 3(-1) = -3$
corner pt. at (−1, −3)

So the corner points are (2, 6), (6, 4), and (−1, −3).

Somebody really smart proved that, for linear systems like this, the maximum and minimum values of the optimization equation will always be on the corners of the feasibility region. So, to find the solution to this exercise, I only need to plug these three points into "$z = 3x + 4y$".

(2, 6): $z = 3(2) + 4(6) = 6 + 24 = 30$
(6, 4): $z = 3(6) + 4(4) = 18 + 16 = 34$
(−1, −3): $z = 3(-1) + 4(-3) = -3 - 12 = -15$

Then **the maximum of $z = 34$ occurs at (6, 4)**, and **the minimum of $z = -15$ occurs at (−1, −3)**.

Given the following constraints, maximize and minimize the value of $z = -0.4x + 3.2y$.

= 0, y >= 0, x <= 5, x + y <= 7, x + 2y >= 4, y <= x + 5″ style=”border: 0px;”>

First I'll solve the fourth and fifth constraints for easier graphing:

= 0, y >= 0, x <= 5, y <= -x + 7, y >= -(1/2)x + 2, y <= x + 5″ style=”border: 0px;”>

The feasibility region looks like this:

From the graph, I can see which lines cross to form the corners, so I know which lines to pair up in order to verify the coordinates. I'll start at the "top" of the shaded area and work my way clockwise around the edges:

$\boldsymbol{y = -x + 7}$	$\boldsymbol{y = -x + 7}$	$\boldsymbol{x = 5}$
$\boldsymbol{y = x + 5}$	$\boldsymbol{x = 5}$	$\boldsymbol{y = 0}$
$-x + 7 = x + 5$		
$2 = 2x$	$y = -(5) + 7 = 2$	[nothing to do]
$1 = xy = (1) + 5 = 6$		
corner at (1, 6)	corner at (5, 2)	corner at (5, 0)
$\boldsymbol{y = 0}$	$\boldsymbol{y = -(1/2)x + 2}$	$\boldsymbol{x = 0}$
$\boldsymbol{y = -(1/2)x + 2}$	$\boldsymbol{x = 0}$	$\boldsymbol{y = x + 5}$
$-(1/2)x + 2 = 0$	$y = -(1/2)(0) + 2$	
$2 = (1/2)x$	$y = 0 + 2$	$y = (0) + 5 = 5$
$4 = x$	$y = 2$	
corner at (4, 0)	corner at (0, 2)	corner at (0, 5)

Now I'll plug each corner point into the optimization equation, $z = -0.4x + 3.2y$:

(1, 6): $z = -0.4(1) + 3.2(6) = -0.4 + 19.2 = 18.8$

(5, 2): $z = -0.4(5) + 3.2(2) = -2.0 + 6.4 = 4.4$

(5, 0): $z = -0.4(5) + 3.2(0) = -2.0 + 0.0 = -2.0$

(4, 0): $z = -0.4(4) + 3.2(0) = -1.6 + 0.0 = -1.6$

(0, 2): $z = -0.4(0) + 3.2(2) = -0.0 + 6.4 = 6.4$

(0, 5): $z = -0.4(0) + 3.2(5) = -0.0 + 16.0 = 16.0$

Then **the maximum is 18.8 at (1, 6) and the minimum is –2 at (5, 0)**.

Given the inequalities, linear-programming exercise are pretty straightforward, if sometimes a bit long. The hard part is usually the word problems, where you have to figure out what the inequalities are. So I'll show how to set up some typical linear-programming word problems.

At a certain refinery, the refining process requires the production of at least two gallons of gasoline for each gallon of fuel oil. To meet the anticipated demands of winter, at least three million gallons of fuel oil a day will need to be produced. The demand for gasoline, on the other hand, is not more than 6.4 million gallons a day.

If gasoline is selling for $1.90 per gallon and fuel oil sells for $1.50/gal, how much of each should be produced in order to maximize revenue?

The question asks for the number of gallons which should be produced, so I should let my variables stand for "gallons produced".
Since this is a "real world" problem, I know that I can't have negative production levels, so the variables can't be negative. This gives me my first two constraints: namely, $x > 0$ and $y > 0$.
Since I have to have at least two gallons of gas for every gallon of oil, then
$x > 2y$.
For graphing, of course, I'll use the more manageable form "$y < (\frac{1}{2})x$".
The winter demand says that $y > 3{,}000{,}000$; note that this constraint eliminates the need for the "$y > 0$" constraint. The gas demand says that $x < 6{,}400{,}000$.
I need to maximize revenue R, so the optimization equation is $R = 1.9x + 1.5y$. Then the model for this word problem is as follows:
$R = 1.9x + 1.5y$, subject to:
$x > 0$
$x < 6{,}400{,}000$
$y > 3{,}000{,}000$
$y < (\frac{1}{2})x$
Using a scale that counts by millions (so "$y = 3$" on the graph means "y is three million"), the above system graphs as follows:
Taking a closer look, I can see the feasibility region a little better:
When you test the corner points at (6.4m, 3.2m), (6.4m, 3m), and (6m, 3m), you should get a maximal solution of R = $16.96m at (x, y) = (6.4m, 3.2m).

ASSESSMENT

1. In linear programming, constraints can be represented by
   (a) equalities
   (b) inequalities
   (c) ratios
   (d) both a and b

2. One subset which satisfies inequality part of equation is graphically represented by
   (a) domain area of y intercept
   (b) range area of x intercept
   (c) straight line
   (d) shaded area around straight line
3. If there is no significant differences in item quality supplied by different sources then it is classified as
   (a) homogenous
   (b) heterogeneous
   (c) indifferent items
   (d) different items
4. One of two subsets for solution set, one subset satisfies equality part of equation and other subset solves
   (a) range part of equation
   (b) domain part of equation
   (c) equality part of equation
   (d) in-equality part of equation
5. Permissible half space is included in side of line only if inequality is satisfied by
   (a) coordinates
   (b) vertex
   (c) intercepts
   (d) quadrants

ANSWERS

1. d
2. d
3. a
4. d
5. d

WEEK 6

Simple and Compound Statement

A statement is a verbal assertion which can determine either true or false. Sometimes, two statements comes together which is known as the compound statement. In this page, we are going to discuss about compound math statement concept.

What is a Compound Statement?

If statements are connected by more than one operators, like, and, or, not etc then it is called a compound statement. If a table contains more than one statement and representing true value, then it is called truth table. In compound statement, there are different rule for Conjunction, Disjunction and Negation. Also, there are different symbols for all these things. In this page, we will explain all the truth table below, so that student can understand easily. An expression which contains more than one operators in it is called a compound statement.

Rule for Solving Compound Statement

First solve the operations inside the parenthesis, then work on others. We will see the basic operators.

Disjunction:

The disjunction of a compound statement is only false if both the combining statements are false, else the disjunction is true. Disjunction is represented by the symbol ” V ” read as “or”.

Conjunction:

The Conjunction of a compound statement is only True if both the combining statements are true. Else, the Conjunction is false. Conjunction is represented by the symbol ” ^ ” read as “and”.

Negation:
The Negation of a compound statement is always opposite of given statement. If statement is false, then its negation will be false and vice verse. Negation is represented by ¬ and read as "not".

Disjunction Truth Table:

P	**Q**	**PvQ**
T	T	T
T	F	T
F	T	T
F	F	F

Conjunction Truth Table:

P	Q	P^Q
T	T	T
T	F	F
F	T	F
F	F	F

Negation Truth Table:

P	¬P
T	F
F	T

Compound Statement Examples
Given below are some of the examples on compound math statement.

Solved Examples

Question 1: (P -> Q) ∧ (Q -> P)
Solution:
Given (P -> Q) ∧ (Q -> P)

P	Q	P -> Q	Q -> P	(P -> Q) ∧ (Q -> P)
T	T	T	T	T

T	F	F	T	F
F	T	T	F	F
F	F	T	T	T

Question 2: (P -> Q) V (Q -> P)

Solution:
Here, it is given that (P -> Q) V (Q -> P)

P	Q	P -> Q	Q -> P	(P -> Q) V (Q -> P)
T	T	T	T	T
T	F	F	T	T
F	T	T	F	T
F	F	T	T	T

Question 3: (⌉P) ∧ (⌉Q)

Solution:
Here, it is given that (⌉P) ∧ (⌉Q)

P	⌉P	Q	⌉Q	(⌉P) ∧ (⌉Q)
T	F	T	F	F
F	T	F	T	T

More on Statements and the Logical Table
A *statement* is an assertion that can be determined to be true or false. The *truth value* of a statement is T if it is true and F if it is false. For example, the statement “2 + 3 = 5” has truth value T. Statements that involve one or more of the *connectives* “and”, “or”, “not”, “if then” and “ if and only if ” are *compound* statements (otherwise they are simple statements). For example, “It is not the case that 2 + 3 = 5” is the negation of the statement above. Of course, it is stated more simply as “2

+ 3 5”. Other examples of compound statements are:

If you finish your homework then you can watch T.V.

This is a question if and only if this is an answer.

I have read this and I understand the concept.

In symbolic logic, we often use letters, such as *p*, *q* and *r* to represent statements and the following symbols to represent the connectives.

Note that the connective “or” in logic is used in the *inclusive* sense (not the exclusive sense as in English). Thus, the logical statement “It is raining or the sun is shining ” means it is raining, or the sun is shining or it is raining and the sun is shining.

If *p* is the statement “The wall is red” and *q* is the statement “The lamp is on”, then is the statement “The wall is red or the lamp is on (or both)” whereas is the statement “If the lamp is on then the wall is red”. The statement translates to “The wall isn’t red and the lamp is on”.

Statements given symbolically have easy translations into English but it should be noted that there are several ways to write a statement in English. For example, with the examples above, the statement directly translates as “If the wall is red then the lamp is on”. It can also be stated as “The wall is red only if the lamp is on” or “The lamp is on if the wall is red”.

Similarly, directly translates as “The wall is red and the lamp is not on” but it would be preferable to say “The wall is red but the the lamp is off”. Click on the microscope for a more extensive list of English equivalents.

The truth value of a compound statement is determined from the truth values of its simple components under certain rules. For example, if *p* is a true statement then the truth value of is F. Similarly, if *p* has truth value F, then the statement has truth value T. These rules are summarized in the following *truth table*.

If *p* and *q* are statements, then the truth value of the statement is T except when both *p* and *q* have truth value F. The truth value of is F except if both *p* and *q* are true. These and the truth values for the other connectives appear in the truth tables below.

From these elementary truth tables, we can determine the truth value of more complicated statements. For example, what is the truth value of given that *p* and *q* are true? In this case, has truth value F and from the second line of the tables above, we see the truth value of the compound statement is F. Had it been the case that *p* was false and *q* true, then again would be false and from the fourth row of the above table we see that is a false statement. To consider all the possible truth values, we construct a truth table.

The lower case t and f were used to record truth values in intermediate steps. Note that while a truth table involving statements *p* and *q* has 4 rows to cover the possibility of each statement being true or false, if we have additional information about either statement this will reduce the number of rows in the truth table. If, for example, the statement *p* is known to be true, then in constructing the truth table of we will only have 2 rows. Truth tables involving *n* statements will have rows unless additional information about the truth values of some of these statements is known.

A statement that is always true is called *logically true* or a *tautology*. A statement that is always false is called *logically false* or a *contradiction*. Symbolically, we denote a tautology by **1** and a contradiction by **0**.

ASSESSMENT

1. Which of the following propositions is tautology?
   (a) (p v q)→q
   (b) p v (q→p)
   (c) p v (p→q)
   (d) Both (b) & (c)
2. Which of the proposition is p^ (~ p v q) is
   (a) A tautulogy
   (b) A contradiction
   (c) Logically equivalent to p ^ q
   (d) All of above
3. Which of the following is/are tautology?
   (a) a v b → b ^ c
   (b) a ^ b → b v c
   (c) a v b → (b → c)
   (d) None of these
4. Logical expression (A^ B) → (C' ^ A) → (A ≡ 1) is
   (a) Contradiction
   (b) Valid
   (c) Well-formed formula
   (d) None of these
5. Identify the valid conclusion from the premises Pv Q, Q → R, P → M, ˥M
   (a) P ^ (R v R)
   (b) P ^ (P ^ R)
   (c) R ^ (P v Q)
   (d) Q ^ (P v R)

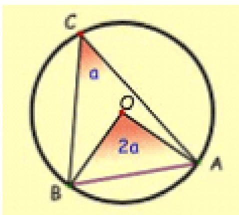
ANSWERS

1. c
2. c
3. b
4. d
5. d

WEEK 7

SSS 2 SECOND TERM MATHEMATICS

Topic: DEDUCTIVE PROOFS – CIRCLE THEOREMS



Centre – the point within the circle where the distance to points on the circumference is the same.

Radius – the distance from the centre to any point on the circle. The diameter is twice the radius.

Circumference (perimeter) – the distance around a circle.

Chord is a straight line joining two points on the circumference.

Diameter – a chord (of max. length) passing through the centre

Sector – a region enclosed by two radii and an arc.

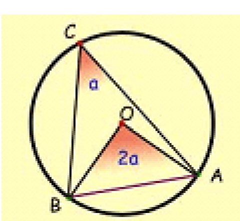
Segment – the region enclosed by a chord and an arc of the circle.

Tangent – a straight line making contact at one point on the circumference, such that the radius from the centre is at right angles to the line.

For example, *AXB* is a **minor arc**, and *AYB* is a **major arc**.

Subtended angles

When a chord subtends an angle on the circumference of a circle, the angle subtended at the centre of the circle is twice the angle.

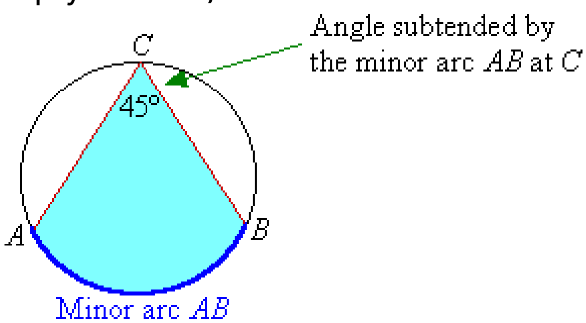


A diameter subtends a right-angle at the circumference.

Angle at the Circumference

If the end points of an arc are joined to a third point on the circumference of a circle, then an angle is formed.

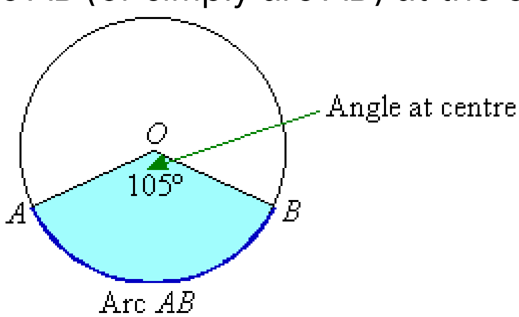
For example, the minor arc *AB* subtends an angle of 45° at *C*. The angle *ACB* is said to be the angle subtended by the minor arc *AB* (or simply arc *AB*) at *C*.



The angle *ACB* is an angle at the circumference standing on the arc *AB*.

Angle at the Centre

If the end points of an arc are joined to the centre of a circle, then an angle is formed. For example, the minor arc *AB* subtends an angle of 105° at *O*. The angle *AOB* is said to be the angle subtended by the minor arc *AB* (or simply arc *AB*) at the centre *O*.

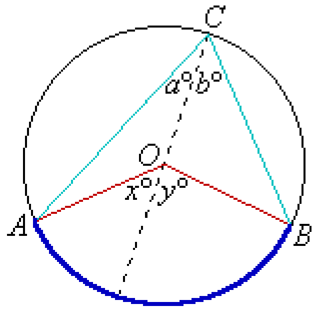


The angle *AOB* is an angle at the centre *O* standing on the arc *AB*.

Angle at Centre Theorem

Theorem

Use the information given in the diagram to prove that the angle at the centre of a circle is twice the angle at the circumference if both angles stand on the same arc.



Given:

Angle AOB and angle ACB stand on the same arc; and O is the centre of the circle.

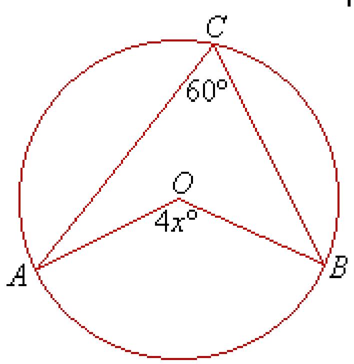
In general:

The angle at the centre of a circle is twice the angle at the circumference if both angles stand on the same arc. This is called the **Angle at Centre Theorem**.

We also call this the **basic property**, as the other angle properties of a circle can be derived from it.

Example

Find the value of the pronumeral in the following circle centred at *O*.



Solution:

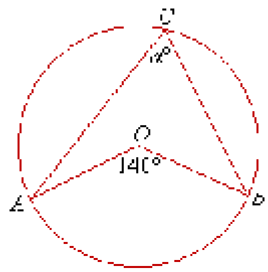
4x = 2 x 60 {Angle at Centre Theorem}

4x = 120

x =120/4 = 30

Example

Find the value of the pronumeral in the following circle centred at *O*.

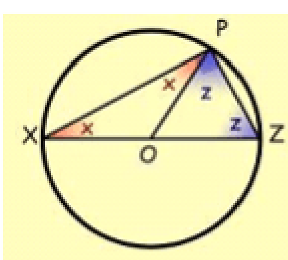


Solution:

2a = 140 (Angle at Centre Theorem)

2a/2 = 140/2

A = 70



angle XPZ = 90^o.

Angle Subtended by a Chord at a Point

Take a line segment PQ and a point R not on the line containing PQ. Join PR and QR (see Fig. 10). Then ∠ PRQ is called the angle subtended by the line segment PQ at the point R. Angles POQ, PRQ and PSQ are called in Fig. 11 below ∠ POQ is the angle subtended by the chord PQ at the centre O, ∠ PRQ and ∠ PSQ are respectively the angles subtended by PQ at points R and S on the major and minor arcs PQ.

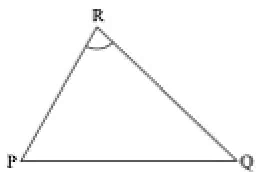


Fig. 10

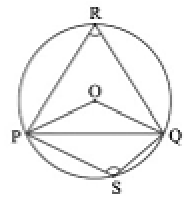


Fig. 11

Let us examine the relationship between the size of the chord and the angle subtended by it at the centre. You may see by drawing different chords of a circle and angles subtended by them at the centre that the longer is the chord, the bigger will be the angle subtended by it at the centre.

Draw two or more equal chords of a circle and measure the angles subtended by them at the centre (see Fig. 12). You will find that the angles subtended by them at the centre are equal. Let us give a proof of this fact.

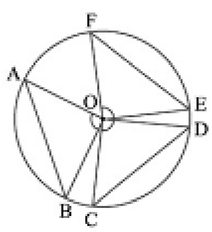


Fig. 12

Theorem: Equal chords of a circle subtend equal angles at the centre.

Proof: You are given two equal chords AB and CD of a circle with centre O (see Fig. 13).

We want to prove that ∠ AOB = ∠ COD.

In triangles AOB and COD,

OA = OC (Radii of a circle)

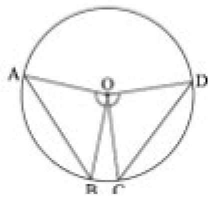
OB = OD (Radii of a circle)

AB = CD (Given)

Therefore, Δ AOB ≅ Δ COD (SSS rule)

This gives ∠ AOB = ∠ COD

(Corresponding parts of congruent triangles)



Remark: For convenience, the abbreviation CPCT will be used in place of 'Corresponding parts of congruent triangles', because we use this very frequently as you will see.

Theorem: *If the angles subtended by the chords of a circle at the centre are equal, then the chords are equal.*

The above theorem is the converse of the Theorem 1. Note that in Fig. 13, if you take ∠AOB = ∠ COD, then

Δ AOB ≅ Δ COD

So, AB = CD

Perpendicular from the Centre to a Chord

Theorem 3: *The perpendicular from the centre of a circle to a chord bisects the chord*.

Given that the perpendicular from the centre of a circle to a chord is drawn and to prove that it bisects the chord. Thus in the converse, what the hypothesis is 'if a line from the centre bisects a chord of a circle' and what is to be proved is 'the line is perpendicular to the chord'. And the converse is:

Theorem 4: *The line drawn through the centre of a circle to bisect a chord is perpendicular to the chord*.

Let AB be a chord of a circle with centre O and O is joined to the mid-point M of AB. We have to prove that OM ⊥ AB. Join OA and OB (see Fig. 16).

In triangles OAM and OBM,

OA = OB (Radii of a circle)

AM = BM (By construction)

OM = OM (Common)

Therefore, ΔOAM ≅ ΔOBM (SSS rule)

This gives ∠OMA = OMB = 90°

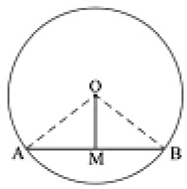
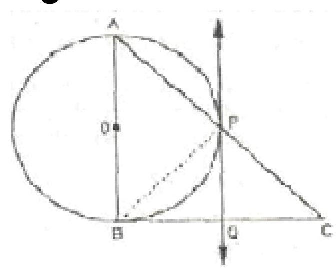


Fig. 16

Angles in Alternative Segment are Equal



The angle between the tangent and chord at the point of contact is equal to the angle in the alternate segment.

Example: In right ABC, a circle with side AB as diameter is drawn to intersect the hypotenuse AC at P. Prove that the tangent to the circle at P bisects the side BC.

Solution: Let O be the centre of the given circle.
Suppose the tangent at P meets BC at Q. Join BP.
You have to prove that BQ = QC.
Since BPQ and BAP = BAC are angles in the alternate segments of chord BP.
BPQ = BAC ...(i)
Since AB is a diameter of the circle and angle in a semi-circle is right angle.
APB = 90
BPC = 90 ...[APB and BPC are linear pairs] ...(ii)
Since A ABC is a right triangle, right angled at B.
BAC + BCA = 90 ...(iii)
From equations (ii) and (iii), you get
BAC + BCA = BPC
BAC + BCA = BPQ + QPC ...[BPC = BPQ + QPC]
BAC + BCA = BAC + QPC ...[BPQ = BAC]

BCA = QPC
QCA = QPC ...[BCA = QCA]
Thus, in A PQC, you have
QCA = QPC
PQ = QC [Sides opposite to equal angles are equal] ... (iv)
Since tangents from an exterior point to a circle are equal in length.
QP = QB i.e. PQ = QB ...(v)
From equations (iv) and (v), you get
BQ = QC.
Thus, Q is a point on BC such that BQ = QC.
Hence, PQ bisects BC.

ASSESSMENT

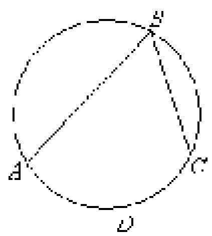
1.
2.
3.
4.
5.

WEEK 8

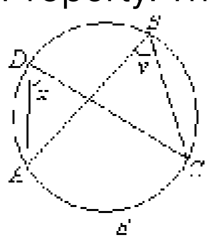
SSS 2 SECOND TERM MATHEMATICS

Topic: SOLVING PROBLEMS ON CIRCLE THEOREMS

An inscribed angle has its vertex on the circle. $\angle ABC$, in the diagram below, is called an inscribed angle or angle at the circumference. The angle is also said to be **subtended by** (i.e. opposite to) **arc** *ADC* or chord *AC*

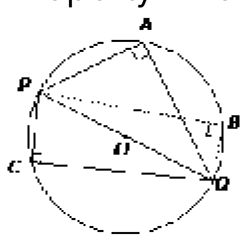


Property: The inscribed angles subtended by the same arc are equal.



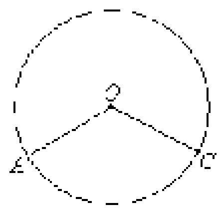
$\angle x = \angle y$ because they are subtended by the same arc *AED*.

Property: The inscribed angles in a semicircle is 90˚.

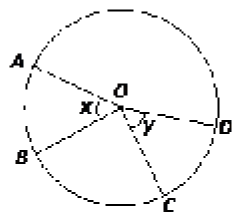


POQ is the diameter. $\angle PAQ = \angle PBQ = \angle PCQ = 90˚$.

A central angle has its vertex is at the centre of the circle. In the diagram below, $\angle AOC$ is called a central angle.



Property: Central angles subtended by arcs of the same length are equal.



$\angle x = \angle y$ because arc AB = arc CD

Inscribed Angle Theorem

Now, we will look at the Inscribed Angle Theorem. It is also called the Central Angle Theorem or Arrow Theorem.

The theorem states

The measure of the central angle is equal to twice the measure of the inscribed angle subtended by the same arc

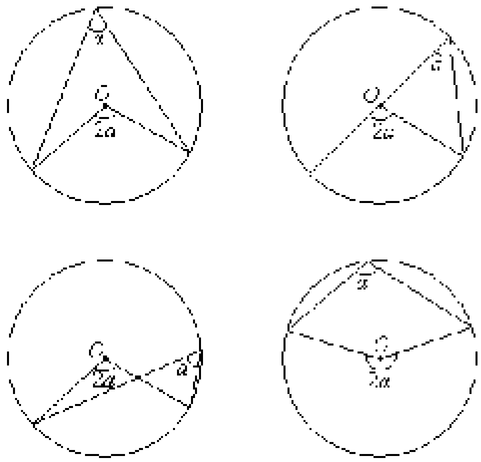
or

An inscribed angle is half of a central angle that subtends the same arc.

or

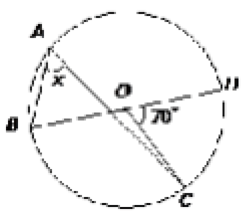
The angle at the centre of a circle is twice any angle at the circumference subtended by the same arc.

The following diagrams illustrates the Inscribed Angle Theorem



Example:

The center of the following circle is *O*. *BOD* is a diameter of the circle. Find the value of *x*.



Solution:

$\angle BOC + 70° = 180°$

$\angle BOC = 110°$

$2x = 110°$

$x = \times 110°$

$= 55°$

ASSESSMENT

1. AD is a diameter of a circle and AB is a chord. If AD = 34 cm, AB = 30 cm, the distance of AB from the centre of the circle is :

   (A) 17 cm (B) 15 cm (C) 4 cm (D) 8 cm

2. In Fig. 10.3, if OA = 5 cm, AB = 8 cm and OD is perpendicular to AB, then CD is equal to:

   (A) 2 cm (B) 3 cm

   (C) 4 cm (D) 5 cm

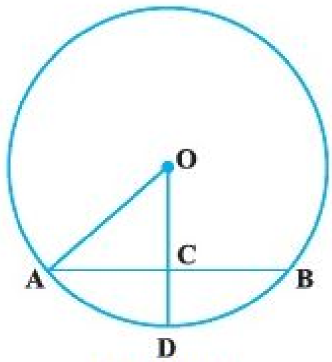


Fig. 10.3

3. If AB = 12 cm, BC = 16 cm and AB is perpendicular to BC, then the radius of the circle passing through the points A, B and C is :

   (A) 6 cm (B) 8 cm

   (C) 10 cm (D) 12 cm

4. In Fig.10.4, if ∠ABC = 20°, then ∠AOC is equal to:

   (A) 20° (B) 40° (C) 60° (D) 10°

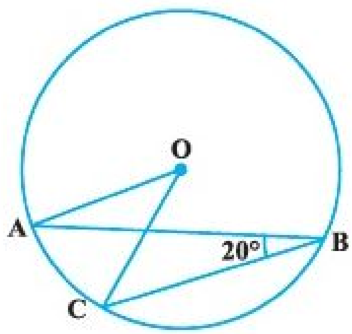


Fig. 10.4

5. In Fig.10.5, if AOB is a diameter of the circle and AC = BC, then ∠CAB is equal to:

   (A) 30° (B) 60°

   (C) 90° (D) 45°

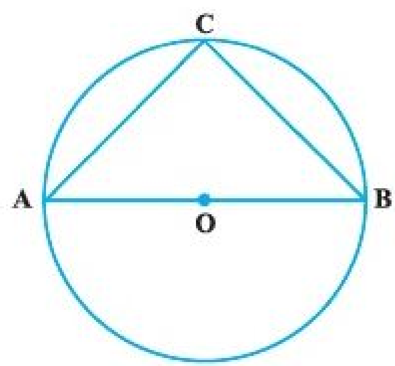


Fig. 10.5

ANSWERS

1. d
2. a
3. c
4. b

5. d

SS 2
THIRD TERM NOTES
ON
MATHEMATICS

TABLE OF CONTENT

THIRD TERM

WEEK 1

SS2 Third Term Mathematics

Introduction to Trigonometric Ratios

Trigonometry is the study of triangles in relation to their sides and angles and many other areas which find applications in many disciplines. In particular, trigonometry functions have come to play great roles in science. For example, in physics, it is used when we want to analyse different kinds of waves, like the sound waves, radio waves, light waves, etc. Also trigonometric ideas are of great importance to surveying, navigation and engineering.

We shall, however at this stage be concerned with the elementary ideas of trigonometry and their application.

Trigonometrical Ratios (Sine, Cosine and Tangent)

The basic trigonometric ratios are defined in terms of the sides of a right-angled triangle. It is necessary to recall that in a right-angled ΔABC, with ∠C = 90^O, the side AB opposite the 90^O is called the hypotenuse.

The notion that there should be some standard correspondence between the lengths of the sides of a triangle and the angles of the triangle comes as soon as one recognizes that similar triangles maintain the same ratios between their sides. That is, for any similar triangle the ratio of the hypotenuse (for example) and another of the sides remains the same. If the hypotenuse is twice as long, so are the sides. It is these ratios that the trigonometric functions express.

To define the trigonometric functions for the angle *A*, start with any right triangle that contains the angle *A*. The three sides of the triangle are named as follows:

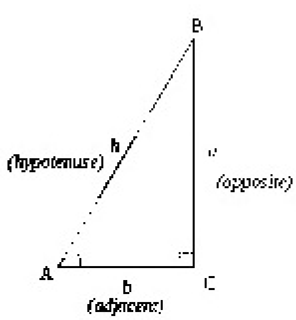
- The *hypotenuse* is the side opposite the right angle, in this case side **h**. The hypotenuse is always the longest side of a right-angled triangle.

- The *opposite side* is the side opposite to the angle we are interested in (angle *A*), in this case side **a**.
- The *adjacent side* is the side having both the angles of interest (angle *A* and right-angle *C*), in this case side **b**.

In ordinary Euclidean geometry, according to the triangle postulate, the inside angles of every triangle total 180° (π radians). Therefore, in a right-angled triangle, the two non-right angles total 90° (π/2 radians), so each of these angles must be in the range of (0°, 90°) as expressed in interval notation. The following definitions apply to angles in this 0° – 90° range. They can be extended to the full set of real arguments by using the unit circle, or by requiring certain symmetries and that they be periodic functions. For example, the figure shows sin θ for angles θ, $\pi - \theta$, $\pi + \theta$, and $2\pi - \theta$ depicted on the unit circle (top) and as a graph (bottom). The value of the sine repeats itself apart from sign in all four quadrants, and if the range of θ is extended to additional rotations, this behavior repeats periodically with a period 2π.

Rigorously, in metric space, one should express angle, defined as scaled arc length, as a function of triangle sides. It leads to inverse trigonometric functions first and usual trigonometric functions can be defined by inverting them back.

The trigonometric functions are summarized in the following table and described in more detail below. The angle θ is the angle between the hypotenuse and the adjacent line – the angle at A in the accompanying diagram.



Function	Abbreviation	Description	Identities (using radians)
Sine	sin	opp/hyp	sin θ = (π/2 − θ) = 1/csc θ
Cosine	cos	adj/hyp	cos θ = (π/2 − θ) = 1/sec θ

Tangent	tan	opp/adj	tan θ = (sin θ/cos θ = cot (π/2 − θ) = 1/cot θ
Cotangent	cot	adj/opp	cot θ = cos θ/sin θ = tan (π/2 − θ) = 1/tan θ
Secant	sec	hyp/adj	sec θ = csc (π/2 − θ) = 1/cos θ
Cosecant	cosec	hyp/opp	csc θ = sec (π/2 − θ) = 1/sinθ

Sine, Cosine and Tangent

The sine of an angle is the ratio of the length of the opposite side to the length of the hypotenuse. (The word comes from the Latin *sinus* for gulf or bay, since, given a unit circle, it is the side of the triangle on which the angle *opens*.) In our case

sin A = opposite/hypotenuse = a/h

This ratio does not depend on the size of the particular right triangle chosen, as long as it contains the angle *A*, since all such triangles are similar.

The **cosine** of an angle is the ratio of the length of the adjacent side to the length of the hypotenuse: so called because it is the sine of the complementary or co-angle. In our case

cos A = adjacent/hypotenuse = b/h

The **tangent** of an angle is the ratio of the length of the opposite side to the length of the adjacent side: so called because it can be represented as a line segment tangent to the circle, that is the line that touches the circle, from Latin *linea tangens* or touching line (cf. *tangere*, to touch). In our case

tan A = opposite/adjacent = a/b

The acronyms “SOHCAHTOA” (“Soak-a-toe”, “Sock-a-toa”, “So-kah-toa”) and “OHSAHCOAT” are commonly used mnemonics for these ratios.

Reciprocal functions

The remaining three functions are best defined using the above three functions.

The **cosecant** csc (*A*), or cosec (*A*), is the reciprocal of sin (*A*); i.e. the ratio of the length of the hypotenuse to the length of the opposite side:
csc A = 1/sin A = hypotenuse/opposite = h/a
The **secant** sec (*A*) is the reciprocal of cos (*A*); i.e. the ratio of the length of the hypotenuse to the length of the adjacent side:
sec A = 1/cos A = hypotenuse/adjacent = h/b.
It is so called because it represents the line that *cuts* the circle (from Latin: *secare*, to cut).
The **cotangent** cot (*A*) is the reciprocal of tan (*A*); i.e. the ratio of the length of the adjacent side to the length of the opposite side:
cot A = 1/tan A = adjacent/opposite = b/a.
Use of Trigonometric Ratio Tables
Apart from finding the trigonometric ratios of angles by construction, the tables of trigonometric ratios in the mathematical tables can be used. These tables consist of both the natural and logarithmic sine, cosine and tangent of angles between 0^0 and 90^0 at intervals of 6^1 or 0.1^0 and having the difference column on the extreme right for intermediate values. Also the table of trigonometric ratio of angles measured in radians are given, for the moment we will consider the natural trigonometric ratios (sine, cosine and tangent) of angles measured in degrees. The tables are usually supplied to save time, since finding the trig ratios of angles or the angles whose trig ratios are given takes a long time.

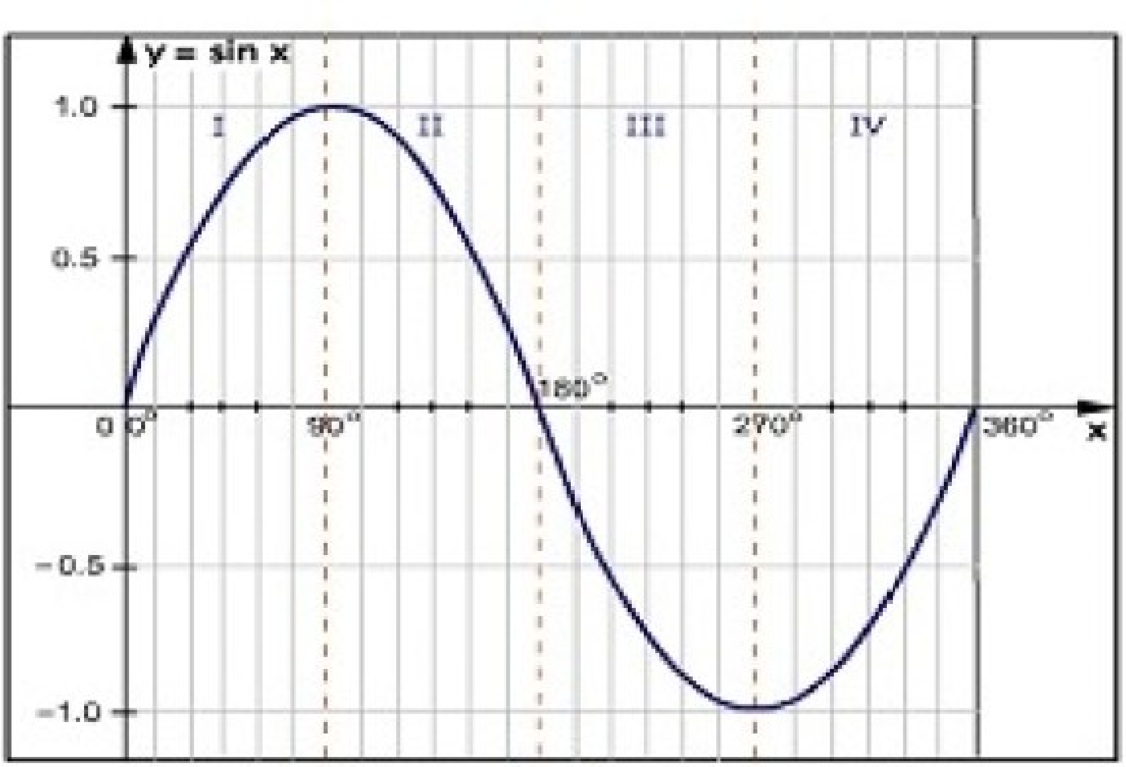
Example
Find sin 32^0
Solution: from the sine table, to find sin 32^0, we look for 32^0 under $0'$ which gives 0.5299. i.e. sin 32^0 = 0.5299.
Find sin $43^0\,30'$

Solution: for sin $43^0\,30'$, we look for sin 43^0 under sin $30'$ and get 0.6884.
i.e. sin $43^0\,30'$ = 0.6884.
Graph of Sine and Cosine for Angles $0^0 \leq x \leq 360^0$
Introduction

We shall consider the graphs of the following functions: sin x and cos x. We usually put y = sin x, y = cos x, to be able to plot points and draw the graphs.

The graph graph of y = sin x for $0^o \leq x \leq 360^o$

x	0^o	30^o	45^o	60^o	90^o	120^o	135^o	150^o	180^o	210^o	225^o	240^o	270^o	300^o	315^o	330^o	360^o
y=sin x	0	0.50	0.71	0.87	1	0.87	0.71	0.50	0	-0.50	-0.71	-0.87	-1	-0.87	-0.71	-0.50	0



From the figure it is evident that the curve repeats itself every 360° or 2p. This fact is expressed by saying that the function has a period of 360° or 2p.

In symbols we write sin (x + n.360°) or sin (x + 2np), sin x = sin (x + n.360°) = sin (x + 2np), where n is any positive or negative integer. This infers that sin x varies and takes a complete ordered range of values once and that sin x is periodic has the period 2p. From the figure we observe that as x increases from 0° to 90°, sin x increases from 0 to 1 and as x increases from 90° to 180°, sin x decreases from 1 to 0.

[A function f(x) is periodic with period T if f (x+T) = f(x) for all values of x]
As x increases from 180° to 270°, sin x decreases from 0 to -1 and as x increases from 270° to 360°, sin x increases from -1 to 0. The maximum absolute value of sin x = 1.

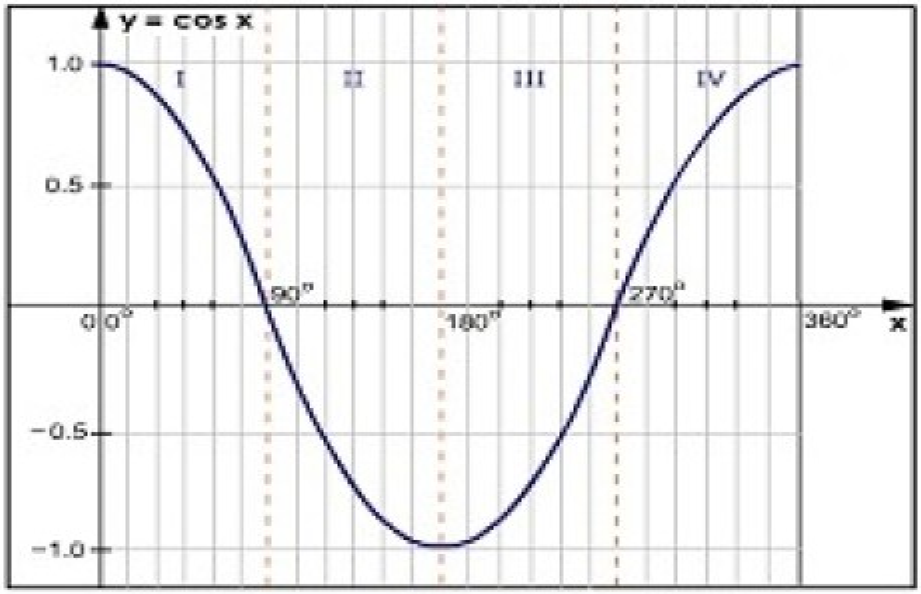
The graph of y = cos x, 0° ≤ x ≤ 360°

In this page we are going to discuss about **Graph of cosine function.** The trigonometric function cosine is defined as the ratio of adjacent side to the hypotenuse. The value of cosine always lies between 1 and -1.Trignometric functions are cyclic functions. It repeats the shape again and again. Study of trigonometric functions are used in many other fields of physics, chemistry and biology like simple harmonic motion, electronics

Properties

Satisfies point symmetry, Domain is all Real Numbers, Range is lies between -1 and 1, Period is 2Pi, From 0 to Pi it decreases and from Pi to 2Pi it increases.

x	0°	30°	45°	60°	90°	120°	135°	150°	180°	210°	225°	240°	270°	300°	315°	330°	360°
y=cosx	1	0.87	0.71	0.50	0	-0.50	-0.71	-0.87	-1	-0.87	-0.71	-0.50	0	0.50	0.71	0.87	1



From the graph it is clear that the curve repeats itself every 360° (2p rad). This fact is expressed by the statement that the function has a period of 360° (2p radians). In symbols we write cos (x + 360°. n) = cos (x + 2pn) = cos x where n is a positive or negative integer.

From the graph we also observe that cos x does not pass through the origin. The maximum and minimum values of cos x are +1 and -1 respectively. As x increases from 0° to 90° cos x decreases from 1 to 0, as x increases from 90° to 180° cos x decreases from 0 to -1, as x increases from 180° to 270° cos x increases from -1 to 0, as x increases from 270° to 360° cos x increases from 0 to 1. cos x is period and has a period 2p.

ASSESSMENT

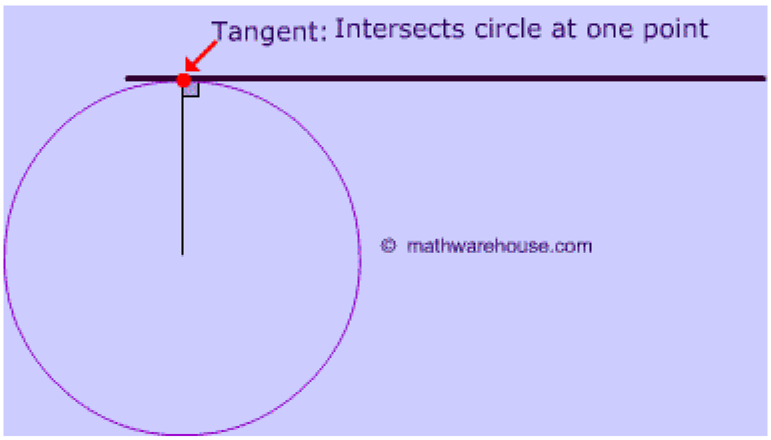
1. What is trigonometry?
2. What are the three sides of a right angle called?
3. What are the three trigonometric ratios?

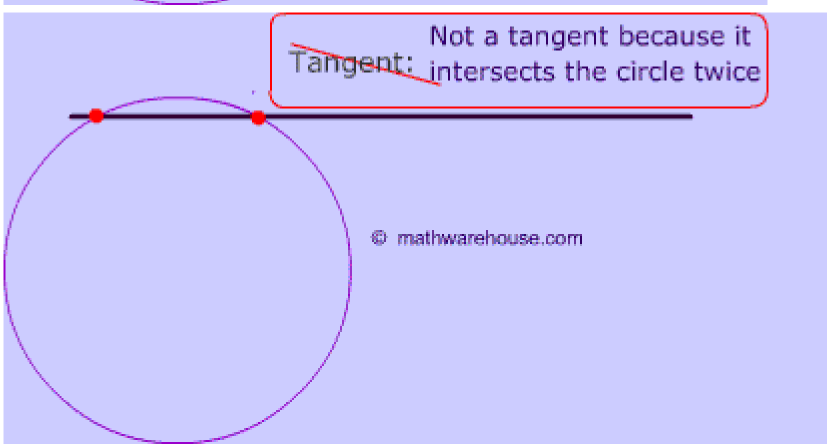
WEEK 2

Introduction

A Tangent of a Circle has two defining properties:

- A tangent intersects a circle in exactly one place
- The tangent intersects the circle's radius at a 90° angle





Since a tangent only touches the circle at exactly one and only one point, that point must be perpendicular to a radius.

To test out the interconnected relationship of these two defining traits of a tangent, try the interactive applet.

The point where the tangent and the circle intersect is called the point of tangency.

Tangent to a circle is perpendicular to the radius at the point of tangency.
 	
• This is a very useful property when the radius that connects to the point of tangency is part of a right angle, because the trigonometry and the Pythagorean Theorem apply to right triangles.
Vocabulary:
 	
• A **tangent** intersects a circle at one point.
    ○ C and **D** are the **points of tangency** to circle O
    ○ AC and AD are tangent to circle O.
• **Perpendicular** means at right angles (meet at 90º).
    ○ OC and OD are radii of the circle O.
    ○ OC is perpendicular to AC.
    ○ OD is perpendicular to AD.
	Interactive Activity
	
	• Adjust the positions of the tangents (see diagram on the left) by dragging the point **A.**
	• Adjust the radius position by dragging points **C** and **D.**
	• <C will always be perpendicular to tangent AC and <D will always be perpendicular to tangent AD.
	• Move point B to overlap radius OC or OD. Radius BO will be perpendicular to the corresponding tangent line.
	• Hold the **SHIFT** key when you drag on the circumference of the circle to change the size of the circle.

2. The tangent segments to a circle from an external point are equal.	**Interactive Activity** • Adjust the positions of the tangents by dragging the **external point A.** • Adjust the radius position by dragging points **C** and **D.** • The length of segment AC will always equal the length of segment AD. • Hold the **SHIFT** key when you drag on the circumference of

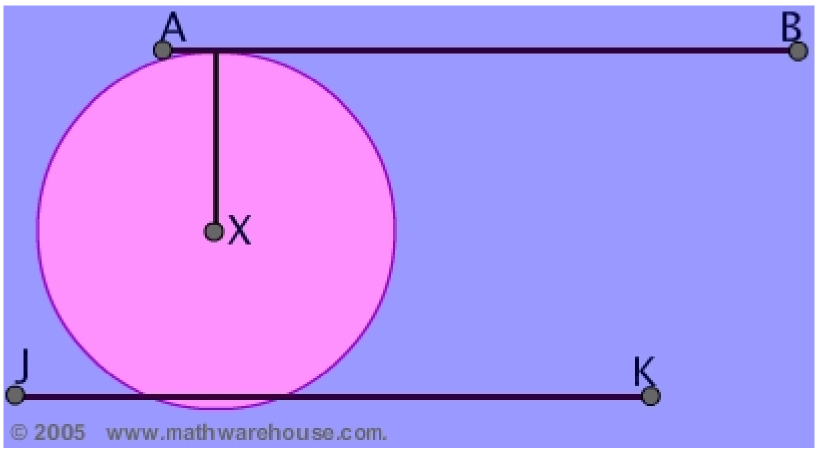
	the circle to change the size of the circle.
3. The angle between a tangent and a chord is equal to the inscribed angle on the opposite side of the chord. • Point A is the **point of tangency** (point where the tangent line touches the circle) of line AX. • Chord AC is a segment with endpoints on the circle. • <XAC is the angle between tangent AX and chord AC. • <ABC is the angle opposite chord AC. • The animation in frame 3/3 shows that the size of <ABC does not change when you move the vertex (point B). • Eventually <ABC will lay on top of <XAC. This shows the two angles must be congruent. • The interactive on the left allows you switch the positions of <ABC and <XAC. In each case <ABC can be moved on top of <XAC. This shows the two angles must be congruent. If you can remember what you see, you will likely remember: **The angle between a tangent and**	

a chord is equal to the inscribed angle on the opposite side of the chord.	

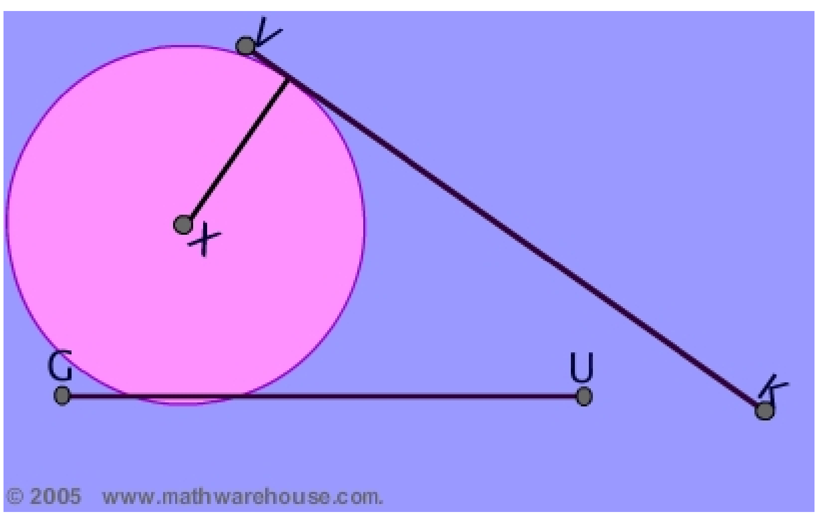
Practice Problems

In the circles below, try to identify which segment is the tangent.

Problem 1



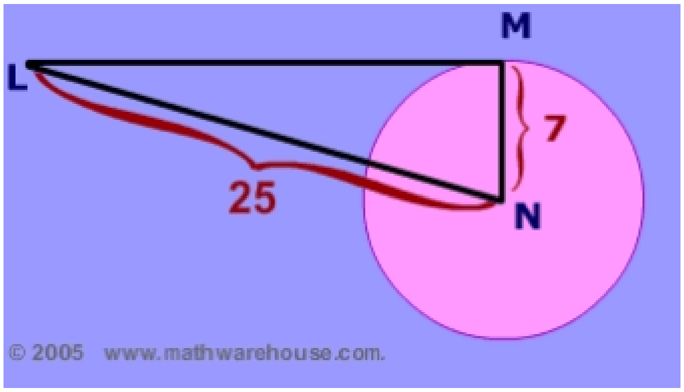
Problem 2



Lengths of Tangents

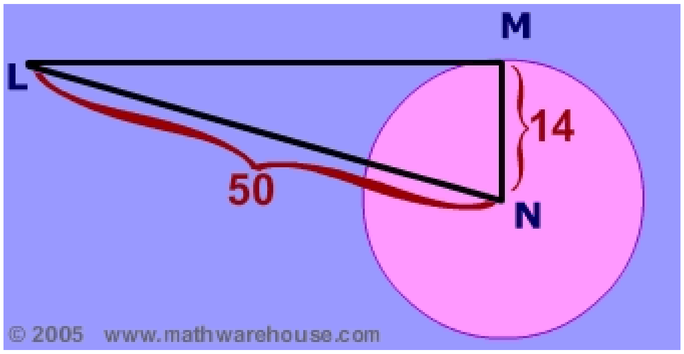
Problem 3

What must be the length of LM for this segment to be tangent line of the circle with center N?



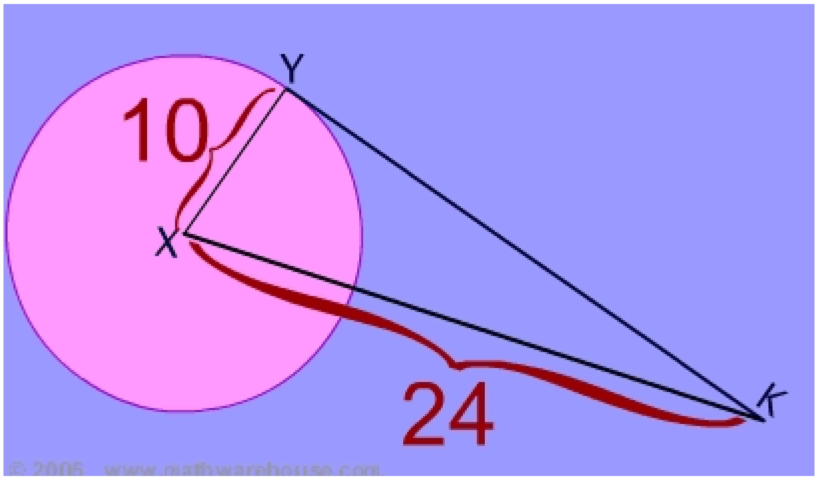
Problem 4

What must be the length of LM for this line to be a tangent line of the circle with center N?



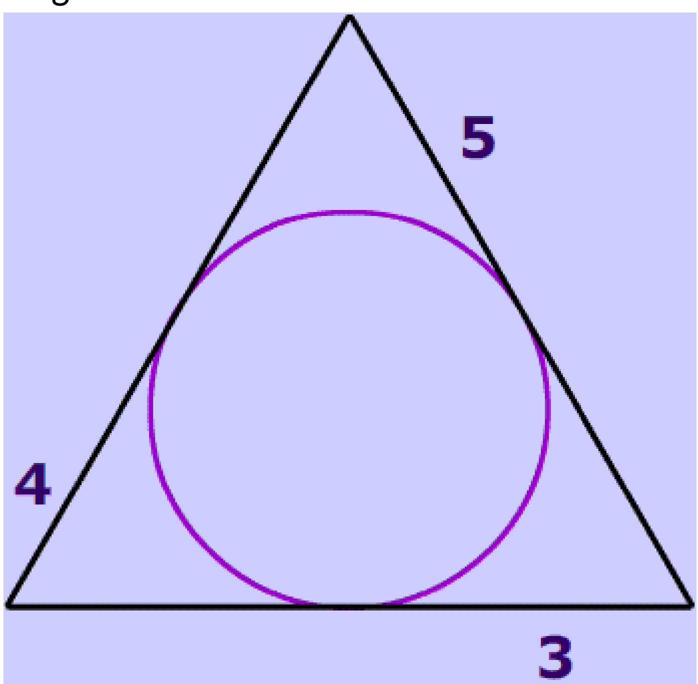
Problem 5

What must be the length of YK for this line to be tangent to the circle with center X?



Problem 6

What is the perimeter of the triangle below? Note: all of the segments are tangent and intersect outside the circle



WEEK 3

Introduction

The sine rule formula states that the ratio of a side to the sine function applied to the corresponding angle is same for all sides of the triangle. For a triangle ABC, sine rule can be stated as given below:

$$\frac{a}{sinA} = \frac{b}{sinB} = \frac{c}{sinC}$$

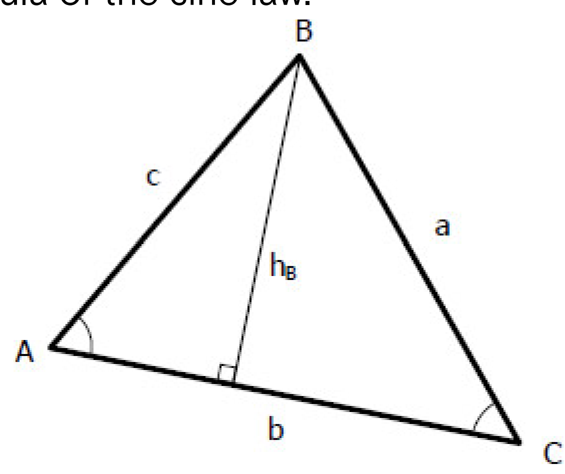
The sine rule formula can be used to find the measure of unknown angle or side of a triangle. It can be used to predict unknown values for two congruent triangles.

If for a given triangle, a, b, and c are the lengths of sides , and A, B, and C are the opposite angles then the sine rule formula is also stated as the reciprocal of this equation:

$$\frac{sinA}{a} = \frac{sinB}{b} = \frac{sinC}{c}$$

Derivation of Sine Rule

To derive the formula, erect an altitude through B and label it h_B as shown below. Expressing h_B in terms of the side and the sine of the angle will lead to the formula of the sine law.



$$\sin A = \frac{h_B}{c}$$

$$h_B = c \sin A$$

$$\sin C = \frac{h_B}{a}$$

$$h_B = a \sin C$$

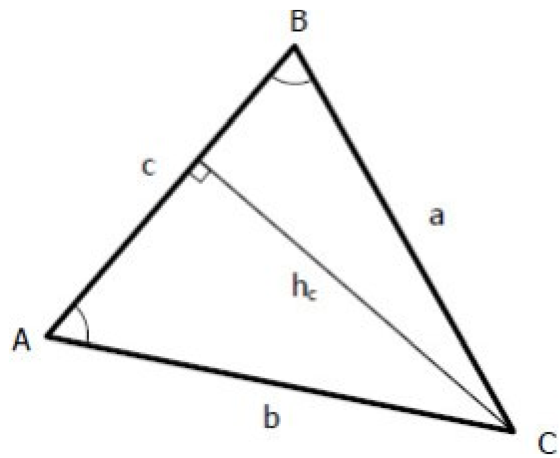
Equate the two h_B's above:

$$h_B = h_B$$

$$c \sin A = a \sin C$$

$$\frac{c}{\sin C} = \frac{a}{\sin A}$$

To include angle B and side b in the above relationship, construct an altitude through C and label it h_C as shown below.



$$\sin A = \frac{h_C}{b}$$

$$h_C = b \sin A$$

$$\sin B = \frac{h_C}{a}$$

$$h_C = a \sin B$$

$$h_C = h_C$$

$$b \sin A = a \sin B$$

$$\frac{b}{\sin B} = \frac{a}{\sin A}$$

Thus,

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Note:

The constant ratio above is the diameter of the circumscribing circle about the triangle. See the proof (*not available for now*) for this note.

Proof of the Sine Rule Formula

From the above triangle AXC,

$$sinA = \frac{h}{b}$$

$$bsin\ A = h$$

From triangle XBC,

$$sin\ B$$

$$asin\ B = h$$

Equating both the Equations we have

$$h = bsin\ A = asin\ B$$

So,

$$\frac{b}{sin\ B} = \frac{a}{sin\ A}$$

If we draw a perpendicular from A to BC, we can show that

$$\frac{b}{sin\ B} = \frac{c}{sin\ C}$$

Hence we have the Sine Rule:

$$\frac{a}{sin\ A} = \frac{b}{sin\ B} = \frac{c}{sin\ C}$$

The area of a triangle:

The Area of any triangle will be $\frac{1}{2}\ ab\ sinC$ using the sine rule formula.

This formula is used in finding the area, base or height of a given triangle if other quantities are given.

$$\frac{1}{2}\ (base)\ \times\ (height)$$

Examples Using the Sine Rule Formula

Example 1: Given side a = 5, side c = 10, and angle C = 30^o. Find the angle A.

Sol: Using the Sine rule formula ,

$\frac{sin\ A}{5}$ = $\frac{sin\ 30}{10}$

$sin\ A$ = $\frac{1}{2}\ \times\ \frac{5}{10}$

A = $sin^{-1}\ \frac{1}{4}$

A = 0.14.47°

The area of a triangle:

The Area of any triangle will be $\frac{1}{2}$ $ab\ sinC$ using the sine rule formula.

This formula is used in finding the area, base or height of a given triangle if other quantities are given.

$$\frac{1}{2}\ (base)\ \times\ (height)$$

Examples Using the Sine Rule Formula

Example 1: Given side a = 5, side c = 10, and angle C = 30^o. Find the angle A.

Sol: Using the Sine rule formula ,

$$\frac{sin\ A}{5} = \frac{sin\ 30}{10}$$

$$sin\ A = \frac{1}{2} \times \frac{5}{10}$$

$$A = sin^{-1}\ \frac{1}{4}$$

A = 0.14.47°

ASSESSMENT

1. What does the Sine rule state?
2. Proof the Sine rule formula
3. Giving side A = 8, side C = 24, and angle C = 60. Find angle A.

WEEK 4

Introduction to Angles of Elevation and Depression

Any surface which is parallel to the surface of the earth is said to be **horizontal**. For example, the surface of liquid in a container is always horizontal, even if the container is held at an angle.

The floor of your classroom is horizontal. Any line drawn on horizontal surface is will also be horizontal. Any line or surface which is perpendicular to a surface is said to be **vertical**. The walls of your classroom are vertical.

A plum-line is a mass which hangs freely on a thread.

1. Say whether the following are horizontal or vertical, or neither:

a. the table top
b. the door
c. the pictures
d. the floor boards
e. the back of the chair
f. the table legs
g. the ruler (on the table)
h. the line where the walls meet
i. the brush handle
j. the top edge of the small

Elevation and depression

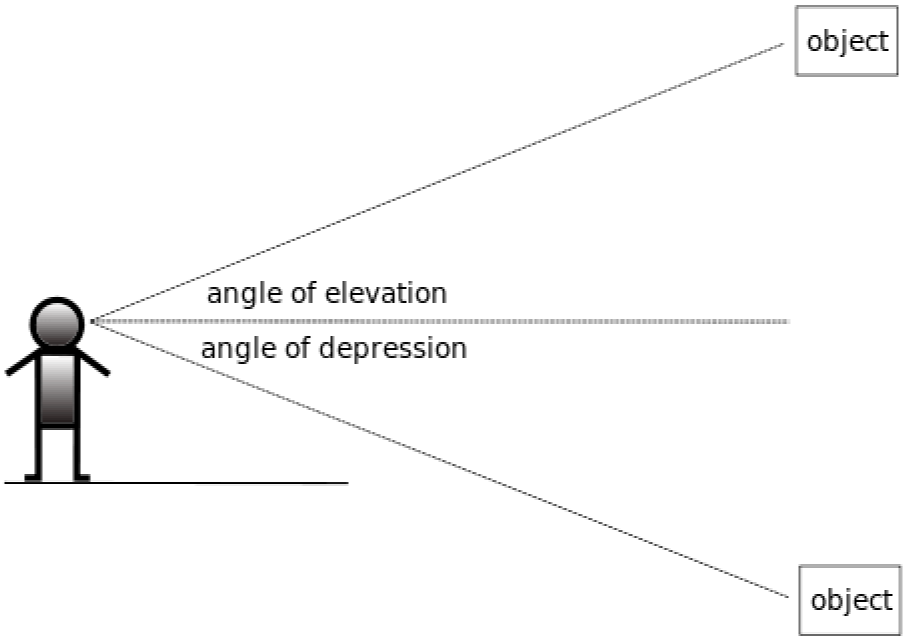
Suppose that you are looking at an object in the distance.

If the object is above you, then the angle of elevation is the angle your eyes look up.

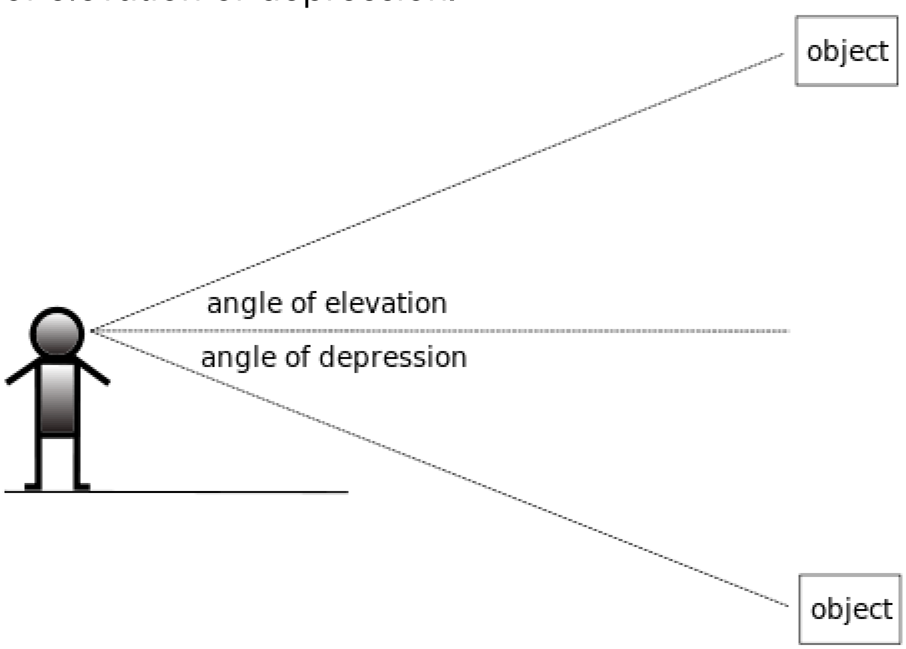
If the object is below you, the angle of depression is the angle your eyes look down.

Angles of elevation and depression are measured from the horizontal.

It is common mistake not to measure the angle of depression from the horizontal.



Using the angle of depression or elevation to an object, and knowing how far away the object is, enables us to find the height of the object using trigonometry.
The advantage of doing this is that it is very difficult to measure the height of a mountain or the depth of a canyon directly; it is much easier to measure how far away it is (horizontal distance) and to measure the angle of elevation or depression.



Suppose that we want to find the height of this tree.
We mark point *A* and measure how far it is from the base of the tree.

Then we measure the angle of elevation from *A* to the top of the tree.

Now,

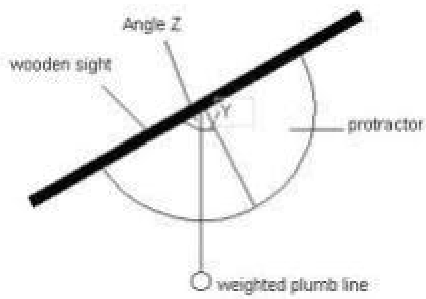
h/x = tan(θ)

h = xtan(θ)

we have measured *x* and θ, so we can calculate tan(θ) and thus we can find *h*, which is the height of the tree.

Angles of elevation and depression can be measured with a simple instrument called a **clinometers**.

A plum-line



Example

Solve for x

Solution

Angle of depression = 34°

But ∠ B = ∠ O (alternate angles)

therefore, ∠ B = 34°

From triangle ABO, we have

tan 34° = 40x

=> 0.6745 = 40x

=> 0.6745 x = 40

=> x = 400.6745

=> x = 59.30

ASSESSMENT

1. What is the common mistake encountered when measuring angle of elevation or depression?

2. If you are looking at the top of a building, what type of angle will be measured?

WEEK 5

Calculation of Class Boundaries and Intervals

Class Boundaries: Class Boundaries are the midpoints between the upper class limit of a class and the lower class limit of the next class in the sequence. Therefore, each class has an upper and lower class boundary.

Example:

Class	**Frequency**
200 – 299	12
300 – 399	19
400 – 499	6
500 – 599	2
600 – 699	11
700 – 799	7
800 – 899	3
Total Frequency	**60**

Using the frequency table above, determine the class boundaries of the first three classes.

For the first class, 200 – 299
The lower class boundary is the midpoint between 199 and 200, that is *199.5*

The upper class boundary is the midpoint between 299 and 300, that is *299.5*

For the second class, 300 – 399
The lower class boundary is the midpoint between 299 and 300, that is *299.5*

The upper class boundary is the midpoint between 399 and 400, that is *399.5*

For the third class, 400 – 499
The lower class boundary is the midpoint between 399 and 400, that is *399.5*

The upper class boundary is the midpoint between 499 and 500, that is *499.5*

Class Intervals: Class interval is the difference between the upper and lower class boundaries of any class.

Example:

Class	**Frequency**
200 – 299	12
300 – 399	19
400 – 499	6
500 – 599	2
600 – 699	11
700 – 799	7
800 – 899	3
Total Frequency	**60**

Using the table above, determine the class intervals for the first class.

For the first class, 200 – 299
The class interval = Upper class boundary – lower class boundary

Upper class boundary = 299.5

Lower class boundary = 199.5

Therefore, the class interval = 299.5 – 199.5

= 100

ASSESSMENT

1. What are class boundaries?
2. What is the lower class boundary of 300-350?
3. What is the upper class boundary of 480-490?
4. What is a class interval?

WEEK 6

Cumulative Frequency Graphs: What is it?

Cumulative frequency is the running total of the frequencies. On a graph, it can be represented by a cumulative frequency polygon, where straight lines join up the points, or a cumulative frequency curve.

Example

Frequency:	Cumulative Frequency:	
4	4	
6	10	(4 + 6)
3	13	(4 + 6 + 3)
2	15	(4 + 6 + 3 + 2)
6	21	(4 + 6 + 3 + 2 + 6)
4	25	(4 + 6 + 3 + 2 + 6 + 4)

This short video shows you how to plotting a cumulative frequency curve from the frequency distribution. How to find the median and inter-quartile range.

The Median Value

The median of a group of numbers is the number in the middle, when the numbers are in order of magnitude. For example, if the set of numbers is 4, 1, 6, 2, 6, 7, 8, the median is 6:

1, 2, 4, **6**, 6, 7, 8 (6 is the middle value when the numbers are in order)

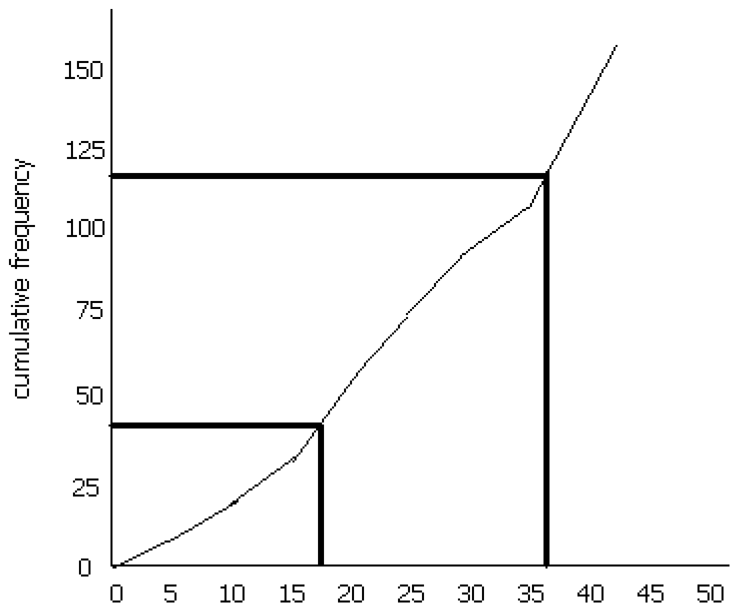
If you have n numbers in a group, the median is the (n + 1)/2 th value. For example, there are 7 numbers in the example above, so replace n by 7 and the median is the (7 + 1)/2 th value = 4th value. The 4th value is 6.

When dealing with a cumulative frequency curve, “n” is the cumulative frequency (25 in the above example). Therefore the median would be the 13th value. To find this, on the cumulative frequency curve, find 13 on the y-axis (which should be labelled cumulative frequency). The corresponding ‘x’ value is an estimation of the median.

Quartiles

If we divide a cumulative frequency curve into quarters, the value at the lower quarter is referred to as the lower quartile, the value at the middle gives the median and the value at the upper quarter is the upper quartile. A set of numbers may be as follows: 8, 14, 15, 16, 17, 18, 19, 50. The mean of these numbers is 19.625 . However, the extremes in this set (8 and 50) distort this value. The interquartile range is a method of measuring the spread of the middle 50% of the values and is useful since it ignore the extreme values.

The lower quartile is (n+1)/4 th value (n is the cumulative frequency, i.e. 157 in this case) and the upper quartile is the 3(n+1)/4 the value. The difference between these two is the interquartile range (IQR).
In the above example, the upper quartile is the 118.5th value and the lower quartile is the 39.5th value. If we draw a cumulative frequency curve, we see that the lower quartile, therefore, is about 17 and the upper quartile is about 37. Therefore the IQR is 20 (bear in mind that this is a rough sketch- if you plot the values on graph paper you will get a more accurate value).



ASSESSMENT

1. What's cumulative Frequency?
2. What is median?
3. What is the inter-quartile range?

WEEK 7

How to Determine the Mean, Median and Mode from Grouped Frequencies

To better explain how to determine the mean, median and mode of grouped frequency data, we will work with common, relatable examples as you can see below-

Ade timed 21 people in the sprint race, to the nearest second:

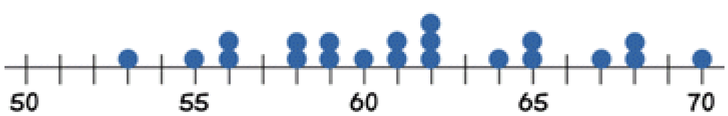
59, 65, 61, 62, 53, 55, 60, 70, 64, 56, 58, 58, 62, 62, 68, 65, 56, 59, 68, 61, 67

To find the Mean, Ade adds up all the numbers, then divides by how many numbers:

$$\textbf{Mean} = \frac{59+65+61+62+53+55+60+70+64+56+58+58+62+62+68+65+56+59+68+61+67}{21}$$

= **61.38095...**

To find the Median Ade places the numbers in value order and finds the middle number.



In this case the median is the 11^{th} number:

53, 55, 56, 56, 58, 58, 59, 59, 60, 61, 61, 62, 62, 62, 64, 65, 65, 67, 68, 68, 70

Median = 61

To find the Mode, or modal value, Ade places the numbers in value order then counts how many of each number. The Mode is the number which appears most often (there can be more than one mode):

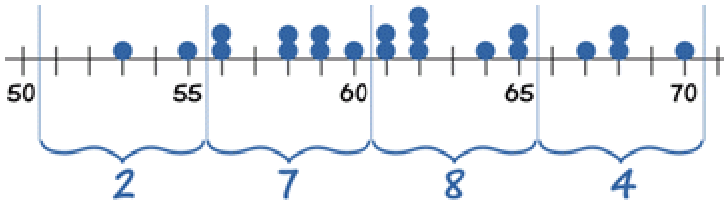
53, 55, 56, 56, 58, 58, 59, 59, 60, 61, 61, 62, 62, 62, 64, 65, 65, 67, 68, 68, 70

62 appears three times, more often than the other values, so **Mode = 62**

Grouped Frequency Table

Alex then makes a Grouped Frequency Table:

Seconds	Frequency
51 – 55	2
56 – 60	7
61 – 65	8
66 – 70	4



So 2 runners took between 51 and 55 seconds, 7 took between 56 and 60 seconds, etc

Suddenly all the original data gets lost (naughty pup!)

Only the Grouped Frequency Table survived …

… can we help Alex calculate the Mean, Median and Mode from just that table?

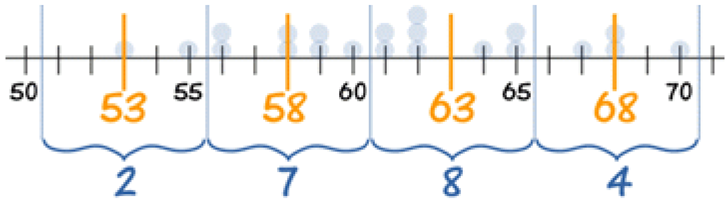
The answer is … no we can't. Not accurately anyway. But, we can make estimates.

Estimating the Mean from Grouped Data

So all we have left is:

Seconds	Frequency
51 – 55	2
56 – 60	7
61 – 65	8
66 – 70	4

- The groups (51-55, 56-60, etc), also called class intervals, are of width 5
- The midpoints are in the middle of each class: 53, 58, 63 and 68



We can estimate the Mean by using the **midpoints**.

So, how does this work?

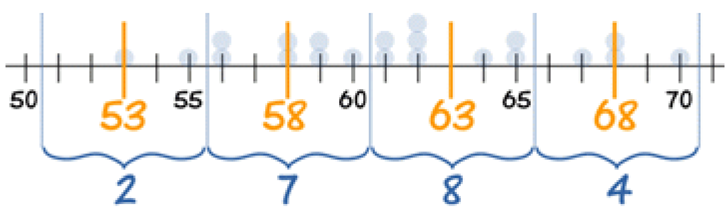
Think about the 7 runners in the group **56 – 60**: all we know is that they ran somewhere between 56 and 60 seconds:

- Maybe all seven of them did 56 seconds,
- Maybe all seven of them did 60 seconds,
- But it is more likely that there is a spread of numbers: some at 56, some at 57, etc

So we take an average and **assume** that all seven of them took 58 seconds.

Let's now make the table using midpoints:

Midpoint	Frequency
53	2
58	7
63	8
68	4



Our thinking is: "2 people took 53 sec, 7 people took 58 sec, 8 people took 63 sec and 3 took 68 sec". In other words we **imagine** the data looks like this:

53, 53, 58, 58, 58, 58, 58, 58, 58, 63, 63, 63, 63, 63, 63, 63, 63, 68, 68, 68, 68

Then we add them all up and divide by 21. The quick way to do it is to multiply each midpoint by each frequency:

Midpoint x	Frequency f	Midpoint × Frequency fx
53	2	106
58	7	406
63	8	504
68	4	272
Totals:	**21**	**1288**

And then our **estimate** of the mean time to complete the race is:

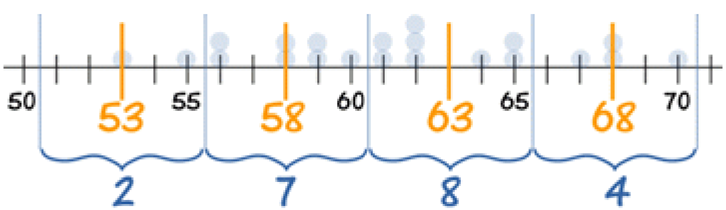
$$\textbf{Estimated Mean} = \frac{1288}{21} = \textbf{61.333...}$$

Very close to the exact answer we got earlier.

Estimating the Median from Grouped Data

Let's look at our data again:

Seconds	Frequency
51 – 55	2
56 – 60	7
61 – 65	8
66 – 70	4



The median is the middle value, which in our case is the 11th one, which is in the 61 – 65 group:

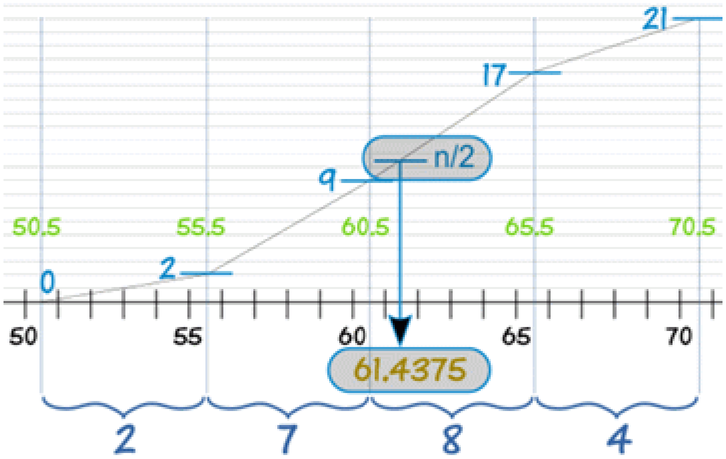
We can say "the **median group** is 61 – 65"

But if we want an estimated **Median value** we need to look more closely at the 61 – 65 group.

We call it "61 – 65", but it really includes values from 60.5 up to (but not including) 65.5.

Why? Well, the values are in whole seconds, so a real time of 60.5 is measured as 61. Likewise 65.4 is measured as 65.

At 60.5 we already have **9** runners, and by the next boundary at 65.5 we have **17** runners. By drawing a straight line in between we can pick out where the median frequency of **n/2** runners is:



And this handy formula does the calculation:

$$\textbf{Estimated Median} = L + \frac{(n/2) - B}{G} \times w$$

where:

- **L** is the lower class boundary of the group containing the median
- **n** is the total number of values
- **B** is the cumulative frequency of the groups before the median group
- **G** is the frequency of the median group
- **w** is the group width

For our example:

- **L** = 60.5
- **n** = 21
- **B** = 2 + 7 = 9
- **G** = 8
- **w** = 5

$$\textbf{Estimated Median} = 60.5 + \frac{(21/2) - 9}{8} \times 5$$

$$= \mathbf{60.5 + 0.9375}$$

$$= \mathbf{61.4375}$$

Estimating the Mode from Grouped Data

Again, looking at our data:

Seconds	Frequency
51 – 55	2
56 – 60	7
61 – 65	8
66 – 70	4

We can easily find the modal group (the group with the highest frequency), which is **61 – 65**

We can say "the **modal group** is 61 – 65"

But the actual Mode may not even be in that group! Or there may be more than one mode. Without the raw data we don't really know.

But, we can estimate the Mode using the following formula:

$$\textbf{Estimated Mode} = L + \frac{f_m - f_{m-1}}{(f_m - f_{m-1}) + (f_m - f_{m+1})} \times w$$

where:

- L is the lower class boundary of the modal group
- f_{m-1} is the frequency of the group before the modal group
- f_m is the frequency of the modal group
- f_{m+1} is the frequency of the group after the modal group
- w is the group width

In this example:

- L = 60.5
- $f_{m-1} = 7$
- $f_m = 8$
- $f_{m+1} = 4$
- w = 5

$$\textbf{Estimated Mode} = 60.5 + \frac{8 - 7}{(8 - 7) + (8 - 4)} \times 5$$

$$= 60.5 + (1/5) \times 5$$

$$= \mathbf{61.5}$$

Our final result is:

- Estimated Mean: **61.333...**
- Estimated Median: **61.4375**

- Estimated Mode: **61.5**

(Compare that with the true Mean, Median and Mode of **61.38..., 61 and 62** that we got at the very start.)

And that is how it is done.
Now let us look at two more examples, and get some more practice along the way!
Baby Carrots Example

Example: You grew fifty baby carrots using special soil. You dig them up and measure their lengths (to the nearest mm) and group the results:

Length (mm)	**Frequency**
150 – 154	5
155 – 159	2
160 – 164	6
165 – 169	8
170 – 174	9
175 – 179	11
180 – 184	6
185 – 189	3

Mean

Length (mm)	**Midpoint x**	**Frequency f**	**fx**
150 – 154	152	5	760
155 – 159	157	2	314
160 – 164	162	6	972
165 – 169	167	8	1336
170 – 174	172	9	1548
175 – 179	177	11	1947

180 – 184	182	6	1092
185 – 189	187	3	561
	Totals:	**50**	**8530**

$$\textbf{Estimated Mean} = \frac{8530}{50} = \textbf{170.6 mm}$$

Median

The Median is the mean of the 25^{th} and the 26^{th} length, so is in the **170 – 174** group:

- **L** = 169.5 (the lower class boundary of the 170 – 174 group)
- **n** = 50
- **B** = 5 + 2 + 6 + 8 = 21
- **G** = 9
- **w** = 5

$$\textbf{Estimated Median} = 169.5 + \frac{(50/2) - 21}{9} \times 5$$

$$= 169.5 + 2.22...$$

$$= \textbf{171.7 mm} \text{ (to 1 decimal)}$$

Mode

The Modal group is the one with the highest frequency, which is **175 – 179**:

- L = 174.5 (the lower class boundary of the 175 – 179 group)
- f_{m-1} = 9
- f_m = 11
- f_{m+1} = 6
- w = 5

$$\textbf{Estimated Mode} = 174.5 + \frac{11 - 9}{(11 - 9) + (11 - 6)} \times 5$$

$$= 174.5 + 1.42...$$

$$= \textbf{175.9 mm} \text{ (to 1 decimal)}$$

Age Example

Age is a special case.

When we say "Sarah is 17" she stays "17" up until her eighteenth birthday.

She might be 17 years and 364 days old and still be called "17".

This changes the midpoints and class boundaries.

Example: The ages of the 112 people who live on a tropical island are grouped as follows:

Age	**Number**
0 – 9	20
10 – 19	21
20 – 29	23
30 – 39	16
40 – 49	11
50 – 59	10
60 – 69	7
70 – 79	3
80 – 89	1

A child in the first group **0 – 9** could be almost 10 years old. So the midpoint for this group is **5** not 4.5

The midpoints are 5, 15, 25, 35, 45, 55, 65, 75 and 85

Similarly, in the calculations of Median and Mode, we will use the class boundaries 0, 10, 20 etc

Mean

Age	**Midpoint x**	**Number f**	**fx**
0 – 9	5	20	100
10 – 19	15	21	315
20 – 29	25	23	575
30 – 39	35	16	560

40 – 49	45	11	495
50 – 59	55	10	550
60 – 69	65	7	455
70 – 79	75	3	225
80 – 89	85	1	85
	Totals:	**112**	**3360**

$$\textbf{Estimated Mean} = \frac{3360}{112} = \mathbf{30}$$

Median

The Median is the mean of the ages of the 56th and the 57th people, so is in the 20 – 29 group:

- **L** = 20 (the lower class boundary of the class interval containing the median)
- **n** = 112
- **B** = 20 + 21 = 41
- **G** = 23
- **w** = 10

$$\textbf{Estimated Median} = 20 + \frac{(112/2) - 41}{23} \times 10$$
$$= 20 + 6.52\ldots$$
$$= \mathbf{26.5} \text{ (to 1 decimal)}$$

Mode

The Modal group is the one with the highest frequency, which is 20 – 29:

- $L = 20$ (the lower class boundary of the modal class)
- $f_{m-1} = 21$
- $f_m = 23$
- $f_{m+1} = 16$
- $w = 10$

$$\textbf{Estimated Mode} = 20 + \frac{23 - 21}{(23 - 21) + (23 - 16)} \times 10$$

$$= 20 + 2.22\ldots$$

$$= \mathbf{22.2} \text{ (to 1 decimal)}$$

Summary

For grouped data, we cannot find the exact Mean, Median and Mode, we can only give estimates.

To estimate the **Mean** use the **midpoints** of the class intervals.

$$\textbf{Estimated Median} = L + \frac{(n/2) - B}{G} \times w$$

where:

- **L** is the lower class boundary of the group containing the median
- **n** is the total number of data
- **B** is the cumulative frequency of the groups before the median group
- **G** is the frequency of the median group
- **w** is the group width

$$\textbf{Estimated Mode} = L + \frac{f_m - f_{m-1}}{(f_m - f_{m-1}) + (f_m - f_{m+1})} \times w$$

where:

- L is the lower class boundary of the modal group
- f_{m-1} is the frequency of the group before the modal group
- f_m is the frequency of the modal group
- f_{m+1} is the frequency of the group after the modal group
- w is the group width

ASSESSMENT

Given the following date 43, 47, 54, 57, 72, 82, 59, 65, 61, 62, 53, 55, 6O, 7O, 64, 56, 58, 58, 62, 62, 68, 65, 56, 59, 68, 61, 67, 65, 76. Find the;

1. Mean
2. Median
3. Mode
4. Range

WEEK 8

SS 2 Third Term Mathematics

Topic: PROBLEM SOLVING ON NUMBER BASES EXPANSION, CONVERSION AND RELATIONSHIP

Converting from base b to base 10

The next natural question is: how do we convert a number from another base into base 10? For example, what does 4201_5 mean? Just like base 10, the first digit to the left of the decimal place tells us how many 5^0's we have, the second tells us how many 5^1's we have, and so forth.
Therefore:

$4201_5 = (4.5^3 + 2.5^2 + 0.5^1 + 1.5^0)_{10}$

$= 4.125 + 2.25 + 1$

$=551_{10}$

From here, we can generalize. Let $x =(a_n a_{n-1} \ldots a_1 a_0)_b$ be an n + 1 -digit number in base b. In our example (2746_{10}) $a_3 = 2$, $a_2 = 7$, $a_1 = 4$ and $a_0 = 6$.
We convert this to base 10 as follows:

$x = (a_n a_{n-1} \ldots a_1 a_0)_b$

$= (b^n.a_n + b^{n-1}. a_{n-1} + \ldots + b.a_1 + a_0)_{10}$

Converting from base 10 to base b

It turns out that converting from base 10 to other bases is far harder for us than converting from other bases to base 10. This shouldn't be a suprise, though. We work in base 10 *all the time* so we are naturally less comfortable with other bases. Nonetheless, it is important to understand how to convert from base 10 into other bases.
We'll look at two methods for converting from base 10 to other bases.

Method 1

Let's try converting 1000 base 10 into base 7. Basically, we are trying to find the solution to the equation $1000 = a_0 + 7a_1 + 49a_2 + 343a_3 + 2401a_4 + \ldots$

where all the are digits from 0 to 6. Obviously, all the a_i from a_4 and up are 0 since otherwise they will add in a number greater than 1000, and all the terms in the sum are nonnegative. Then, we wish to find the largest a_3 such that $343a_3$ does not exceed 1000. Thus, $a_3 = 2$ since $2a_3 = 686$ and $3a_3 = 1029$.This leaves us with $1000 = a_0 + 7a_1 + 7a_1 + 49a_2 + 343(2) \leftrightarrow 314 = a_0 + 7a_1 + 49a_2$.

Using similar reasoning, we find that $a_2 = 6$, leaving us with $20 = a_0 + 7a_1$. We use the same procedure twice more to get that $a_1 = 2$ and $a_0 = 6$. Finally, we have that $1000_{10} = 2626_7$.

An alternative version of method 1 is to find the "digits" $a_0, a_1, \ldots$ starting with a_0. Note that is just the remainder of division of 1000 by 7. So, to find it, all we need to do is to carry out one division with remainder. We have 1000:7 = 142(R6). How do we find a_1, now? It turns out that all we need to do is to find the remainder of the division of the quotient 142 by 7: 142:7 = 20(R2), so $a_1 = 2$. Now, 20:7 = 2(R6), so $a_3 = 6$. Finally, 2:7 = 0(R2), so . We may continue to divide beyond this point, of course, but it is clear that we will just get 0:7 = 0(R0) during each step.

Note that both versions of this method use computations in base 10. It's often a good idea to double check by converting your answer back into base 10, since this conversion is easier to do. We know that $2626_7 = 343.2 + 6.49 + 2.7 = 1000$, so we can rest assured we got the right answer.

Method 2

We'll exhibit the second method with the same problem used to exhibit the first method.

The second method is just like how we converted from other bases into base 10. To do this, we pretend that our standard number system is base 7. In base 7, however, there is no digit 7. So 7 is actually represented as "10." Also, the multiplication rules we know do not hold. For example, $3.3 \neq 9$ (in base 7). For one, there is no 9 in base 7. Second, we need to go back to the definition of multiplication to fully understand what's happening. Multiplication is a shorthand for repeated addition. So, $3.3 = 3 + 3 + 3 = 12_7$.

In base 7, we have that 10 (the decimal number 10) is 13. Thus, if we view everything from base 7, we are actually converting 1000_{13} to base 10. So, this is just 13^3. Remember that we aren't doing this in our regular decimal system, so $13^3 \neq 2197$. Instead, we have to compute 13 x 13 x 13 as (13 x 13) x 13 = 202 x 13 = 2626.

This method can be *very* confusing unless you have a very firm grasp on the notion of number systems.

Binary and Hexadecimal

Two of the most common number bases are binary (base 2) and hexadecimal (base 16). Both of these are used in computer science. A binary number consists entirely of 0s and 1s. Each binary digit is called a bit. A base 16 number requires additional symbols after 9 since any positive integer less than 16 is a single digit. The standard notation is to use the letters a=10,b=11,c=12,d=13,e=14,f=15. In order to indicate that a number is written in hexadecimal, the prefix 0x is used. For instance, 0x95 is the same as 95_{16} and is equivalent to $9 \times 16^1 + 5 = 149$.

Examples

1. Convert the binary number 1111 1110 to decimal. (A space is often inserted every 4 digits to improve readability of binary numbers. A group of 4 digits (bits) is called a nible and a group of 8 bits is called a byte)
We could simply add all of the digit values: 0 x 1 + 1 x 2 + 1 x 4 + 1 x 8 + 1 x 16 + 1 x 32 + 1 x 64 + 1 x 128. But, there is a much shorter way. Note that in base 10, $99 = 10^2 - 1$ and $999 = 10^3 - 1$. Similarly, in base 2, 11 = 22 – 1, 111 = 23 – 1, and so on. Therefore, 1111 1111 = 28 – 1 = 255. The number we are trying to convert is one less, so it must be 254.
2. Convert the hexadecimal number 0xA7 into decimal.
Since A means ten and the A is in the "16s place," we have 0x A7 = 10 x 16 + 7 = 167
3. Convert 700 into hexadecimal. $16^2 = 256$, so 700 will require three digits. Using Method 1 above, we want to solve:
$700 = a_0 + 16a_1 + 256a_2$
$700/256 = 2$ remainder 188, so $a_2 = 2$

$188 = a_0 + 16a_1$

$188/16 = 11$ remainder 12, so $a_1 = 11$ and $a_0 = 12$ we have

$700 = 12 + 16 \times 11 + 256 \times 2$

Using standard hexadecimal notation, we can write

700 = 0x2BC

Numeral systems conversion table

DecimalBase-10	BinaryBase-2	OctalBase-8	HexadecimalBase-16
0	0	0	0
1	1	1	1
2	10	2	2
3	11	3	3
4	100	4	4
5	101	5	5
6	110	6	6
7	111	7	7
8	1000	10	8
9	1001	11	9
10	1010	12	A
11	1011	13	B
12	1100	14	C
13	1101	15	D
14	1110	16	E
15	1111	17	F
16	10000	20	10
17	10001	21	11
18	10010	22	12
19	10011	23	13
20	10100	24	14
21	10101	25	15
22	10110	26	16

23	10111	27	17
24	11000	30	18
25	11001	31	19
26	11010	32	1A
27	11011	33	1B
28	11100	34	1C
29	11101	35	1D
30	11110	36	1E
31	11111	37	1F
32	100000	40	20

ASSESSMENT

1. Convert the following to base 10; 11011, 100011, 101010, 1110111
2. Convert the following to base 2; $549_{2,}$ $6219_{2,}$ $7543_{2,}$ $1397_{2,}$ $5463_{2,}$ 6741_{2}

WEEK 9

SS 2 Third Term Mathematics
Topic: INDICIAL EQUATION

Concept and Relationship with Quadratic Equation
Exponential or Indicial Equation is a combination of indices and all other forms of equations, it is very easy to solve provided you have excellent knowledge of the laws of Indices.

Rules for Solving Exponential (Indicial) Equations
1. The two sides i.e LHS and RHS of the equation must be expressed in index form.
2. The two sides of the equation must also have the same values for you to cancel them out.
3. You'll always solve for an unknown value which can be represented by any letter of the alphabet.

Note
You will need to master all the laws of indices, if you must properly understand exponential equations.

Examples
1. If $3^2 = 3^2$, find x.

Solution
$3^2 = 3^2$

Step 1
Note that the equations above are already expressed in index form, so just cancel the similar ones out;
3 cancels 3.
x = 2.
2. If $2^{x+1} = 2^3$, find x.

Solution
$2^{x+1} = 2^3$

Step 1
The equation is already in index form, so just cancel out the similar ones;
2 cancels 2;
$x + 1 = 3$

Step 2
Make x the subject by carrying +1 to the RHS;
$x = 3 - 1$
$x = 2$.
3. If $3^x = 9$, solve for x.

Solution
$3^x = 9$

Step 1
Express 9 in index form; expressing 9 in index form is 3^2 which gives (3 3).
$3^2 = 3^2$

Step 2
3 cancels 3;
$x = 2$.
4. $16^x = 0.125$. Solve for x.

Solution
$16^x = 0.125$

Step 1
Express 0.125 in fraction by carrying the points out;
$16^x = 125 / 1000$

Step 2
Cancellation Process;
5 goes in 125 gives 25 while 5 goes in 1000 gives 200;
$16^x = 25 / 200$
Cancellation Process;
5 goes in 25 gives 5 while 5 goes in 200 gives 40;
$16^x = 5 / 40$
Cancellation Process;
5 goes in 5 gives 1 while 5 goes in 40 gives 8;
$16^x = 1 / 8$

Step 3
Apply Negative index law to the fraction at the right;
$16^x = 8^{-1}$

Step 4
Take both values to index form;
$2^{4(x)} = 2^{3(-1)}$

Step 5
2 cancels 2;
4(x) = 3(-1)

Step 6
Remove the brackets;
4x = -3

Step 7
Divide both sides by 4;
4x/4 = -3/4
Cancellation Process;
4 cancels 4;
x = -3 / 4
5. $8(4^x) = 32$, solve for x.

Solution

$8(4^x) = 32$

Step 1

Remove the bracket at the LHS;

$8 \times 4^x = 32$

$32^x = 32$

Step 2

32 cancels 32;

$x = 1$.

6. $2^{x(x-3)} = 1/4$ Solve for x.

Solution

$2^{x(x-3)} = 1/4$

Step 1

Apply negative index law to the fraction at the RHS;

$2^{x(x-3)} = 4^{-1}$

Step 2

Express the value 4^{-1} in index from;

$2^{x(x-3)} = 2^{2(-1)}$

Step 3

2 cancels 2;

$x(x - 3) = 2(-1)$

Step 4

Remove the brackets;

$x^2 - 3x = -2$

Step 5
Rearrange;
$x^2 - 3x + 2 = 0$. —- quadratic equation.
As you can see, this is a quadratic equation. So let's factorise!
$x^2 - 3x + 2 = 0$
$+2x^2$ (x)
$-3x$ (+)
($-2x$ & $-1x$)
$x^2 - 2x - 1x + 2 = 0$
$x(x - 2) - 1(x - 2) = 0$
$(x - 2)(x - 1) = 0$
$(x - 2) = 0$ OR $(x - 1) = 0$
$x - 2 = 0$ OR $x - 1 = 0$
$x = 0 + 2$ OR $x = 0 + 1$
$x = 2$ OR $x = 1$.
7. $2^{2(x-3)} = 1$. Solve for x.

Solution
$2^{2(x-3)} = 1$

Step 1
Apply zero index law to 1;
$2^{2(x-3)} = 2^0$

Step 2
2 cancels 2;
$2(x - 3) = 0$

Step 3
Remove bracket;
$2x - 6 = 0$

Step 4
Rearrange;
$2x = 0 + 6$
$2x = 6$

Step 5
Divide both sides by 2;
$2x/2 = 6/2$
Cancellation Process;
2 cancels 2 while 2 goes in 6 gives 3;
$x = 3$.

ASSESSMENT
Solve the following equations;
i. $27^x = 81$
ii. $32^x = 0.0625$
iii. $4^{6(x-3)} = 1$
iv. $16(4^x) = 64$
v. $9^x / 27 = 81$
vi. $4^{x+2} = 8^{x-1}$
vii. $8^x = 0.25$
viii. $2^x(1/4) = 16$
ix. $2^x = 16$
x. $2^{3x-1} = 4^{x+3}$