

1

2

## Generative Thermodynamic Computing

Stephen Whitelam<sup>\*</sup>

Molecular Foundry, Lawrence Berkeley National Laboratory, 1 Cyclotron Road, Berkeley, California 94720, USA

5

(Received 25 June 2025; accepted 14 November 2025)

6

7 We introduce a generative modeling framework for thermodynamic computing, in which structured data  
 8 are synthesized from noise by the natural time evolution of a physical system governed by Langevin  
 9 dynamics. While conventional diffusion models use neural networks to perform denoising, here the  
 10 information needed to generate structure from noise is encoded by the dynamics of a thermodynamic  
 11 system. Training proceeds by maximizing the probability with which the computer generates the reverse  
 12 of a noising trajectory, which ensures that the computer generates data with minimal heat emission. We  
 13 demonstrate this framework within a digital simulation of a thermodynamic computer. If realized in analog  
 14 hardware, such a system would function as a generative model that produces structured samples without the  
 15 need for artificially injected noise or active control of denoising.

16

DOI: 10.1103/kwy-1xln

17 *Introduction*—In this Letter we describe a generative  
 18 modeling framework for thermodynamic computing.  
 19 Thermodynamic computing is closely related to the field  
 20 of probabilistic computing [1–3]. It is based on the ideas that  
 21 we can do energy-efficient computation by using small  
 22 physical devices whose microscopic states change with time  
 23 in response to thermal fluctuations, and that the fluctuations  
 24 of a suitably designed device can encode the outcome of a  
 25 desired calculation [4–8]. Here, we show that thermodynamic  
 26 computers can perform generative modeling in a  
 27 manner analogous to diffusion models, with key differences.

28 In a diffusion model, structured inputs are degraded by  
 29 the gradual addition of noise. A neural network is trained  
 30 to enact the reverse process, allowing the generation of  
 31 structure from noise [9–12]. This process is implemented  
 32 on a digital computer, where noise is introduced in the form  
 33 of artificially generated pseudorandom numbers. Here,  
 34 we use analytic calculations and digital simulations to  
 35 suggest an alternative approach, in which the noise-driven  
 36 dynamics of a thermodynamic computer—the noise  
 37 arising naturally from the system's interaction with its  
 38 environment—generates structure from noise. If realized in  
 39 analog hardware, such a system would generate structured  
 40 outputs simply by evolving with time under its natural  
 41 dynamics. It would not require added pseudorandom noise,  
 42 or the guidance of a digital neural network.

43 In more detail, we consider a model thermodynamic  
 44 computer, a set of fluctuating nonlinear degrees of freedom  
 45 coupled by bilinear interactions. The computer's degrees  
 46 of freedom evolve according to overdamped Langevin  
 47 dynamics. This design is inspired by existing hardware  
 that can perform linear algebra [7], and by our recent work

48 showing that a nonlinear version of such hardware  
 49 can function as the thermodynamic version of a neural  
 50 network [13]. We provide input to the computer to make it  
 51 display images of digits from the MNIST data set [14], and  
 52 allow these images to degrade by running the dynamics  
 53 of the computer with its interunit couplings set to zero.  
 54 Such degradation is called *noising* in the diffusion model  
 55 literature. As we do so, we compute from the Langevin  
 56 equation the probability that a computer with hypothetical  
 57 nonzero couplings would have generated the *reverse* of this  
 58 noising trajectory, and we adjust the values of these  
 59 hypothetical couplings by gradient descent in order to  
 60 maximize that probability. After running several such  
 61 noising trajectories, we construct a denoising computer  
 62 using the trained couplings, and verify that its natural  
 63 dynamics, starting from noisy initial conditions, leads to the  
 64 generation of structured MNIST-like digits. Independent  
 65 dynamical trajectories of the same computer produce a  
 66 variety of outcomes, some of which are not contained in the  
 67 training set.

68 In this approach the denoising dynamics is encoded by  
 69 the couplings of the trained thermodynamic computer,  
 70 which plays the role of a denoising neural network in a  
 71 diffusion model. If realized in analog hardware—for  
 72 example, using networks of mechanical [15], electrical [8],  
 73 or superconducting [16] oscillators—the information  
 74 required for denoising would be encoded in the energy  
 75 landscape of the computer, rather than in a digital neural  
 76 network. As a result, denoising would not be simulated, but  
 77 physically enacted.

78 In this mode of operation the thermodynamic computer  
 79 resembles a nonequilibrium, continuous-spin analog of a  
 80 Boltzmann machine, a statistical mechanical model that  
 81 represents probability distributions over binary variables  
 82 [17,18]. The key difference is that a Boltzmann machine

<sup>\*</sup>Contact author: swhitelam@lbl.gov

83 encodes information in its equilibrium Boltzmann distribution, whereas our device runs on a physical clock: the  
 84 computation is carried out by the dynamics of the system at a designated time, with no requirement to attain equilibrium.  
 85 We therefore refer to the device considered here, which uses Langevin trajectories to perform a calculation,  
 86 as a *Langevin computer*.

87 We also show that the training process has a direct  
 88 physical interpretation: it adjusts the computer's couplings  
 89 in order to minimize the thermodynamic irreversibility of  
 90 the generative process. By finding the computer most likely  
 91 to have generated the reverse of a noising trajectory, we  
 92 minimize the expected heat emission and entropy produc-  
 93 tion of the denoising computer. Our results therefore link  
 94 the design of generative thermodynamic models to funda-  
 95 mental physical principles, and broaden our understanding  
 96 of the capabilities of thermodynamic computers.

97 *Training a generative Langevin computer*—Consider a  
 98 model of a thermodynamic computer. The computer is  
 99 composed of  $N$  classical, real-valued fluctuating degrees  
 100 of freedom  $\mathbf{x} = \{x_i\}$ , which could represent voltage states  
 101 in electrical circuits [8], oscillator positions in a mechanical  
 102 system [15], or phases in Josephson junction devices  
 103 [16,19]. The computer's units  $x_i$  evolve in time according  
 104 to the overdamped Langevin dynamics

$$\dot{x}_i = -\mu\partial_i V_{\theta}(\mathbf{x}) + \sqrt{2\mu k_B T}\eta_i(t). \quad (1)$$

105 Here,  $\mu$  is the mobility parameter, which sets the basic time  
 106 constant of the computer. For the thermodynamic com-  
 107 puters of Refs. [7,8],  $\mu^{-1}$  is of order a microsecond. For  
 108 damped oscillators made from mechanical elements [15] or  
 109 Josephson junctions [16,19],  $\mu^{-1}$  is of order a millisecond or a  
 110 nanosecond, respectively. The first term on the right-  
 111 hand side of Eq. (1) is the force arising from the computer's  
 112 potential energy  $V_{\theta}(\mathbf{x})$ , given a set of parameters (couplings  
 113 and biases)  $\theta = \{J_{ij}\}, \{b_i\}$ ; note that  $\partial_i \equiv \partial/\partial x_i$ . The  
 114 second term on the right-hand side of Eq. (1) models  
 115 temporally uncorrelated thermal fluctuations:  $k_B T$  is the  
 116 thermal energy scale, and the Gaussian white noise terms  
 117 satisfy  $\langle \eta_i(t) \rangle = 0$  and  $\langle \eta_i(t)\eta_j(t') \rangle = \delta_{ij}\delta(t-t')$ .

118 The potential energy  $V_{\theta}(\mathbf{x})$  of the computer is

$$V_{\theta}(\mathbf{x}) = \sum_{i=1}^N (J_2 x_i^2 + J_4 x_i^4) + \sum_{i=1}^N b_i x_i + \sum_{(ij)} J_{ij} x_i x_j. \quad (2)$$

123 The first sum in Eq. (2), which runs over the  $N$  units, sets  
 124 the intrinsic couplings of the computer. For  $J_4 = 0$  we have  
 125 a linear model [7], whose unit activations are linear  
 126 functions of their inputs, while for  $J_4 > 0$ , the case we  
 127 consider, we have a nonlinear model that can act as the  
 128 thermodynamic analog of a neural network [13] (positive  
 129  $J_4$  also ensures the thermodynamic stability of the  
 130 computer as the  $J_{ij}$  are adjusted). We consider the case  $J_2 > 0$ ,

132 which creates units with one stable state, analogous to the  
 133  $s$ -units of thermodynamic computing [8]. The alternative  
 134 choice,  $J_2 < 0$ , creates bistable units, analogous to the  
 135  $p$ -spins in the field of probabilistic computing [1].

136 The remaining terms in (2) contain the trainable param-  
 137 eters of the computer. The parameters  $b_i$  are input signals or  
 138 biases applied to each unit. The parameters  $J_{ij}$  are pairwise  
 139 couplings between units, inspired by the bilinear inter-  
 140 actions of the thermodynamic computers of Refs. [7,8],  
 141 with the sum running over all interunit connections.  
 142 Equation (2) describes a thermodynamic computer of  
 143 arbitrary connectivity, and the following discussion applies  
 144 to the same.

145 Imagine that we observe a dynamical trajectory of the  
 146 computer at a series of discrete times,  $\omega = \{\mathbf{x}(t_k)\}_{k=0}^K$ ,  
 147 where  $t_k = k\Delta t$ . The probability that any step of this  
 148 trajectory was generated by a thermodynamic computer  
 149 with parameters  $\theta$  can be calculated from the Onsager-  
 150 Machlup action associated with the Langevin equation  
 151 [20,21]. A time-discretized version of this action can be  
 152 derived by first considering a standard Euler integration  
 153 scheme for Eq. (1),

$$x_i(t + \Delta t) = x_i(t) - \mu\partial_i V_{\theta}(\mathbf{x})\Delta t + \sqrt{2\mu k_B T\Delta t}\eta_i. \quad (3)$$

154 Here,  $\Delta t$  is the integration time step, and  $\eta_i$  is a Gaussian  
 155 random variable with zero mean and unit variance. Writing  
 156  $\Delta x_i \equiv x_i(t + \Delta t) - x_i(t)$ , we can rearrange (3) to read

$$\eta_i = \frac{\Delta x_i + \mu\partial_i V_{\theta}(\mathbf{x})\Delta t}{\sqrt{2\mu k_B T\Delta t}}. \quad (4)$$

157 Next, note that the probability of generating the step  
 158  $\mathbf{x} \rightarrow \mathbf{x} + \Delta\mathbf{x}$  is that of drawing  $N$  noise values  $\eta_i$ ,

$$P_{\theta}^{\text{step}}(\Delta\mathbf{x}) = (2\pi)^{-N/2} \prod_{i=1}^N \exp(-\eta_i^2/2), \quad (5)$$

161 with the  $\eta_i$  given by Eq. (4). Hence, the negative log-  
 162 probability that a computer with parameters  $\theta$  generated the  
 163 step  $\mathbf{x} \rightarrow \mathbf{x} + \Delta\mathbf{x}$  is

$$-\ln P_{\theta}^{\text{step}}(\Delta\mathbf{x}) = \sum_{i=1}^N \frac{[\Delta x_i + \mu\partial_i V_{\theta}(\mathbf{x})\Delta t]^2}{4\mu k_B T\Delta t}, \quad (6)$$

166 up to an unimportant constant term. The negative log-  
 167 probability that a computer with parameters  $\theta$  would  
 168 generate the *reverse* step,  $\mathbf{x} + \Delta\mathbf{x} \equiv \mathbf{x}' \rightarrow \mathbf{x}$ , is

$$-\ln \tilde{P}_{\theta}^{\text{step}}(\Delta\mathbf{x}) = \sum_{i=1}^N \frac{[-\Delta x_i + \mu\partial_i V_{\theta}(\mathbf{x}')\Delta t]^2}{4\mu k_B T\Delta t}. \quad (7)$$

169 To increase the probability with which the computer  
 170 would have generated the entire reverse trajectory

172  $\tilde{\omega} = \{\mathbf{x}(t_{K-k})\}_{k=0}^K$ , we can sum Eq. (7) over all steps of the  
 173 trajectory, differentiate that expression with respect to each  
 174 parameter of the computer, and update the parameters as

$$J_{ij} \rightarrow J_{ij} + \alpha \sum_{k=1}^K \frac{\partial}{\partial J_{ij}} \ln \tilde{P}_{\theta}^{\text{step}}[\Delta \mathbf{x}(t_k)], \quad (8)$$

$$b_i \rightarrow b_i + \alpha \sum_{k=1}^K \frac{\partial}{\partial b_i} \ln \tilde{P}_{\theta}^{\text{step}}[\Delta \mathbf{x}(t_k)], \quad (9)$$

178 where  $\alpha$  is a learning rate. Recall that  $\Delta \mathbf{x}(t_k)$  is the  
 179 displacement generated at time step  $k$  in the observed  
 180 (forward) trajectory. The gradient terms in (8) and (9) can  
 181 be calculated analytically, and are

$$\begin{aligned} -\frac{\partial}{\partial J_{ij}} \ln \tilde{P}_{\theta}^{\text{step}}(\Delta \mathbf{x}) &= \frac{-\Delta x_i + \mu \partial_i V_{\theta}(\mathbf{x}') \Delta t}{2k_B T} x_j \\ &+ \frac{-\Delta x_j + \mu \partial_j V_{\theta}(\mathbf{x}') \Delta t}{2k_B T} x_i, \end{aligned} \quad (10)$$

183 and

$$-\frac{\partial}{\partial b_i} \ln \tilde{P}_{\theta}^{\text{step}}(\Delta \mathbf{x}) = \frac{-\Delta x_i + \mu \partial_i V_{\theta}(\mathbf{x}') \Delta t}{2k_B T}, \quad (11)$$

184 where

$$\partial_i V_{\theta}(\mathbf{x}) = 2J_2 x_i + 4J_4 x_i^3 + b_i + \sum_{j \in \mathcal{N}(i)} J_{ij} x_j. \quad (12)$$

185 Here,  $\mathcal{N}(i)$  denotes the set of units connected to unit  $i$ .

186 When the forward trajectories depict noising processes,  
 187 training over many such trajectories identifies couplings  
 188 that allow a thermodynamic computer to transform noise  
 189 into structured data.

190 *Numerical illustration of generative thermodynamic  
 191 computing*—To illustrate this result we carried out a  
 192 digital simulation of a thermodynamic computer. We set  
 193  $J_2 = J_4 = 10k_B T$ , and consider a computer with  $N_v = 28^2$   
 194 visible units and  $N_h = 512$  hidden units. The visible units  
 195 will be used as a display, and the hidden units used to do  
 196 computation. The computer has  $N_v N_h$  trainable couplings  
 197  $J_{ij}$  between visible and hidden units, and  $N_h(N_h - 1)/2$   
 198 trainable couplings between hidden units. Hidden units  
 199 have trainable biases  $b_i$ . Visible units have no trainable  
 200 biases, but during training we impose visible-unit biases  
 201  $b_i \propto P_i$ , where  $P_i$  denotes the  $i^{\text{th}}$  pixel of an MNIST digit  
 202 (each digit's pixels were adjusted to have zero mean and  
 203 unit variance). We display the visible units in a  $28 \times 28$   
 204 grid, matching the presentation of an MNIST digit.

205 To construct noising trajectories we project an MNIST  
 206 image onto the visible units, via their nontrainable biases.  
 207 We project part of the same image onto the hidden-unit  
 208



FIG. 1. (a) Example noising trajectory. (b) The remaining digits used in the training set.

210 biases, in order to provide them with some signal, and set  
 211 all couplings  $J_{ij}$  to zero. We then let the computer come to  
 212 equilibrium, by simulating Eq. (1) for a sufficiently long  
 213 time. We then run a dynamical trajectory of time  $t_f = 2.5$ ,  
 214 slowly diminishing the intensity of the imposed digit. The  
 215 result is an image that becomes increasingly noisy, as  
 216 shown in Fig. 1(a). Positive values of the unit activations  $x_i$   
 217 are shown blue, while negative values are shown white.

218 As we run each noising trajectory, we update Eqs. (8)  
 219 and (9). This process, repeated over many trajectories,  
 220 identifies the parameters  $\theta$  of the computer that would,  
 221 with maximum likelihood, generate the reverse of the  
 222 trajectory, and so convert noise into signal. In this small-  
 223 scale example we trained the computer using only three Q1  
 224 digits, shown in Figs. 1(a) and 1(b).

225 In Fig. 2(a) we show three independent trajectories of the  
 226 thermodynamic computer trained in this way. Trajectories  
 227 begin from a noisy initial state prepared by bringing the  
 228 coupling-free thermodynamic computer to equilibrium. In  
 229 each case, the trained thermodynamic computer gradually  
 230 transforms noise into structure, illustrating its ability to  
 231 perform noise-to-structure generation. These results indicate  
 232 that the computer has internalized representations of  
 233 the digits and can reproduce them via physical evolution.  
 234 In the case of the digit “1”, the generated image appears

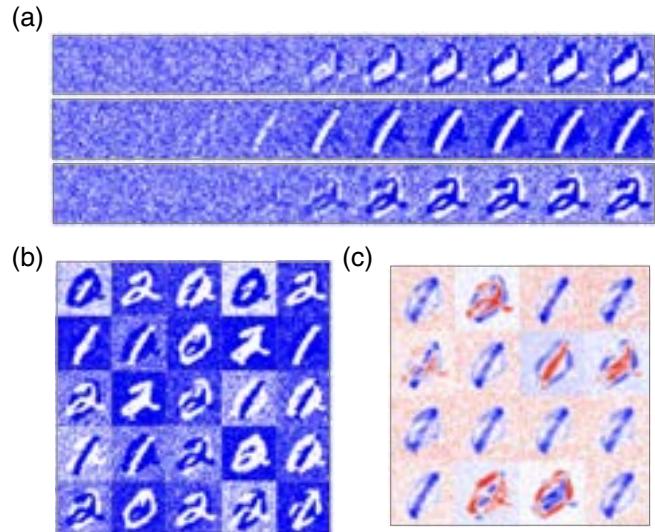


FIG. 2. (a) Three independent dynamical trajectories of the trained denoising thermodynamic computer. (b) The outcome at time  $t = t_f$  of 25 independent trajectories of the trained computer. (c) Coupling patterns between 16 representative hidden units and the visible layer.

235 inverted. This behavior reflects a breaking of the visible  
236 layer's approximate symmetry under the transformation  
237  $x_i \rightarrow -x_i$  (the digits presented to the computer have pixel  
238 values that are roughly symmetric about zero).

239 In Fig. 2(b) we show the output at time  $t = t_f$  of 25  
240 independent trajectories generated by the trained thermody-  
241 namic computer. The outputs exhibit diversity in style and  
242 form, behavior that is typical of diffusion models trained to  
243 approximate, rather than exactly replicate, the data distribu-  
244 tion. Not all outputs are necessarily desirable or clearly  
245 interpretable—some display “mode mixing,” in the language  
246 of diffusion models—but this simple example is intended as  
247 a proof of principle, a demonstration that structure can be  
248 generated from noise by physical dynamics alone.

249 Some insight into the behavior of the computer is  
250 provided by Fig. 2(c), which shows 16 of the computer's  
251 hidden units' learned couplings to the visible layer. Each  
252 panel represents the pattern of couplings  $J_{ij}$  between one  
253 hidden unit and all visible units: blue indicates positive  
254 coupling, red negative (either can be excitatory or inhibi-  
255 tory, via the interaction  $J_{ij}x_i x_j$ , depending on the values of  
256 the unit activations  $x_i$ ). These patterns act as receptive fields  
257 that determine how each hidden unit responds to features in  
258 the visible layer. Several of the units have learned localized,  
259 digitlike structures, suggesting that the computer decom-  
260 poses input patterns into interpretable visual components.  
261 Some hidden units exhibit complementary structure—  
262 activating for certain strokes while suppressing others—  
263 enabling the system to encode multiple features. Together,  
264 these receptive fields determine the energy landscape that  
265 guides the computer's dynamics, allowing the system to  
266 transform noise into structured outputs.

267 *Physical interpretation of the learning process*—The  
268 generative model we have considered is a thermodynamic  
269 system, and we can interpret the learning algorithm used  
270 to train it in physical terms. To do so, consider one step  
271 of a noising trajectory  $\omega = \{\mathbf{x}(t_k)\}_{k=0}^K$ , generated by any  
272 means. Consider the ratio of two probabilities (a standard  
273 device in nonequilibrium statistical mechanics [22]): the  
274 probability that the step was generated by a computer with  
275 a set of reference couplings (call them  $\theta = \mathbf{0}$ ), and the  
276 probability that our denoising computer (with parameters  $\theta$ )  
277 generated the reverse of that step. This ratio is

$$\begin{aligned} \ln \frac{P_{\mathbf{0}}^{\text{step}}(\Delta \mathbf{x})}{\tilde{P}_{\theta}^{\text{step}}(\Delta \mathbf{x})} &= \sum_{i=1}^N \frac{(-\Delta x_i + \mu \partial_i V_{\theta}(\mathbf{x}') \Delta t)^2}{4\mu k_B T \Delta t} \\ &\quad - \sum_{i=1}^N \frac{(\Delta x_i + \mu \partial_i V_{\theta}(\mathbf{x}) \Delta t)^2}{4\mu k_B T \Delta t} \\ &\approx - \sum_{i=1}^N \frac{\Delta x_i \partial_i V_{\theta}(\mathbf{x}) + \Delta x_i \partial_i V_{\theta}(\mathbf{x}')}{2k_B T} \\ &\approx - \frac{\Delta Q_{\mathbf{0}} + \Delta Q_{\theta}}{2k_B T}, \end{aligned} \quad (13)$$

278 to leading order in  $\Delta t$ . Here,  $\Delta Q_{\mathbf{0}}$  and  $\Delta Q_{\theta}$  denote the  
279 incremental heat dissipated during the forward step by  
280 the reference and trained computers, respectively. Over the  
281 entire trajectory, in the limit  $\Delta t \rightarrow 0$ , we have  
282

$$\ln \frac{P_{\mathbf{0}}[\omega]}{P_{\theta}[\tilde{\omega}]} = -\frac{1}{2} [\beta Q_{\mathbf{0}}(\omega) + \beta Q_{\theta}(\omega)]. \quad (14)$$

283 Here,  $\beta \equiv 1/(k_B T)$ , and  $P_{\mathbf{0}}[\omega]$  and  $P_{\theta}[\tilde{\omega}]$  denote the  
284 probabilities of observing the forward trajectory with  
285 the reference computer, and the reverse trajectory with  
286 the denoising computer, respectively. The terms  $Q_{\mathbf{0}}(\omega)$  and  
287  $Q_{\theta}(\omega)$  represent the total heat dissipated along the forward  
288 trajectory by the two computers.

289 Training proceeds by minimizing the negative log-  
290 probability  $-\ln P_{\theta}[\tilde{\omega}]$ . According to the fluctuation  
291 relation (14), and given that the reference process is fixed,  
292 this is equivalent to minimizing  $-Q_{\theta}(\omega)$ , the negative total  
293 heat dissipated by the denoising computer when generating  
294 the noising trajectory  $\omega$ . Given that heat changes sign upon  
295 time reversal, the learning process therefore minimizes the  
296 heat  $Q_{\theta}(\tilde{\omega}) = -Q_{\theta}(\omega)$  emitted by the trained computer  
297 along the trajectory  $\tilde{\omega}$ , i.e., as it generates structure from  
298 noise. In this sense, the trained dynamics is thermody-  
299 nically optimal: it is the dynamics that reconstructs  
300 the imposed data with the least heat emitted or entropy  
301 produced.

302 *Thermodynamic advantage*—We can estimate the  
303 thermodynamic advantage, the ratio of the energy costs  
304 of digital to thermodynamic computation, by considering the  
305 energy scales of denoising using a digital neural network and  
306 a hardware version of our simulated thermodynamic com-  
307 puter. The basic energy scale of a digital neural network is  
308 set by a multiply accumulate (MAC) operation, which differs  
309 by hardware implementation but is typically about 1 pJ [23],  
310 or  $2.4 \times 10^8 k_B T$  at room temperature. For a modest multi-  
311 layer perceptron denoiser ( $784 \rightarrow 128 \rightarrow 128 \rightarrow 784$ ),  
312 a single denoising step requires  $\sim 2.2 \times 10^5$  MACs. If we  
313 make the very conservative assumption that the denoiser is  
314 used only 10 times within a denoising trajectory, the order-  
315 of-magnitude energy budget of denoising using a neural  
316 network is not less than  $5 \times 10^{14} k_B T$ .

317 The energy cost of the thermodynamic computer is much  
318 smaller. We can calculate the heat emitted by the computer  
319 from the difference of the potential energy (2) between the  
320 start and end of a trajectory,  $Q = V_{\theta}[\mathbf{x}(0)] - V_{\theta}[\mathbf{x}(t_f)]$  (no  
321 work is done on the computer after the hidden-unit biases  
322 are established). Over 1000 independent denoising tra-  
323 jectories of the trained computer we calculate a mean heat  
324 emission of  $\langle Q \rangle = 2.9 \times 10^3 k_B T$ , with standard deviation  
325  $3.5 \times 10^2 k_B T$ . Comparing this value to the digital estimate  
326 gives a ratio of more than  $10^{11}$ . That is, if implemented in  
327 hardware, the thermodynamic computer would be more  
328 than 10 orders of magnitude more energy efficient than  
329 a digital neural network. The example shown here is  
330

331 rudimentary by the standards of state-of-the-art diffusion  
 332 models, but shows the potential, and potential energy  
 333 savings, of thermodynamic computation.

334 *Conclusions*—We have proposed a generative modeling  
 335 framework in which structure is produced from noise by  
 336 the physical dynamics of a thermodynamic system. This  
 337 approach follows the logic of a diffusion model, but instead  
 338 of using a digital neural network and artificially injected  
 339 noise, the information required for generation is encoded in  
 340 the system’s energy landscape and emerges from a physical  
 341 dynamics. Used in this way, the thermodynamic computer  
 342 is a Langevin computer, a nonequilibrium, continuous-spin  
 343 analog of a Boltzmann machine [17,18].

344 The learning process, maximizing the likelihood that a  
 345 trained system could have produced the reverse of a noising  
 346 trajectory, admits a natural interpretation in terms of  
 347 entropy production: the model learns to reverse the forward  
 348 dynamics in the most thermodynamically reversible way.  
 349 Thermodynamic learning therefore links generative mod-  
 350 eling to fundamental physical principles.

351 Reference [24] proposed the idea of making a generative  
 352 model by controlling analog physical dynamics with a  
 353 digital neural network. Here, we have shown that analog  
 354 hardware on its own can be generative. However, having a  
 355 neural network control the couplings of the thermodynamic  
 356 computer would indeed make it more expressive: for  
 357 instance, the neural network could adjust the couplings  
 358 of the computer as a function of time, or set the computer’s  
 359 couplings so as to produce conditioned outputs.

360 We have used digital simulation to demonstrate that  
 361 nonlinear, nonequilibrium analog hardware can learn to  
 362 generate structured outputs from noise. This Letter pro-  
 363 vides another example of a thermodynamic computer  
 364 trained to operate under nonequilibrium conditions [13].  
 365 Realized physically—trained digitally, with the learned  
 366 couplings implemented in hardware—such systems could  
 367 perform autonomous generative computation without  
 368 external control or artificial randomness, opening new  
 369 avenues of exploration for physically grounded, energy-  
 370 efficient machine learning.

371 *Acknowledgments*—I thank Isaac Tamblyn and Adrienne  
 372 Zhong for discussions. This work was done at the  
 373 Molecular Foundry, supported by the Office of Science,  
 374 Office of Basic Energy Sciences, of the U.S. Department of  
 375 Energy under Contract No. DE-AC02-05CH11231, and  
 376 partly supported by U.S. DOE Office of Science Scientific  
 377 User Facilities AI/ML project “A digital twin for spatio-  
 378 temporally resolved experiments.”

380 *Data availability*—The data that support the findings of  
 381 this article are openly available [25].

382

[1] J. Kaiser and S. Datta, *Appl. Phys. Lett.* **119** (2021).  
 383 [2] N. A. Aaudit, A. Grimaldi, M. Carpentieri, L. Theogarajan,  
 384 J. M. Martinis, G. Finocchio, and K. Y. Camsari, *Natl.*  
 385 *Electron. Rev.* **5**, 460 (2022).  
 386 [3] S. Misra, L. C. Bland, S. G. Cardwell, J. A. C. Incorvia,  
 387 C. D. James, A. D. Kent, C. D. Schuman, J. D. Smith, and  
 388 J. B. Aimone, *Adv. Mater.* **35**, 2204569 (2023).  
 389 [4] T. Conte, E. DeBenedictis, N. Ganesh, T. Hylton, G. E.  
 390 Crooks, and P. J. Coles, *arXiv:1911.01968*.  
 391 [5] T. Hylton, *Entropy* **22**, 256 (2020).  
 392 [6] G. Wimsatt, Olli P. Saira, Alexander B. Boyd, Matthew H.  
 393 Matheny, S. Han, Michael L. Roukes, and James P.  
 394 Crutchfield, *Phys. Rev. Res.* **3**, 033115 (2021).  
 395 [7] M. Aifer, K. Donatella, M. H. Gordon, S. Duffield, T. Ahle,  
 396 D. Simpson, G. Crooks, and P. J. Coles, *npj Unconv.*  
 397 *Comput.* **1**, 13 (2024).  
 398 [8] D. Melanson, M. Abu Khater, M. Aifer, K. Donatella, M.  
 399 Hunter Gordon, T. Ahle, G. Crooks, A. J. Martinez, F.  
 400 Sbahi, and P. J. Coles, *Nat. Commun.* **16**, 3757 (2025).  
 401 [9] J. Sohl-Dickstein, E. Weiss, N. Maheswaranathan, and S.  
 402 Ganguli, in *International Conference on Machine Learning*  
 403 (PMLR, 2015), pp. 2256–2265.  
 404 [10] L. Yang, Z. Zhang, Y. Song, S. Hong, R. Xu, Y. Zhao, W.  
 405 Zhang, B. Cui, and M.-H. Yang, *ACM Comput. Surv.* **56**, 1  
 406 (2023).  
 407 [11] G. Biroli, T. Bonnaire, V. De Bortoli, and M. Mézard,  
 408 *Nat. Commun.* **15**, 9957 (2024).  
 409 [12] Z. Yu and H. Huang, *Phys. Rev. E* **111**, 014111 (2025).  
 410 [13] S. Whitelam and C. Casert, *arXiv:2412.17183*.  
 411 [14] Y. LeCun, C. Cortes, and C. J. Burges, MNIST leaderboard,  
 412 <http://yann.lecun.com/exdb/mnist/> (1998).  
 413 [15] S. Dago, J. Pereda, N. Barros, S. Ciliberto, and L. Bellon,  
 414 *Phys. Rev. Lett.* **126**, 170601 (2021).  
 415 [16] Kyle J. Ray and James P. Crutchfield, *Phys. Rev. Appl.* **19**,  
 416 014049 (2023).  
 417 [17] G. Hinton, in *Encyclopedia of Machine Learning and Data*  
 418 *Mining* (Springer, New York, 2017), pp. 164–168.  
 419 [18] R. Salakhutdinov and G. Hinton, in *Artificial Intelligence*  
 420 *and Statistics* (PMLR, 2009), pp. 448–455.  
 421 [19] C. Z. Pratt, K. J. Ray, and J. P. Crutchfield, *Chaos* **35**,  
 422 043112 (2025).  
 423 [20] H. Risken and H. Risken, *Fokker-Planck Equation*  
 424 (Springer, New York, 1996).  
 425 [21] L. F. Cugliandolo and V. Lecomte, *J. Phys. A* **50**, 345001  
 426 (2017).  
 427 [22] U. Seifert, *Rep. Prog. Phys.* **75**, 126001 (2012).  
 428 [23] M. Horowitz, in *Proceedings of the 2014 IEEE International*  
 429 *Solid-State Circuits Conference Digest of Technical Papers*  
 430 (ISSCC) (IEEE, New York, 2014), pp. 10–14.  
 431 [24] P. J. Coles, C. Szczepanski, D. Melanson, K. Donatella,  
 432 A. J. Martinez, and F. Sbahi, in *Proceedings of the 2023*  
 433 *IEEE International Conference on Rebooting Computing*  
 434 (ICRC) (IEEE, New York, 2023), pp. 1–10.  
 435 [25] S. Whitelam, The code used in this Letter can be found at the  
 436 GitHub repository  $\theta$ , 2025, [https://github.com/swhitelam/generative\\_thermodynamic\\_computing](https://github.com/swhitelam/generative_thermodynamic_computing).  
 437

Q3