Complete Solutions to Examination Questions 1

1. Expanding the given quadratic by using FOIL we have

$$(2x-5)(x-2)=\underbrace{2x^2}_{\text{F}}-\underbrace{4x}_{\text{O}}-\underbrace{5x}_{\text{I}}+\underbrace{10}_{\text{L}}$$
$$=2x^2-9x+10$$

2. Writing the square root as an index ½ we have

$$\frac{a^3\sqrt{b^5}}{a^{-1/3}b^{3/5}}=\frac{a^3b^{5/2}}{a^{-1/3}b^{3/5}}$$
$$=a^{3-(-1/3)}b^{5/2-3/5}$$
$$=a^{10/3}b^{25/10-6/10}=a^{10/3}b^{19/10}$$

3. Substituting $v=3.5$, $u=2.3$ and $a=-9.8$ into the given equation $v^2=u^2+2as$:

$$3.5^2=2.3^2+\left[2\times(-9.8)\times s\right]$$
$$12.25=5.29-19.6s$$
$$19.6s=5.29-12.25 \quad \text{gives} \quad s=\frac{5.29-12.25}{19.6}=-0.3551$$

4. *How do we solve the given simultaneous equations?*

$$2x + 3y = 14 \qquad (1)$$
$$5x - 2y = 25.5 \qquad (2)$$

Multiply equation (1) by 2 and equation (2) by 3:

$$4x + 6y = 28 \qquad (3)$$
$$15x - 6y = 76.5 \qquad (4)$$

Adding equations (3) and (4) yields:

$$19x=104.5$$

Dividing through by 19 gives $x=5.5$. Substituting $x=5.5$ into equation (1):

$$2(5.5) + 3y = 14 \quad \text{which gives} \quad 11 + 3y = 14$$

Solving $11 + 3y = 14$ gives $y=1$. Hence our solution is $x=5.5$ and $y=1$.

5. Multiplying $\frac{2}{x}+\frac{3}{x+1}=\frac{4}{x}$ through by $x(x+1)$:

$$2(x+1)+3x=4(x+1)$$
$$2x+2+3x=4x+4$$
$$2x+3x-4x=4-2 \quad \text{yields} \quad x=2$$

Our solution is $x=2$.

6. *How do we form a quadratic out of* $\frac{2}{3x}-\frac{3x}{4}=5$ *?*

Multiply this $\frac{2}{3x}-\frac{3x}{4}=5$ by $4\times 3x=12x$:

$$12x\left(\frac{2}{3x}-\frac{3x}{4}\right)=12x(5)$$

$$4(2)-(3x)^2=60x$$

$$8-9x^2=60x$$

Rearranging this gives the quadratic equation

$$9x^2+60x-8=0$$

How do we solve this quadratic?

Use the quadratic formula $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$. *What are the values of a, b and c in this case?*

$a=9,\ b=60$ and $c=-8$. Substituting this $a=9,\ b=60$ and $c=-8$ into the above formula gives

$$x=\frac{-60\pm\sqrt{60^2-\left[4\times 9\times(-8)\right]}}{2\times 9}$$

$$=\frac{-60\pm\sqrt{3600+288}}{18}$$

$$=\frac{-60\pm 62.3538}{18}$$

$$=\frac{-60+62.3538}{18},\ \frac{-60-62.3538}{18}$$

$$=0.1308,\ -6.7974$$

Our solution is $x=0.1308,\ -6.7974$.

7. We need to write $\frac{y^3\sqrt{y}}{y^4}$ as a single index. Writing square root $\sqrt{\ }$ as an index ½:

$$\frac{y^3\sqrt{y}}{y^4}=\frac{y^3y^{1/2}}{y^4}$$

$$=y^{3+1/2-4}=y^{-1/2}=\frac{1}{\sqrt{y}}$$

Single index is $y^{-1/2}$.

8. Expanding the given equation $2(x+7)=5(x-1)+8$ yields

$$2x+14=5x-5+8$$

$$14+5-8=5x-2x \qquad \text{[Collecting Like terms]}$$

$$11=3x \qquad \text{implies that } x=\frac{11}{3}$$

Hence we have $x=\frac{11}{3}$.

9. *How do we solve the given quadratic equation* $3x^2-4x-1=0$ *?*

Using the quadratic formula $x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$ with $a=3$, $b=-4$ and $c=-1$:

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}=\frac{-(-4)\pm\sqrt{(-4)^2-[4\times3\times(-1)]}}{2\times3}$$
$$=\frac{4\pm\sqrt{28}}{6}$$
$$=\frac{4+5.2915}{6},\ \frac{4-5.2915}{6}$$
$$=1.5486,\ -0.2153$$

Our solution correct to 3 decimal places is $x=-0.215,\ 1.549$.

10. *How do we make m the subject of the formula* $t=\dfrac{M-2m}{3m-5}$ *?*

Multiplying through by $3m-5$ gives

$$(3m-5)t=M-2m$$
$$3mt-5t=M-2m \qquad [\text{Expanding the Left Hand Side}]$$
$$3mt+2m=M+5t \qquad [\text{Collecting like terms}]$$
$$m(3t+2)=M+5t \qquad [\text{Factorising the Left Hand Side}]$$
$$m=\frac{M+5t}{3t+2} \qquad \text{Provided } t\neq-\frac{2}{3}$$

We have $m=\dfrac{M+5t}{3t+2}$. Substituting the given values $M=0.5$ and $t=0.004$ into $m=\dfrac{M+5t}{3t+2}$:

$$m=\frac{0.5+5(0.004)}{3(0.004)+2}=0.2584$$

11. *How do we solve the given simultaneous equations?*

$$2x = 3 + \frac{4}{y} \qquad (*)$$
$$5y - \frac{2}{x} = 4 \qquad (**)$$

Dividing equation (*) by 2 gives

$$x=\frac{3}{2}+\frac{4}{2y}=\frac{3}{2}+\frac{2}{y}=\frac{3y+4}{2y}$$

Substituting $x=\dfrac{3y+4}{2y}$ into (**) gives

$$5y-\frac{2}{(3y+4)/2y}=5y-\frac{4y}{3y+4}=4$$

$$5y(3y+4)-4y=4(3y+4) \qquad [\text{Multiplying by } 3y+4]$$

$$15y^2+20y-4y=12y+16$$

$$15y^2+4y-16=0$$

Solving this quadratic equation $15y^2+4y-16=0$ by using the quadratic formula $y=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$ with $a=15$, $b=4$ and $c=-16$:

$$y=\frac{-4\pm\sqrt{4^2-[4\times15\times(-16)]}}{2\times15}$$

$$=\frac{-4\pm\sqrt{16+960}}{30}$$

$$=\frac{-4-\sqrt{976}}{30},\ \frac{-4+\sqrt{976}}{30}$$

We have $y=\frac{-4-\sqrt{976}}{30}$, $\frac{-4+\sqrt{976}}{30}$. *What are the corresponding values of x?*

Substituting $y=\frac{-4-\sqrt{976}}{30}$ into the given equation $2x=3+\frac{4}{y}$ (*) gives

$$2x=3+\frac{4}{\frac{-4-\sqrt{976}}{30}}$$

$$=3+\frac{120}{-4-\sqrt{976}}=\frac{-12-3\sqrt{976}+120}{-4-\sqrt{976}}=\frac{108-3\sqrt{976}}{-4-\sqrt{976}}$$

What is x equal to?

Dividing by 2 gives $x=\frac{108-3\sqrt{976}}{2\left(-4-\sqrt{976}\right)}=\frac{108-3\sqrt{976}}{-8-2\sqrt{976}}$.

Similarly substituting $y=\frac{-4+\sqrt{976}}{30}$:

$$2x=3+\frac{4}{\frac{-4+\sqrt{976}}{30}}$$

$$=3+\frac{120}{-4+\sqrt{976}}=\frac{-12+3\sqrt{976}+120}{-4+\sqrt{976}}=\frac{108+3\sqrt{976}}{-4+\sqrt{976}}$$

which gives $x=\frac{108+3\sqrt{976}}{2\left(-4+\sqrt{976}\right)}=\frac{108+3\sqrt{976}}{-8+2\sqrt{976}}$.

We have the solutions $x=\frac{108+3\sqrt{976}}{-8+2\sqrt{976}}$, $y=\frac{-4+\sqrt{976}}{30}$ and $x=\frac{108-3\sqrt{976}}{-8-2\sqrt{976}}$,

$y = \frac{-4-\sqrt{976}}{30}$.

You may check these answers by substituting these values into the given equations (*) and (**).

12. (i) We need to apply the rules of indices to $\sqrt[3]{\frac{9^{x-1}2^{-2x-1}}{5^{x+2}3^{2x+4}20^{7-x}}}$:

$$\sqrt[3]{\frac{9^{x-1}2^{-2x-1}}{5^{x+2}3^{2x+4}20^{7-x}}} = \left(\frac{9^{x-1}2^{-2x-1}}{5^{x+2}3^{2x+4}20^{7-x}}\right)^{1/3} \qquad \left[\text{Remember } \sqrt[3]{a} = a^{1/3}\right]$$

$$= \left(\frac{\left(3^2\right)^{x-1} 2^{-2x-1}}{5^{x+2}3^{2x+4}\left(2^2 5\right)^{7-x}}\right)^{1/3} \qquad \left[\text{Rewriting } 9 = 3^2 \text{ and } 20 = 2^2 5\right]$$

$$= \left(\frac{3^{2x-2}2^{-2x-1}}{5^{x+2}3^{2x+4}\left(2^{2(7-x)}5^{7-x}\right)}\right)^{1/3} \qquad \left[\text{Applying } \left(a^m\right)^n = a^{mn}\right]$$

$$= \left(\frac{3^{2x-2-(2x+4)}2^{-2x-1-2(7-x)}}{5^{x+2}5^{7-x}}\right)^{1/3} = \left(\frac{3^{-6}2^{-15}}{5^9}\right)^{1/3} = \frac{3^{-2}2^{-5}}{5^3} = \frac{1}{2^5 3^2 5^3} = \frac{1}{36000}$$

The horrendous expression simplifies to $\frac{1}{36000}$ or $\frac{1}{2^5 3^2 5^3}$.

(ii) *How do we simplify* $\frac{3^{(3x+2)}2^{(4x+1)}}{4^{(2x-5)}27^{(x-1)}}$ *?*

Again apply the rules of indices

$$\frac{3^{(3x+2)}2^{(4x+1)}}{4^{(2x-5)}27^{(x-1)}} = \frac{3^{(3x+2)}2^{(4x+1)}}{\left(2^2\right)^{(2x-5)}\left(3^3\right)^{(x-1)}} \qquad \left[\text{Writing } 4 = 2^2 \text{ and } 27 = 3^3\right]$$

$$= \frac{3^{(3x+2)}2^{(4x+1)}}{2^{4x-10}3^{3x-3}} = 2^{4x+1-(4x-10)}3^{3x+2-(3x-3)} = 2^{11}3^5$$

The given expression simplifies to $2^{11}3^5$.

13. *How do make x the subject of* $v = \frac{1}{k}\sqrt{\left[\frac{1+\left(\frac{1}{x}+1\right)^2}{Lg}\right]}$ *?*

Squaring both sides gives:

$$v^2 = \frac{1}{k^2}\left[\frac{1+\left(\frac{1}{x}+1\right)^2}{Lg}\right]$$

Multiplying both sides by $k^2 L\text{g}$:

$$k^2v^2Lg = 1 + \left(\frac{1}{x} + 1\right)^2$$

Subtracting 1 and taking the positive square root:

$$\sqrt{k^2v^2Lg - 1} = \frac{1}{x} + 1 \qquad \text{gives} \qquad \frac{1}{x} = \sqrt{k^2v^2Lg - 1} - 1$$

Therefore we have

$$x = \frac{1}{\sqrt{k^2v^2Lg - 1} - 1}$$

Substituting the given values $k = 1.26,\ L = 48.55,\ g = 32.21,\ v = 0.0483$ into the above $x = \frac{1}{\sqrt{k^2v^2Lg - 1} - 1}$:

$$x = \frac{1}{\sqrt{\left[(1.26)^2(0.0483)^2 \times 48.55 \times 32.21\right] - 1} - 1}$$

$$= \frac{1}{\sqrt{5.7918 - 1} - 1} = \frac{1}{\sqrt{4.7918} - 1} = \frac{1}{1.1890} = 0.8410$$

Our x value is 0.8410.