Complete Solutions to Examination Questions 2

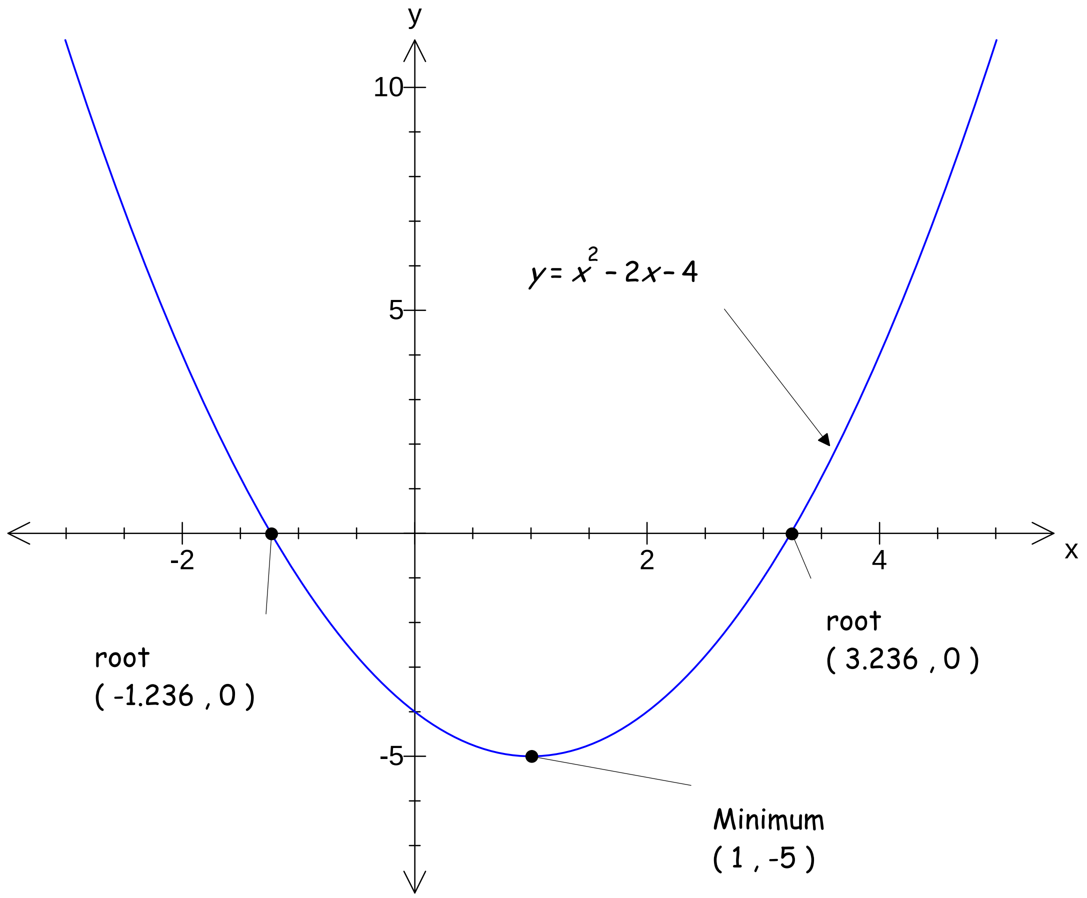
1. (a) To find the roots of $x^2-2x-4=0$ we use the quadratic formula $x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$ with $a=1$, $b=-2$ and $c=-4$:

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}=\frac{-(-2)\pm\sqrt{(-2)^2-[4\times1\times(-4)]}}{2\times1}$$
$$=\frac{2\pm\sqrt{4+16}}{2}$$
$$=\frac{2\pm\sqrt{20}}{2}$$
$$=1\pm\sqrt{5} \qquad \left[\text{Because } \sqrt{20}=\sqrt{4\times5}=2\sqrt{5}\right]$$
$$=1-\sqrt{5},\ 1+\sqrt{5}=-1.236,\ 3.236$$

(b) We can complete the square on x^2-2x-4 to find where the minimum occurs:

$$x^2-2x-4=(x-1)^2-1-4=(x-1)^2-5$$

Hence the minimum occurs at $x=1$ with a y value of -5:

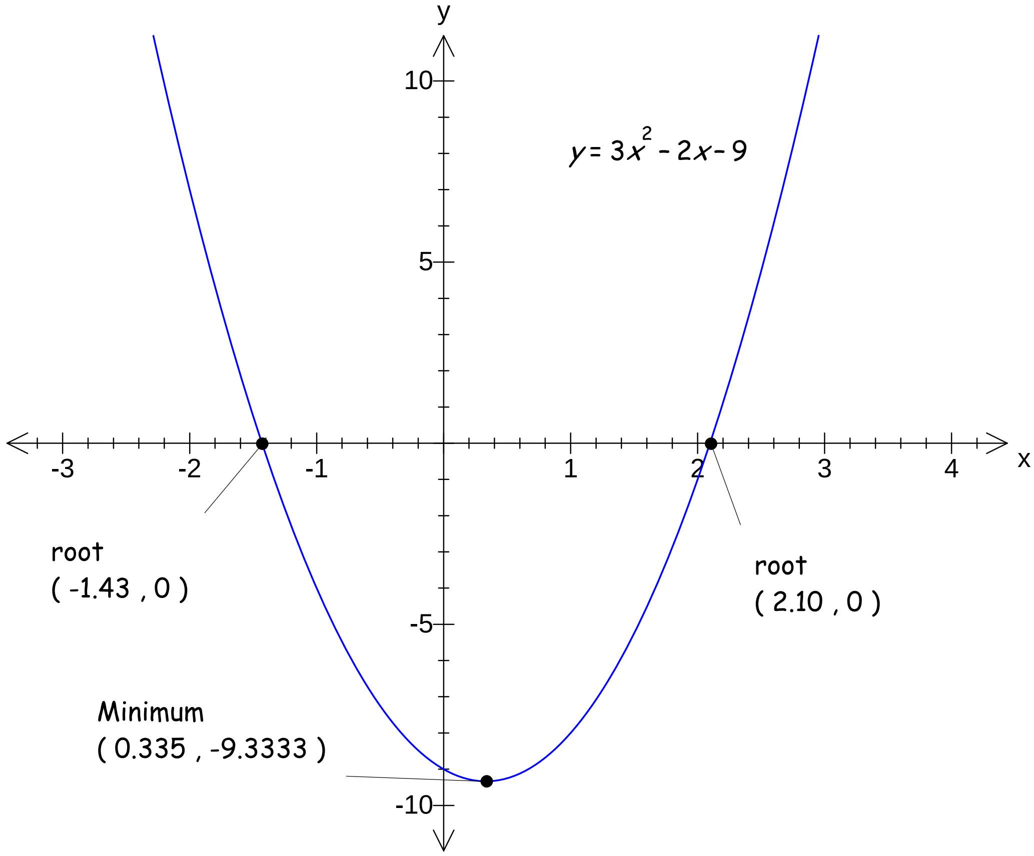


2. (a) To find the roots of $3x^2-2x-9=0$ we use the formula $x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$ with $a=3$, $b=-2$ and $c=-9$:

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}=\frac{-(-2)\pm\sqrt{(-2)^2-[4\times3\times(-9)]}}{2\times3}$$
$$=\frac{2\pm\sqrt{4+108}}{6}$$
$$=\frac{2\pm\sqrt{112}}{6}$$
$$=\frac{2\pm10.583}{6}$$
$$=\frac{2-10.583}{6},\ \frac{2+10.583}{6}=-1.43,\ 2.10$$

(b) Completing the square on the given quadratic $3x^2-2x-9$ is more difficult because of the coefficient of x^2 is 3 which means we need to divide by 3. However it is easier to see that the minimum value occurs halfway between the roots, -1.43 and 2.10, which is

$$\frac{-1.43+2.10}{2}=0.335$$



3. The expansion of $(1+x)^6$ was carried out in question 1a Exercise 2f:

$$(1+x)^6=1+6x+15x^2+20x^3+15x^4+6x^5+x^6$$

How do we find an approximation to $(11/10)^6$ *?*

Use the above result of the expansion of $(1+x)^6$ with $x=\frac{1}{10}=0.1$ because

$$\frac{11}{10}=1+\frac{1}{10}=1+0.1$$

To give the final answer correct to three decimal places we need to work to four decimal places:

$$\begin{aligned}(1+0.1)^6 &= 1+6(0.1)+15(0.1)^2+20(0.1)^3+15(0.1)^4+6(0.1)^5+(0.1)^6\\ &= 1+0.6+0.15+0.02+0.0015+0+0\\ &= 1.7715 = 1.772 \text{ (3 dp)}\end{aligned}$$

4. By using Pascal's triangle for $n=5$ we have 1, 5, 10, 5 and 1. We have

$$(a+b)^5 = a^5+5a^4b+10a^3b^2+10a^2b^3+5ab^4+b^5 \qquad (*)$$

How do we determine $\left(x^2+\frac{1}{x}\right)^5$ *?*

Substitute $a=x^2$ and $b=\frac{1}{x}$ into the above (*):

$$\left(x^2+\frac{1}{x}\right)^5 = \left(x^2\right)^5+5\left(x^2\right)^4\left(\frac{1}{x}\right)+10\left(x^2\right)^3\left(\frac{1}{x}\right)^2+10\left(x^2\right)^2\left(\frac{1}{x}\right)^3+5x^2\left(\frac{1}{x}\right)^4+\left(\frac{1}{x}\right)^5$$

$$\underbrace{=}_{\substack{\text{Using the rules}\\ \text{of indices } \left(a^m\right)^n=a^{mn}}} x^{10}+5x^8\left(\frac{1}{x}\right)+10x^6\left(\frac{1}{x^2}\right)+10x^4\left(\frac{1}{x^3}\right)+5x^2\left(\frac{1}{x^4}\right)+\frac{1}{x^5}$$

$$\underbrace{=}_{\substack{\text{Using the rules}\\ \text{of indices } a^m/a^n=a^{m-n}}} x^{10}+5x^7+10x^4+10x+\frac{5}{x^2}+\frac{1}{x^5}$$

Hence we have $\left(x^2+\frac{1}{x}\right)^5 = x^{10}+5x^7+10x^4+10x+\frac{5}{x^2}+\frac{1}{x^5}$.

5. *How do we complete the square on* $3x^2-6x+1$?

Take the coefficient of x^2, which is 3, out:

$$\begin{aligned}3x^2-6x+1 &= 3\left(x^2-2x+\frac{1}{3}\right)\\ &= 3\left[(x-1)^2-1+\frac{1}{3}\right] = 3\left[(x-1)^2-\frac{2}{3}\right]=0\end{aligned}$$

How do we solve $3\left[(x-1)^2-\frac{2}{3}\right]=0$ *?*

Divide both sides by 3 and then transpose to make *x* the subject of the formula:

$$(x-1)^2-\frac{2}{3}=0$$

$$(x-1)^2=\frac{2}{3} \quad \text{taking the square root gives } x-1=\pm\sqrt{\frac{2}{3}}$$

Therefore $x=1\pm\sqrt{\frac{2}{3}}=1-\sqrt{\frac{2}{3}},\ 1+\sqrt{\frac{2}{3}}$. Using our calculator gives

$$x=0.184,\ 1.817$$

6. (a) We need to factorize $f(x) = 9x^2 - 12x - 5$ and equate to zero.

$$9x^2 - 12x - 5 = (3x+1)(3x-5) = 0$$

Solving this $(3x+1)(3x-5) = 0$ gives $3x+1 = 0,\ 3x-5 = 0$ which means that

$$x = -\frac{1}{3},\quad x = \frac{5}{3}$$

The *x*-intercepts are $\left(-\frac{1}{3},\ 0\right)$ and $\left(\frac{5}{3},\ 0\right)$.

(b) *How do we complete the square on the given quadratic expression* $9x^2 - 12x - 5$*?*
Take out 9:

$$9x^2 - 12x - 5 = 9\left(x^2 - \frac{12}{9}x - \frac{5}{9}\right)$$

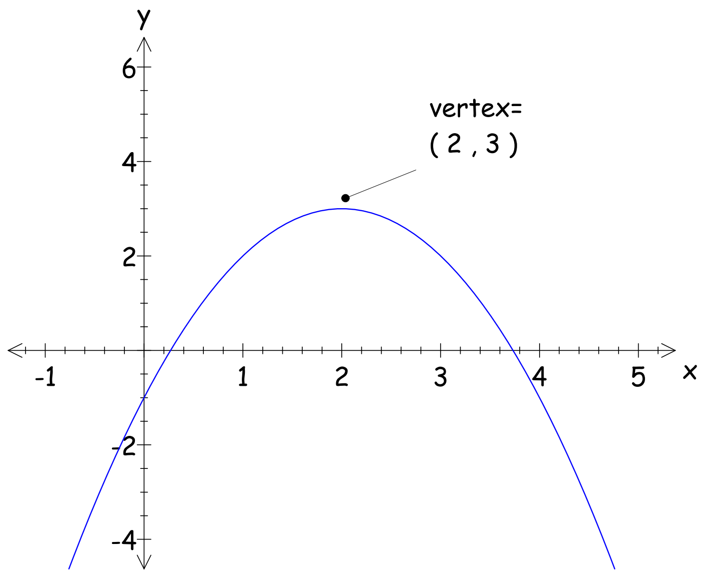
$$= 9\left(x^2 - \frac{4}{3}x - \frac{5}{9}\right) \qquad \left[\text{Because } \frac{12}{9} = \frac{4}{3}\right]$$

$$= 9\left[\left(x - \frac{2}{3}\right)^2 - \frac{4}{9} - \frac{5}{9}\right] \qquad \left[\text{Because } \frac{2}{3} \text{ is half of } \frac{4}{3}\right]$$

$$= 9\left[\left(x - \frac{2}{3}\right)^2 - 1\right] = 9\left(x - \frac{2}{3}\right)^2 - 9$$

Since x^2 coefficient is positive (9) therefore we have a minimum. The minimum value is -9 when $x = \frac{2}{3}$. Hence the vertex of the graph is $\left(\frac{2}{3},\ -9\right)$.

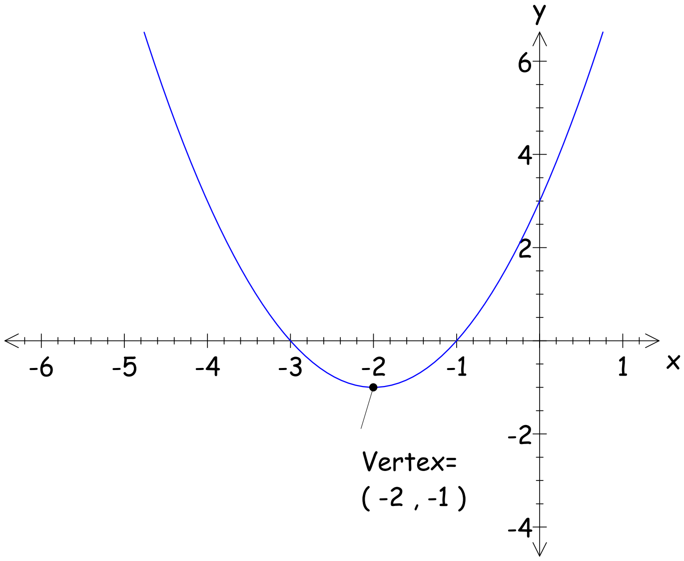
7. (a) *How do we find the quadratic equation of the given graph?*



This means that the maximum value is 3 at $x = 2$. Since we have maximum therefore the x^2 coefficient is negative. This means we have $-(x-2)^2$ because the maximum value occurs at $x = 2$. Hence we have the quadratic $3-(x-2)^2$ because the maximum value is 3.
Expanding this out yields

$$3-(x-2)^2 = 3-(x^2-4x+4)$$
$$= 3-x^2+4x-4 = -1+4x-x^2$$

(b) Similarly for the given quadratic:



This time we have a minimum which means that the coefficient of x^2 is positive. Since the minimum occurs at $(-2,\ -1)$ therefore the quadratic is given by:

$$(x+2)^2-1 = x^2+4x+4-1$$
$$= x^2+4x+3$$