1. Find the area of the region bounded by the curve $y = 3x - x^2$ and the x axis.

2. R is the region bounded by the curves $y = 3 - \sin(x)$, $y = \sin(x)$, $x = 0$ and $x = \pi$. Find the area of R.

<div align="right">University of Manchester UK 2009</div>

3. Using Simpson's rule with 4 strips, find an approximate value of the integral $\int_0^1 e^{x^2} dx$.

<div align="right">University of Manchester UK 2007</div>

4. Write down the Trapezoidal Rule approximation T_3 for $\int_1^4 x \cos(\pi / x) dx$. Leave your answer expressed as a sum involving cosines.

[T_3 in this question means consider 3 intervals].

<div align="right">University of British Columbia Canada 2008</div>

5. Approximate $\int_0^2 \dfrac{x}{1 + x^2} dx$ using the Trapezoidal Rule with $n = 4$ subintervals.

<div align="right">North Carolina State University USA 2010</div>

6. Find the area under the graph of $y = \ln x$ between $x = 1$ and $x = 3$.

<div align="right">University of Aberdeen UK 2004</div>

7. Integrals like $\int \sin(x^2) dx$ arise in the diffraction of light but they cannot be evaluated exactly. Find the Maclaurin series for $\sin(x^2)$ up to and including the term in x^{10}, and use it to find an approximate value for

$$\int_0^1 \sin(x^2) dx$$

giving your answer to four decimal places.

<div align="right">University of Surrey UK 2008</div>

8. You are standing on the top of a building which is 256 ft. high. You drop a large orange pumpkin and watch as it falls helplessly to the ground.

(a) Assume that the acceleration due to gravity is $s''(t) = -32$ ft./sec.^{2}.
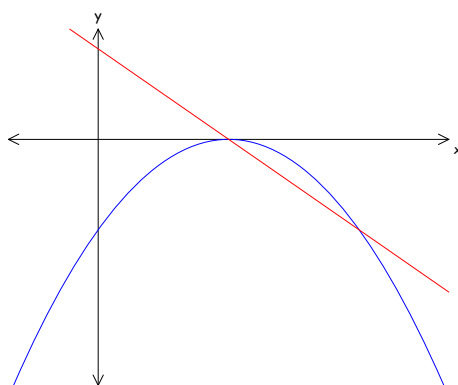
Derive the instantaneous velocity, $s'(t)$, and height (above ground), $s(t)$, formulas for this pumpkin.

(b) In how many seconds will the pumpkin strike the ground?

(c) What is the pumpkin's velocity as it strikes the ground?

<div align="right">University of California Davis USA</div>

9. Consider the region bounded by the curves $y = -(x-2)^2$ and $y = 4 - 2x$.

(a) Express the area A of this region as an integral taken with respect to x.
(b) Evaluate the integral to find A.

10. Calculate an approximation to the integral

$$\int_1^3 \frac{x}{\sqrt{1+x^2}}\, dx$$

using the Simpson's Rule by dividing the range into four equal intervals. You should work to an accuracy of three decimal places throughout. What is the approximate percentage error in the result (you may compute the integral directly or use the formula which approximates the error)?

11. Find the following:
(a) the average value of $f(x) = x^3 + x$ on the interval $1 \le x \le 3$

(b) the area of the region between the graphs of $f(x) = x^3$ and $g(x) = x$ for $0 \le x \le 2$.

12. Find the first three nonzero terms in the power series representation in powers of x

(i.e. the Maclaurin series) for $\int_0^x t \cos(t^3)\, dt$.

13. Find the numbers b such that the average value of the function $f(x) = 3x^2 - 6x + 2$

on the interval $[0, \; b]$ is equal to 0.

14. Let $I = \int_0^1 \cos(x^2)\, dx$. Write down the first three nonzero terms obtained by using the

Maclaurin series to estimate I, and explain why the error in using this estimate is less than 0.001.

1. $9/2$
2. $3\pi - 4$
3. $1.464 \ (3\text{dp})$
4. $\dfrac{1}{2}\Big[\cos(\pi) + 2\big(2\cos(\pi/2) + 3\cos(\pi/3)\big) + 4\cos(\pi/4)\Big]$
5. $0.78077 \ (5\text{dp})$
6. $\ln(27) - 2$
7. $0.3103 \ (4\text{dp})$
8. (a) $s'(t) = -32t$, $s(t) = 16\big(16 - t^2\big)$ (b) $t = 4s$ (c) $-128\,\text{ft/sec}$.
9. (a) $A = \displaystyle\int_{2}^{4}\Big[-(x-2)^2 - (4-2x)\Big]\,dx$ (b) $4/3$
10. 1.747 , % error is 0.057%.
11. (a) 12 (b) $5/2$
12. $\dfrac{x^2}{2} - \dfrac{x^8}{16} + \dfrac{x^{14}}{336} - \ldots$
13. $b = 1$ and 2
14. $1 - \dfrac{1}{10} + \dfrac{1}{216} - \ldots$ and the fourth non-zero term is $\dfrac{1}{9360} < \dfrac{1}{1000} = 0.001$.