Complete solutions to Exercise 1(b)

1. $V=\sqrt{PR}$

2. We have $\sqrt{\frac{\gamma P}{\rho}}=c$ and need to extract P, how?

Square both sides to remove the square root:

$$\left(\sqrt{\frac{\gamma P}{\rho}}\right)^2=c^2$$

$$\frac{\gamma P}{\rho}=c^2$$

Multiply both sides by ρ:

$$\gamma P=\rho c^2$$

Divide both sides by γ:

$$P=\frac{\rho c^2}{\gamma}$$

3. $A=\frac{2D}{\rho C v^2}$. Similar to **EXAMPLE 6**.

4. Substituting $R_1=100$ and $R_2=270$ into $\frac{1}{R}=\frac{1}{R_1}+\frac{1}{R_2}$ gives

$$\frac{1}{R}=\frac{1}{100}+\frac{1}{270}=\frac{37}{2700}$$

To find R we invert $\frac{37}{2700}$ to give

$$R=\frac{2700}{37}=72.97\Omega \text{ (correct to 2 d.p.)}$$

5. We have $T=2\pi\sqrt{\frac{l}{g}}$. Transposing gives

$$\sqrt{\frac{l}{g}}=\frac{T}{2\pi}$$

$$\frac{l}{g}=\left(\frac{T}{2\pi}\right)^2$$

$$l=g\frac{T^2}{4\pi^2}$$

Substituting $T=0.5$ and $g=9.81$ into the above gives

$$l=9.81\times(0.5)^2/4\pi^2$$

$$=0.062 \text{ m (correct to 2 s.f.)}$$

6. Very similar to **EXAMPLE 8**, thus $C=1.4\times10^{-5}F=14\mu F$.

7. Need to obtain D from $2\pi k\rho n^3D^5=P$. Divide both sides by $2\pi k\rho n^3$, to remove all of the Left Hand Side apart from D^5:

$$D^5=\frac{P}{2\pi k\rho n^3}$$

How do we remove the 5th power of D? Take the fifth root

$$\underbrace{\sqrt[5]{D^5}}_{=D} = \sqrt[5]{\frac{P}{2\pi k\rho n^3}}$$

$$D = \underbrace{\left(\frac{P}{2\pi k\rho n^3}\right)^{\frac{1}{5}}}_{\text{by (1.2)}}$$

8. First consider the bracket term

$$\frac{1}{A} - \beta = \frac{1}{A} - \frac{A\beta}{A} = \frac{1-A\beta}{A}$$

Substituting this into $\left(\frac{1}{A} - \beta\right)v_{out} = v_{in}$ gives

$$\left(\frac{1-A\beta}{A}\right)v_{out} = v_{in}$$

Divide both sides by v_{in} to give

$$\frac{v_{out}}{v_{in}}\left(\frac{1-A\beta}{A}\right) = 1$$

$$\left(\frac{v_{in}}{v_{in}} = 1\right)$$

Need to get $\frac{v_{out}}{v_{in}} = ---$ Divide both sides by $\left(\frac{1-A\beta}{A}\right)$:

$$\frac{v_{out}}{v_{in}} = \frac{1}{\left(\frac{1-A\beta}{A}\right)} \underbrace{=}_{\text{by (1.3)}} \left(\frac{1-A\beta}{A}\right)^{-1} \underbrace{=}_{\text{by (1.4)}} \frac{A}{1-A\beta}$$

9. We have $\tau = \frac{T}{\frac{1}{2}\pi r^3} = \frac{2T}{\pi r^3}$ because $1 \div \left(\frac{1}{2}\right) = 2$. How do we dig out r from

$$\tau = \frac{2T}{\pi r^3}?$$

First find r^3 then we take the cube root, $\sqrt[3]{\ }$, of both sides.

Multiply both sides by πr^3:

$$\pi r^3 \tau = 2T$$

Divide both sides by $\pi\tau$:

$$r^3 = \frac{2T}{\pi\tau}$$

(1.2) $\sqrt[n]{a} = a^{\frac{1}{n}}$

(1.3) $\frac{1}{x} = x^{-1}$

(1.4) $\left(\frac{a}{b}\right)^{-1} = \frac{b}{a}$

Take the cube root of both sides:

$$\underbrace{\sqrt[3]{r^3}}_{=r} = \sqrt[3]{\frac{2T}{\pi\tau}}$$

$$r = \underbrace{\left(\frac{2T}{\pi\tau}\right)^{\frac{1}{3}}}_{\text{by (1.2)}}$$

Question 10 (a), (b) and (c) are straightforward transposition.

10. (a) $r = \frac{ER}{V} - R$ (b) $u = \sqrt{v^2 - 2as}$ (c) $K = \rho v^2 - \frac{4a}{3}$

(d) Multiply both sides by $8vL$ to give

$$\pi \Pr^4 t = 8vL\eta$$

Divide both sides by πPt to obtain only r^4 on the Left Hand Side:

$$r^4 = \frac{8vL\eta}{\pi Pt}$$

How can we extract r from r^4?

Take the 4th root, $\sqrt[4]{\ }$, of both sides:

$$\sqrt[4]{r^4} = \sqrt[4]{\frac{8vL\eta}{\pi Pt}}$$

$$r = \underbrace{\left(\frac{8vL\eta}{\pi Pt}\right)^{\frac{1}{4}}}_{\text{by (1.2)}}$$

(e) Dividing both sides by 2π, so we have

$$\sqrt{\frac{l}{g}} = \frac{T}{2\pi}$$

Need to find l. Square both sides and then multiply by g

$$\frac{l}{g} = \left(\frac{T}{2\pi}\right)^2 = \frac{T^2}{4\pi^2}$$

$$l = \frac{T^2 g}{4\pi^2}$$

(f) We have

$$\left(P + \frac{a}{V^2}\right)(V - b) = RT$$

How do we find P?

Divide both sides by $(V - b)$ and then take away $\frac{a}{V^2}$:

$$P + \frac{a}{V^2} = \frac{RT}{V - b}$$

$$P = \frac{RT}{V - b} - \frac{a}{V^2}$$

(1.2) $$\sqrt[n]{a} = a^{\frac{1}{n}}$$

10. (g) To remove the denominator $(n-1)$ we multiply both sides by $(n-1)$:

$$P_1V_1 - P_2V_2 = W(n-1)$$

Add P_2V_2 and divide through by P_1:

$$P_1V_1 = W(n-1) + P_2V_2$$

$$V_1 = \frac{W(n-1)+P_2V_2}{P_1}$$

(h) Divide by P and take the nth root to give $V = \left(\frac{C}{P}\right)^{\frac{1}{n}}$.

(i) We are given $\frac{CW}{D}\sqrt{\frac{h}{u}} = f$.

We need to remove C, W, D and h from the Left Hand Side to get u on its own, how?

First multiply by D and then divide through by CW:

$$\sqrt{\frac{h}{u}} = \frac{fD}{CW}$$

How do we remove the square root? Square both sides

$$\left(\sqrt{\frac{h}{u}}\right)^2 = \left(\frac{fD}{CW}\right)^2$$

$$\frac{h}{u} = \frac{f^2D^2}{C^2W^2}$$

Next we dismiss h on the Left Hand Side by dividing both sides by h:

$$\frac{1}{u} = \frac{f^2D^2}{C^2W^2h}$$

Taking $(\ )^{-1}$ of both sides we have

$$u = \frac{C^2W^2h}{f^2D^2}$$