Complete solutions to Exercise 1(e)

1. Expand as in **EXAMPLE 16** and **EXAMPLE 17**:

(a) $6x+2$ (b) $-2x-1$ (c) $-15y-3$ (d) $3x^2+5x$

(e) $3(y-1)-(2y+1)=3y-3-2y-1=3y-2y-3-1=\underbrace{y}_{=3y-2y}-4$

(f) We have

$$\begin{aligned}x(x-3)+x(3x+2) &= (x\times x)-(x\times 3)+(x\times 3x)+(x\times 2)\\ &= x^2-3x+3x^2+2x\\ &= 4x^2-x\end{aligned}$$

2.(a) We have

$$\frac{w}{2EI}\left(Lx^3-x^4\right)=\frac{wLx^3}{2EI}-\frac{wx^4}{2EI}$$

(b) Expanding gives

$$\begin{aligned}\frac{wx^3}{8EI}(2L-3x) &= \frac{2Lwx^3}{8EI}-\frac{3wx^3x}{8EI}\\ &= \frac{wLx^3}{4EI}-\frac{3wx^4}{8EI}\end{aligned}$$

(c) Similarly we have

$$\begin{aligned}\frac{wx^2}{48EI}\left(3L^2-2x^2\right) &= \frac{3L^2wx^2}{48EI}-\frac{2wx^2x^2}{48EI}\\ &= \frac{wL^2x^2}{16EI}-\frac{wx^4}{24EI}\end{aligned}$$

(d) Taking a negative sign inside the bracket changes the signs inside:

$$-\frac{w}{12EI}\left(Lx^3-\frac{x^4}{2}-\frac{L^3x}{2}\right)=-\frac{wLx^3}{12EI}+\frac{wx^4}{24EI}+\frac{wL^3x}{24EI}$$

3. (a) $(x+1)(x+2)=x^2+3x+2$ (b) $(2x+3)(3x+5)=6x^2+19x+15$

(c) We have

$$\begin{aligned}(2x-1)^2 &\underbrace{=}_{\text{by (1.14)}} (2x)^2-(2\times 2x\times 1)+1^2\\ &= 4x^2-4x+1\end{aligned}$$

(d) Expanding

$$\begin{aligned}(a+b)^2-(a-b)^2 &= \underbrace{a^2+2ab+b^2}_{\text{by (1.13)}}-\underbrace{\left(a^2-2ab+b^2\right)}_{\text{by (1.14)}}\\ &= a^2+2ab+b^2-a^2+2ab-b^2\\ &= \left(a^2-a^2\right)+\left(b^2-b^2\right)+2ab+2ab\\ &= 0+0+4ab\\ &= 4ab\end{aligned}$$

(1.13) $(a+b)^2=a^2+2ab+b^2$

(1.14) $(a-b)^2=a^2-2ab+b^2$

(e) In a similar manner we have

$$(xy+1)^2 - x(y+1) = \underbrace{(xy)^2 + (2 \times xy \times 1) + 1}_{\text{by (1.13)}} - xy - x$$

$$= x^2y^2 + 2xy + 1 - xy - x$$

$$= x^2y^2 + \underbrace{xy}_{=2xy-xy} - x + 1$$

4.(a) $(x-5)(x+5) = x^2 + \underbrace{5x - 5x}_{=0} - 25 = x^2 - 25$. Also (b) and (c) are similar; the middle terms cancel each other out. There is no x term.

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5. (a) $(R+\omega L)(R-\omega L) = R^2 - \omega^2 L^2$ (by (1.15)).

(b) Similar to **EXAMPLE 20**.

$$\frac{1}{R^2} + \left(\omega C - \frac{1}{\omega L}\right)^2 = \frac{1}{R^2} + \underbrace{\left[(\omega C)^2 - \left(2 \times \cancel{\omega} C \times \frac{1}{\cancel{\omega} L}\right) + \left(\frac{1}{\omega L}\right)^2\right]}_{\text{by (1.14)}}$$

$$= \frac{1}{R^2} + \omega^2 C^2 - \frac{2C}{L} + \frac{1}{\omega^2 L^2}$$

6. At some point you will reach $(x-x)$ which is zero. Any number multiplied by zero gives zero, hence

$$(x-a)(x-b)\ldots(x-x)\ldots(x-z) = 0$$

(1.13) $(a+b)^2 = a^2 + 2ab + b^2$

(1.14) $(a-b)^2 = a^2 - 2ab + b^2$

(1.15) $(a-b)(a+b) = a^2 - b^2$