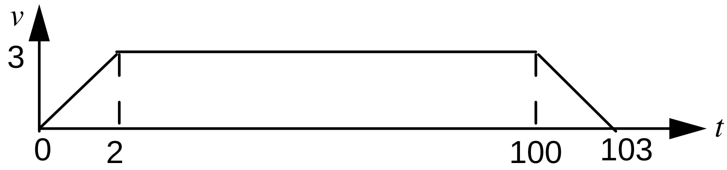
Complete solutions to Exercise 2(b)

1. Similar to **EXAMPLE 5** of chapter 2.



(a) The gradient $=0$ between $t=2$s and $t=100$s. Hence by (2.2)

$$\text{acceleration}=0$$

(b) The total displacement = Area under the graph

$$\text{Area of trapezium}=\frac{1}{2}(103+98)\times 3=301.5$$

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2. For t between 0 and 60 we have gradient$=\frac{10}{60}=\frac{1}{6}$. Since the straight line goes through the origin, so the v-intercept=0. Substituting $m=\frac{1}{6}, c=0$ into $v=mt+c$ gives: $v=\frac{1}{6}t$.

For $60<t\le 240$ we have a horizontal line with a value of 10. So v=10.
For $240<t\le 360$ the straight line slopes downwards to the right so we have a negative gradient:

$$\text{gradient}=-\frac{10}{360-240}=-\frac{1}{12}$$

We do not know the v-intercept, c, because it is not labelled on the graph. By (2.1) we have

$$v=-\frac{1}{12}t+c \qquad (*)$$

where c is the v-intercept. From the graph we see that at $t=360$, $v=0$. We can find c by substituting this into (*):

$$0=-\left(\frac{1}{12}\times 360\right)+c$$

$$0=-30+c$$

Hence $c=30$. So substituting $c=30$ into (*):

$$v=-\frac{1}{12}t+30$$

Combining all three equations for v gives:

$$v=\begin{cases}\frac{1}{6}t & 0\le t\le 60\\ 10 & 60<t\le 240\\ -\frac{1}{12}t+30 & 240<t\le 360\end{cases}$$

(2.1) $y=mx+c$ where $m=\text{gradient}$, $c=y-\text{intercept}$

(2.2) acceleration = gradient of the $v-t$ graph

3. For t between 0 and 10 we have a slope of $\frac{5-2}{10}=\frac{3}{10}=0.3$. Hence $m=0.3$ and from the graph the v-intercept, c, is equal to 2. Thus:

$$v=0.3t+2$$

For $10<t\le 20$, slope=0, and v-intercept is 5, so $v=5$.

For $20<t\le 23$, slope $=-\frac{5}{23-20}=-\frac{5}{3}$. By (2.1)

$$v=-\frac{5}{3}t+c \qquad (\dagger)$$

where c is the v-intercept. From the graph in question we have when t=23, v=0. Substituting these into $(\dagger)$:

$$0=\left(-\frac{5}{3}\times 23\right)+c \text{ gives } c=\frac{5\times 23}{3}$$

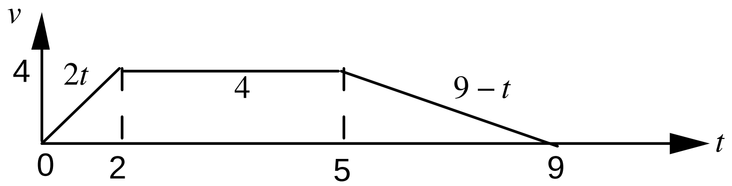
Substituting $c=\frac{5\times 23}{3}$ into (†) gives:

$$\begin{aligned} v &= \frac{5\times 23}{3}-\frac{5t}{3} \\ &= \frac{5}{3}(23-t) \quad \left[\text{taking out the common factor } \frac{5}{3}\right] \end{aligned}$$

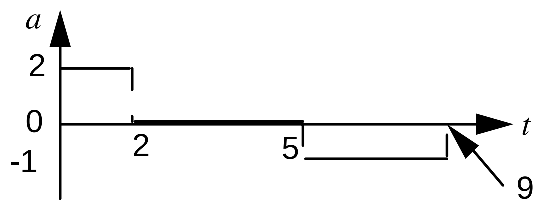
Combining these three equations :

$$v=\begin{cases} 0.3t+2 & 0\le t\le 10 \\ 5 & 10<t\le 20 \\ \frac{5}{3}(23-t) & 20<t\le 23 \end{cases}$$

4. (a) We have

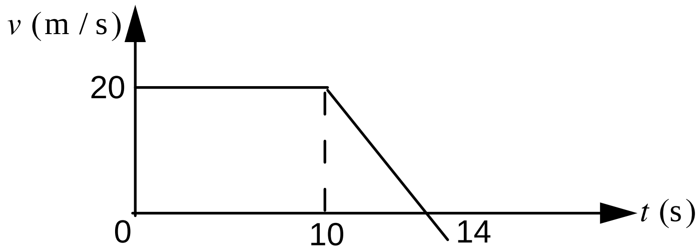


(b) For t between 0 and 2, the gradient is 2.
For $2<t\le 5$, the gradient is 0.
For $5<t\le 9$, the gradient is -1. So we have the acceleration-time graph given by:



5. (i) Initial velocity of $20m/s$ at t=0 means the graph starts at 20 on the v axis. No acceleration means that the velocity-time graph has a gradient of zero for these times, $0\le t\le 10$. Then after $t>10$ the graph has a gradient of -5 so it slopes downwards to the right of t=10.

(2.1) $y=mx+c$ where m=gradient, c=y-intercept



(ii) The gradient of the velocity-time graph is -5 after 10s. This means the value of v is decreasing by 5 for every second. So after 4 seconds the value of v is decreased by 20. Since at $t = 10,\ v = 20$ so at $t = 14,\ v = 0$. Hence the velocity is zero at $t = 14$ s.

6.(a) The acceleration a=0 when the gradient is 0. By observing the graph in question this occurs during $0 \le t \le 6$ and $14 \le t \le 20$.

(b) The acceleration between 6 and 14 is determined by the gradient

$$\text{gradient} = -\frac{10+15}{14-6} = -\frac{25}{8} = -3.125$$

Thus acceleration$= -3.125$.

7. For $t \le 3$, gradient$=\frac{9}{3}=3$.

For $3 < t \le 8$, gradient=0.

For $8 < t \le 10$, gradient$=-\frac{9}{10-8}=-\frac{9}{2}$. Thus the $a-t$ graph is: