Complete solutions to Exercise 3(a)

1. We have

$$f(1) = 1^2 = 1$$
$$f(2) = 2^2 = 4$$
$$f(3) = 3^2 = 9$$
$$f(-1) = (-1)^2 = 1$$
$$f(-2) = (-2)^2 = 4$$
$$f(-3) = (-3)^2 = 9$$

It is a many $\to$ one function.

2. Given $f(t) = \frac{9}{5}t + 32$, we have:

$$\text{f}(0) = \left(\frac{9}{5} \times 0\right) + 32 = 32$$
$$\text{f}(100) = \left(\frac{9}{5} \times 100\right) + 32 = 212$$
$$\text{f}(24) = \left(\frac{9}{5} \times 24\right) + 32 = 75.2$$

3. Similar to question 1 but we find x^3 in each case. We have $g(1) = 1, g(-1) = -1$, $g(2) = 8, g(-2) = -8, g(3) = 27$ and $g(-3) = -27$. We notice that

$$g(-a) = -g(a)$$

A function with this property for all numbers a is called an odd function.

4. Given $\text{g}(t) = -4.9t^2$,

$$\text{g}(1) = -4.9 \times 1^2 = -4.9$$
$$\text{g}(\pi) = -4.9 \times \pi^2 = -48.36 \quad \text{(correct to } 2\text{d.p.)}$$
$$\text{g}\left(\frac{1}{\sqrt{4.9}}\right) = -4.9 \times \left(\frac{1}{\sqrt{4.9}}\right)^2 = -4.9 \times \frac{1}{4.9} = -1$$

5. $f(h) = \pm\sqrt{20h}$ is <u>not</u> a function because you have 2 outputs for any given input.

6. (a) $v(0) = 9.8 \times 0 = 0$

$v(1) = 9.8 \times 1 = 9.8$

$v(2) = 9.8 \times 2 = 19.6$

$v(5) = 9.8 \times 5 = 49$

(b) $v(t^2) = 9.8t^2$

$v(t+1) = 9.8 \times (t+1) = 9.8t + 9.8$

7. (a) We are given $s(t) = t^3 - t^2 + 5$:

$$s(0) = 0^3 - 0^2 + 5 = 5$$
$$s(1.5) = 1.5^3 - 1.5^2 + 5 = 6.125$$
$$s(2.5) = 2.5^3 - 2.5^2 + 5 = 14.375$$

(b) We are given

$$s(t+1) = (t+1)^3 - (t+1)^2 + 5$$

From **EXAMPLE 3** of chapter 3 we have:

$$(t+1)^3 = t^3 + 3t^2 + 3t + 1$$

$$(t+1)^2 = t^2 + 2t + 1$$

Putting these into $s(t+1)$ gives:

$$\begin{aligned} s(t+1) &= t^3 + 3t^2 + 3t + 1 - (t^2 + 2t + 1) + 5 \\ &= t^3 + 3t^2 + 3t + 1 - t^2 - 2t - 1 + 5 \\ &= t^3 + \underbrace{3t^2 - t^2}_{=2t^2} + \underbrace{3t - 2t}_{=t} + 1 - 1 + 5 \\ &= t^3 + 2t^2 + t + 5 \end{aligned}$$

8. We replace the V with IR,

$$P(IR) = \frac{(IR)^2}{R} = \frac{I^2R^2}{R} = I^2R$$

9. We substitute r and $2r$ in place of R:

$$V(r) = \frac{Er}{r+r} = \frac{E\cancel{r}}{2\cancel{r}} = \frac{E}{2}$$

$$V(2r) = \frac{E2r}{2r+r} = \frac{2E\cancel{r}}{3\cancel{r}} = \frac{2E}{3}$$

10. (a) For $h(t) = 0$ we have:

$$\begin{aligned} &200t - 4.9t^2 = 0 \\ &t(200 - 4.9t) = 0 \\ &t = 0 \text{ or } 200 - 4.9t = 0 \end{aligned}$$

How do we solve $200 - 4.9t = 0$?

$$\begin{aligned} &200 = 4.9t \\ &\frac{200}{4.9} = t \\ &t = 40.82 \text{ (correct to } 2 \text{ d.p.)} \end{aligned}$$

(b) $h(t+2) = 200(t+2) - 4.9(t+2)^2$

$$\begin{aligned} &= 200t + 400 - 4.9\underbrace{(t^2 + 4t + 4)}_{\text{by (1.13)}} \\ &= 200t + 400 - 4.9t^2 - 19.6t - 19.6 \\ &= (400 - 19.6) + \underbrace{200t - 19.6t}_{=180.4t} - 4.9t^2 \\ h(t+2) &= 380.4 + 180.4t - 4.9t^2 \end{aligned}$$

(1.13) $$(a+b)^2 = a^2 + 2ab + b^2$$