**Complete solutions to Exercise 3(b)**

1. (a) $f^{-1}(x)=\dfrac{x+2}{7}$ (b) $f^{-1}(x)=\dfrac{1}{x}$ (provided $x\neq 0$) (c) $f^{-1}(x)=1-x$

(d) $f^{-1}(x)=5-\dfrac{3}{x}$ $(x\neq 0)$

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2. (a) Let $y=\dfrac{t}{t+2}$ $(t\neq -2)$. Then we need to obtain $t=\ldots$ Multiplying both sides by $(t+2)$:

$$(t+2)y=t$$

Expanding the Left Hand Side gives:

$$ty+2y=t$$

Collecting the $t$' s on one side:

$$2y=t-ty$$

Factorizing the Right Hand Side:

$$2y=t(1-y)$$

How do we remove $1-y$ on the Right Hand Side to obtain $t$ on its own? Divide both sides by $1-y$:

$$t=\frac{2y}{1-y}\ \left(\text{provided}\quad 1-y\neq 0\right)$$

So the inverse function is

$$g^{-1}(t)=\frac{2t}{1-t}\ \left(\text{provided}\quad 1-t\neq 0\right)$$

(b) and (c) can be obtained in a similar way. We have:
(b) $h^{-1}(x)=\dfrac{2x+1}{x+1}$ $(x\neq -1)$ and (c) $f^{-1}(x)=\dfrac{x+1}{2-x}(x\neq 2)$.

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3. (i) Similar to above.
(ii) Note that $g(t)=f^{-1}(t)$ where $f^{-1}$ is same as part (i).
Remember you are taking the inverse of the inverse so it must go back to the start,
$g^{-1}(t)=\dfrac{t+1}{3t+1}=f(t)$.

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4. (i) Let $y=\dfrac{x^3+3}{2-x^3}$ (provided $2-x^3\neq 0$) We need to make $x$ the subject of the formula, $x=\ldots$ Multiply both sides by $2-x^3$:

$$y\left(2-x^3\right)=x^3+3$$

Expanding the Left Hand Side gives:

$$2y-x^3y=x^3+3$$

Collecting the $x^3$'s on one side:

$$\begin{aligned}2y-3&=x^3+x^3y\\&=x^3(1+y)\end{aligned}$$

How do you find $x$ on its own?
Divide both sides by $(1+y)$ where $y\neq -1$:

$$x^3=\frac{2y-3}{1+y}$$

Take the cube root of both sides:

$$x=\sqrt[3]{\frac{2y-3}{1+y}}=\left(\frac{2y-3}{1+y}\right)^{\frac{1}{3}}$$

Replacing $y$ with $x$ gives the inverse function $f^{-1}(x)=\left(\frac{2x-3}{1+x}\right)^{\frac{1}{3}}$ (provided $x\neq-1$).

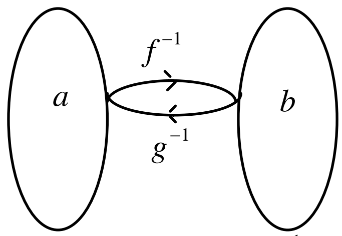
(ii) Note that $g(t)=f^{-1}(t)$ where $f^{-1}$ is same as part (i):

$$f^{-1}(t)=\left(\frac{2t-3}{1+t}\right)^{\frac{1}{3}} \quad \text{(provided } 1+t\neq 0\text{)}$$

Hence $g^{-1}(t)$ must reverse the whole process so we end up at the start with

$$g^{-1}(t)=\frac{t^3+3}{2-t^3}=f(t) \quad \text{(provided } 2-t^3\neq 0\text{)}$$

5. In both cases we have:




$f^{-1}$ takes $a$ to $b$ but $g$ takes $b$ back to $a$, so our $g^{-1}$ is the inverse of the inverse, $g^{-1}=\left(f^{-1}\right)^{-1}$.