Complete solutions to Exercise 3(e)

1. In (a) substitute values close to 0 for x. For (b) substitute large values of x.
(a) 1 (b) 0

2. We can substitute $t = 1,\ 10,\ 1000,\ 1\times 10^{6}$ and 1×10^{10}:

t	$10\left(1-2.71828^{-0.5t}\right)$
1	$10\left(1-2.71828^{-(0.5\times 1)}\right)= 6.3212$
10	$10\left(1-2.71828^{-(0.5\times 10)}\right)= 9.9995$
1000	$10\left(1-2.71828^{-(0.5\times 1000)}\right)= 10.0000$
1×10^{6}	$10\left(1-2.71828^{-\left(0.5\times 1\times 10^{6}\right)}\right)=10.0000$
1×10^{10}	$10\left(1-2.71828^{-\left(0.5\times 1\times 10^{10}\right)}\right)=10.0000$

Hence $v = 10m/s.$

3. (a) $\lim_{x\to 3}\dfrac{x^2-9}{x-3} = \lim_{x\to 3}\dfrac{(x-3)(x+3)}{x-3} = \lim_{x\to 3}(x+3) = 6$

(b) $\lim_{x\to -\frac{1}{2}}\dfrac{2x^2-x-1}{2x+1} \underbrace{=}_{\text{factorizing the numerator}} \lim_{x\to -\frac{1}{2}}\dfrac{(2x+1)(x-1)}{2x+1} = \lim_{x\to -\frac{1}{2}}(x-1) = -\dfrac{3}{2}$

(c) $\lim_{t\to 1}\dfrac{t-1}{\sqrt{t}-1} \underbrace{=}_{\text{using (1.15) on numerator}} \lim_{t\to 1}\dfrac{\left(\sqrt{t}-1\right)\left(\sqrt{t}+1\right)}{\left(\sqrt{t}-1\right)} = \lim_{t\to 1}\left(\sqrt{t}+1\right) = 2$

4. Similar to **EXAMPLE 22**; $e_{ss} = \dfrac{2}{3}$

5. Substituting for $G(s)$ gives:

$$e_{ss} = \lim_{s\to 0}\frac{s}{1+\dfrac{5(s+2)}{s(s+5)}} \qquad (*)$$

Multiplying the numerator and denominator of (*) by $s(s+5)$ gives:

$$\begin{aligned} e_{ss} &= \lim_{s\to 0}\frac{ss(s+5)}{s(s+5)+5(s+2)} \\ &= \lim_{s\to 0}\frac{s^2(s+5)}{s^2+5s+5s+10} \\ &= \lim_{s\to 0}\frac{s^2(s+5)}{s^2+10s+10} \\ e_{ss} &= 0 \end{aligned}$$

As $s\to 0$ the numerator $s^2(s+5)\to 0$ and the denominator $s^2+10s+10\to 10.$ So $e_{ss} = 0$.

(1.15) $\left(a^2-b^2\right) = (a-b)(a+b)$

6. We have:

$$e_{ss} = \lim_{s\to 0} \frac{1}{s\left(1+\frac{k}{s}\right)}$$

$$= \lim_{s\to 0} \frac{1}{s + \cancel{s}\frac{k}{\cancel{s}}}$$

$$= \lim_{s\to 0} \frac{1}{s+k}$$

$$e_{ss} = \frac{1}{k}$$

7. Multiplying $G(s) = \frac{(k_1+k_2)\times 1}{s\times(1+s\tau)} = \frac{k_1+k_2}{s(1+s\tau)}$. Substituting this into $e_{ss} = \lim_{s\to 0} \frac{M}{1+G(s)}$ gives:

$$e_{ss} = \lim_{s\to 0} \frac{M}{1+\frac{k_1+k_2}{s(1+s\tau)}}$$

$$= \lim_{s\to 0} \frac{Ms(1+s\tau)}{s(1+s\tau)+k_1+k_2}$$

$$= 0$$

because as $s \to 0,$ the numerator goes to 0 and denominator goes to $k_1 + k_2$.

8. What is $f(x+h)$?

$$f(x+h) = (x+h)^2$$

$$\underbrace{=}_{\text{by } (1.13)} x^2 + 2hx + h^2$$

Substituting this gives:

$$\lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{(x^2+2hx+h^2)-x^2}{h} = \lim_{h\to 0} \frac{2hx+h^2}{h}$$

$$= \lim_{h\to 0} \frac{h(2x+h)}{h}$$

$$\underbrace{=}_{\substack{\text{cancelling} \\ \text{the } h\text{'s}}} \lim_{h\to 0} (2x+h)$$

$$= 2x$$

9. Similar to question 8 with cubic expansion:

$$(x+h)^3 = x^3 + 3x^2h + 3xh^2 + h^3$$

Hence we end up with $\lim_{h\to 0} \frac{g(x+h)-g(x)}{h} = 3x^2$

(1.13) $$(a+b)^2 = a^2 + 2ab + b^2$$