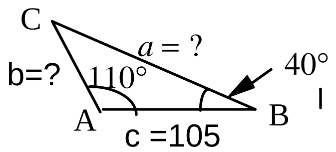
Complete Solutions to Exercise 4(d)

1.



Angle $C = 180° - 40° - 110° = 30°$

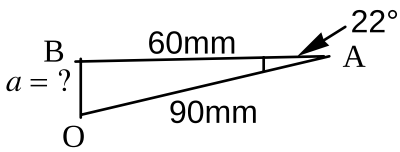
Substituting $A = 110°$, $C = 30°$ and $c = 105$ into (4.19) gives:

$$\frac{a}{\sin(110°)} = \frac{105}{\sin(30°)} = 210$$

$$a = 210 \times \sin(110°) = 197 \text{ mm} \quad (3 \text{ s.f.})$$

Similarly $b = 210 \times \sin(40°) = 135$ mm (3 s.f.)

2.



We need to apply the cosine rule (4.20) because we are given two sides and the angle between them. We can rewrite O as C and call the side opposite this angle, c.

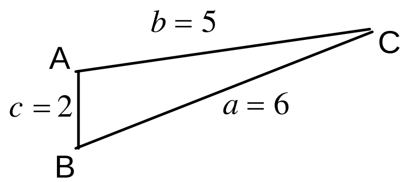
Substituting $b = 90$, $c = 60$ and $A = 22°$ into (4.20):

$$a^2 = 90^2 + 60^2 - [2 \times 90 \times 60 \times \cos(22°)]$$

$$= 1686$$

$$a = \sqrt{1686} = 41 \text{ mm} \quad (2 \text{ s.f.})$$

3. We have



How can we find angle A?

We apply the cosine rule (4.20) or (4.23) because we have 3 sides and no angle.

Substituting $a = 6$, $b = 5$ and $c = 2$ into (4.20) gives:

$$6^2 = 5^2 + 2^2 - [2 \times 5 \times 2 \times \cos(\text{A})]$$

$$36 = 29 - 20\cos(\text{A})$$

Rearranging to make $\cos(\text{A})$ the subject : $\cos(\text{A}) = \frac{29 - 36}{20} = -\frac{7}{20}$

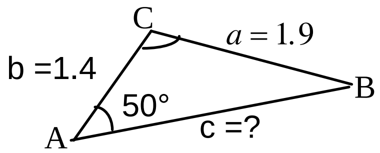
How do we find angle A?

Take the inverse cos of both sides: $\text{A} = \cos^{-1}\left(-\frac{7}{20}\right) = 110°$

(4.19) $$\frac{a}{\sin(\text{A})} = \frac{c}{\sin(\text{C})}$$

(4.20) $$a^2 = b^2 + c^2 - 2bc\cos(\text{A})$$

4. We need to find length AB in the following:



By (4.19) we have

$$\frac{c}{\sin(\mathrm{C})} = \frac{1.9}{\sin(50°)} = 2.48$$

$$c = 2.48 \times \sin(\mathrm{C}) \qquad (*)$$

We don't know the size of angle C, so how do we find length c?
We can find angle B first by using the sine rule, (4.19), again:

$$\frac{1.4}{\sin(\mathrm{B})} = 2.48$$

$$\mathrm{B} = \sin^{-1}\left(\frac{1.4}{2.48}\right) = 34.37°$$

To find angle C: $C = 180° - 50° - 34.37° = 95.63°$
Substituting $C = 95.63°$ into (*) gives $c = 2.48 \times \sin(95.63°) = 2.47$ m (2 d.p.)
(Alternatively we could use the cosine formula $c^2 = 2bc\cos(A) + a^2 - b^2$ and then solve for c)

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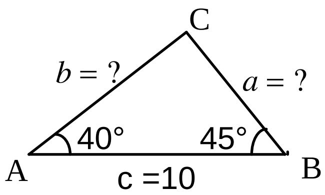
5. Need to find length a and b in the following:



We need to find angle C: angle $C = 180° - 40° - 45° = 95°$
Substituting $B = 45°$, $C = 95°$ and $c = 10$ into the sine rule, (4.19):

$$\frac{b}{\sin(45°)} = \frac{10}{\sin(95°)} = 10.038$$

$$b = 10.038 \times \sin(45°) = 7.10 \text{ m} \quad (3 \text{ s.f.})$$

Similarly $a = 10.038 \times \sin(40°) = 6.45$ m (3 s.f.)

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6. Since we are given 3 sides we use the cosine rule. Applying (4.23) :

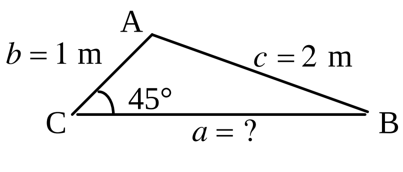
$$\cos(\mathrm{A}) = \frac{b^2 + c^2 - a^2}{2bc}$$

Substituting $a = 200$, $b = 60$ and $c = 180$ into this formula gives

$$\cos(\mathrm{A}) = \frac{60^2 + 180^2 - 200^2}{2 \times 60 \times 180} = -0.185$$

So angle $\mathrm{A} = \cos^{-1}(-0.185) = 101°$.

7.



By (4.19) we have

$$\frac{a}{\sin(\mathrm{A})} = \frac{2}{\sin(45°)} = \frac{2}{\underbrace{1/\sqrt{2}}_{\text{by TABLE 1}}} = 2\sqrt{2}$$

So $a = 2\sqrt{2}\sin(\mathrm{A})$ (†)

The problem is we do not know the size of angle A. Using (4.19) with $b = 1$ gives:

$$\frac{1}{\sin(\mathrm{B})} = \frac{a}{\sin(\mathrm{A})} = 2\sqrt{2} \text{ transposing gives } \sin(\mathrm{B}) = \frac{1}{2\sqrt{2}}$$

To find angle B we take inverse sin of both sides:

$$\mathrm{B} = \sin^{-1}\left(\frac{1}{2\sqrt{2}}\right) = 20.70°$$

Hence angle $\mathrm{A} = 180° - 20.7° - 45° = 114.3°$

Substituting this into (†) gives $a = 2\sqrt{2}\sin(114.3°) = 2.6 \text{ m}$ (2 s.f.)

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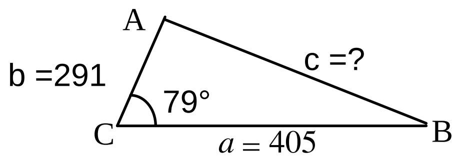
8. $\frac{1}{8}$ of a revolution is $\frac{1}{8} \times 360° = 45°$.

Same as the previous solution with $b = 30$ mm and $c = 60$ mm. Length $\mathrm{OB} = 77$ mm. (Side c is opposite O).

9.



Since we are given 2 sides and the angle between them we use the cosine rule (4.22).

$$c^2 = 405^2 + 291^2 - [2 \times 291 \times 405 \times \cos(79°)]$$

$$= 203730$$

$$c = \sqrt{203730} = 451 \text{ m} \quad (3 \text{ s.f.})$$

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10. Consider triangle ODE to find length DE. Angle $\mathrm{D} = 180° - 70° - 30° = 80°$.

Applying (4.19) gives:

$$\frac{\mathrm{DE}}{\sin(30°)} = \frac{2.5}{\sin(80°)} = 2.539$$

$$\mathrm{DE} = 2.539 \times \sin(30°) = 1.27 \text{ m} \quad (3 \text{ s.f.})$$

Similarly $\mathrm{OD} = 2.39$ m.

Now consider triangle ODC to find length CD. How can we find length CD?

Angle DOC=$90° - 30° = 60°$. We use the cosine rule (4.22) because we are given 2 sides and the angle (DOC) between them:

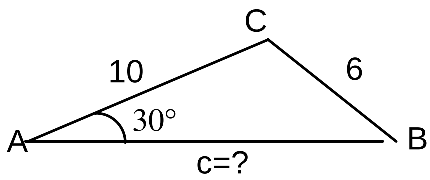
$$\mathrm{CD}^2 = 2.39^2 + 3^2 - [2 \times 3 \times 2.39 \times \cos(60°)] = 7.542$$

$$\mathrm{CD} = \sqrt{7.542} = 2.75 \text{ m} \quad (3 \text{ s.f.})$$

(4.19) $a/\sin(\mathrm{A}) = b/\sin(\mathrm{B}) = c/\sin(\mathrm{C})$

(4.22) $c^2 = a^2 + b^2 - 2ab\cos(\mathrm{C})$

11. (a) We have



Labelling according to the sine rule: $a = 6$, $\mathrm{A} = 30°$ and $b = 10$. To find angle B we apply (4.19)

$$\frac{10}{\sin(\mathrm{B})} = \frac{6}{\sin(30°)} = 12$$

$$\sin(\mathrm{B}) = \frac{10}{12} \text{ implies that } \mathrm{B} = \sin^{-1}\left(\frac{10}{12}\right) = 56.44°$$

Also $\mathrm{B} = 180° - 56.44° = 123.56°$. We consider the 2 cases.

CASE I. With $\mathrm{B} = 56.44°$ we have

$$\text{angle } \mathrm{C} = 180° - (30° + 56.44°) = 93.56°$$

To find length c we apply the sine rule again:

$$\frac{c}{\sin(93.56°)} = 12$$

$$c = 12 \times \sin(93.56°) = 11.98$$

We have angle $\mathrm{B} = 56.44°$, angle $\mathrm{C} = 93.56°$ and length AB=11.98. This triangle is shown above.

CASE II. With $\mathrm{B} = 123.56°$. Similar to case I.

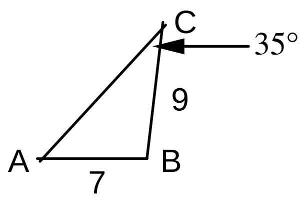
$$\text{angle } \mathrm{C} = 180° - (123.56° + 30°) = 26.44°$$

As before

$$c = 12 \times \sin(26.44°) = 5.34$$

Thus angle $\mathrm{B} = 123.56°$, angle $\mathrm{C} = 26.44°$ and length AB=5.34.

(b) We have



Angle A can be found by using the sine rule:

$$\frac{9}{\sin(\mathrm{A})} = \frac{7}{\sin(35°)} = 12.204$$

$$\sin(\mathrm{A}) = \frac{9}{12.204} \text{ gives } \mathrm{A} = \sin^{-1}\left(\frac{9}{12.204}\right) = 47.52°$$

Also angle $\mathrm{A} = 180° - 47.52° = 132.48°$. We have 2 cases $\mathrm{A} = 47.52°$ or $\mathrm{A} = 132.48°$

The remaining procedure is similar to (a).

CASE I. Angle $\mathrm{A} = 47.52°$, angle $\mathrm{B} = 97.48°$ and AC=12.1. (See diagram).

CASE II. Angle $\mathrm{A} = 132.48°$, angle $\mathrm{B} = 12.52°$ and AC=2.65.

(4.19) $$\frac{a}{\sin(\mathrm{A})} = \frac{b}{\sin(\mathrm{B})} = \frac{c}{\sin(\mathrm{C})}$$