

Complete solutions to Exercise 4(e)
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1. Using (4.26) throughout this question.

$$(i) \frac{123 \times \pi}{180} = 2.15 \text{ rad (3 s.f.)}$$

$$(ii) \frac{13 \times \pi}{180} = 0.23 \text{ rad (2 s.f.)}$$

$$(iii) \frac{131.67 \times \pi}{180} = 2.2981 \text{ rad (5 s.f.)}$$

$$(iv) \frac{333.3 \times \pi}{180} = 5.817 \text{ rad (4 s.f.)}$$

2. We apply (4.26) throughout this question.

$$(i) 90^\circ = \frac{90 \times \pi}{180} = \frac{\pi}{2} \text{ rad}$$

$$(ii) 30^\circ = \frac{\pi}{6} \text{ rad}$$

$$(iii) 330^\circ = \frac{330 \times \pi}{180} = \frac{33}{18} \pi = \frac{11}{6} \pi \text{ rad}$$

$$(iv) 22.5^\circ = \frac{22.5 \times \pi}{180} = \frac{\pi}{8} \text{ rad (because } \frac{22.5}{180} = \frac{1}{8} \text{)}$$

$$(v) 10^\circ = \frac{\pi}{18} \text{ rad}$$

$$(vi) 27^\circ = \frac{27 \times \pi}{180} = \frac{3\pi}{20} \text{ rad}$$

$$(vii) 3^\circ = \frac{3 \times \pi}{180} = \frac{\pi}{60} \text{ rad}$$

$$(viii) 144^\circ = \frac{144 \times \pi}{180} \stackrel{\substack{\equiv \\ \text{divide numerator and} \\ \text{denominator by } 18}}{=} \frac{8\pi}{10} = \frac{4\pi}{5} \text{ rad}$$

(To put a fraction in its simplest form use your calculator).

3. Applying (4.27) throughout this question.

$$(i) \frac{\pi}{6} = \frac{\cancel{\pi} \times 180}{6 \cancel{\pi}} = 30^\circ$$

$$(ii) \frac{3\pi}{10} = \left(\frac{3 \cancel{\pi} \times 180}{10 \cancel{\pi}} \right)^\circ = (3 \times 18)^\circ = 54^\circ$$

$$(iii) 1.45 = \left(\frac{1.45 \times 180}{\pi} \right)^\circ = 83.1^\circ \text{ (3 s.f.)}$$

$$(iv) \frac{\pi}{126} = \left(\frac{\cancel{\pi} \times 180}{126 \cancel{\pi}} \right)^\circ = \left(\frac{10}{7} \right)^\circ \text{ (Simplification of fraction by calculator).}$$

$$(v) 200 = \left(\frac{200 \times 180}{\pi} \right)^\circ = (1.15 \times 10^4)^\circ \text{ (3 s.f.)}$$

$$(vi) \frac{\pi}{9} = \left(\frac{\cancel{\pi} \times 180}{9 \cancel{\pi}} \right)^\circ = 20^\circ$$

$$(4.26) \quad x^\circ = \frac{x \times \pi}{180} \text{ rad}$$

$$(4.27) \quad x \text{ rad} = \left(\frac{x \times 180}{\pi} \right)^\circ$$

$$(vii) \frac{7\pi}{81} = \left(\frac{7\cancel{\pi} \times 180}{81\cancel{\pi}} \right)^\circ \quad \begin{array}{c} \equiv \\ \text{divide numerator} \\ \text{and denominator} \\ \text{by 9} \end{array} \quad \left(\frac{140}{9} \right)^\circ$$

$$(viii) \frac{13\pi}{120} = \left(\frac{13\cancel{\pi} \times 180}{120\cancel{\pi}} \right)^\circ = \left(\frac{13 \times 18}{12} \right)^\circ = \left(\frac{13 \times 3}{2} \right)^\circ = \left(\frac{39}{2} \right)^\circ = 19.5^\circ$$

4. Place your calculator into radian mode and determine the values:

$$(i) \sin\left(\frac{\pi}{10}\right) = 0.31 \qquad (ii) \cos\left(\frac{7\pi}{2}\right) = 0$$

(iii) For $\frac{\cos(7\pi)}{2}$ we evaluate $\cos(7\pi)$ first and then divide by 2:

$$\frac{\cos(7\pi)}{2} = \frac{-1}{2} = -\frac{1}{2}$$

5. (i) $\frac{\pi}{4}$ is 45° so using TABLE 1 we have $\sin\left(\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}$

$$\frac{\pi}{2} \sin\left(\frac{\pi}{4}\right) = \frac{\pi}{2} \cdot \frac{1}{\sqrt{2}} = \frac{\pi}{2\sqrt{2}}$$

(ii) Note that $\frac{\pi}{3} = 60^\circ$, so by TABLE 1 $\tan\left(\frac{\pi}{3}\right) = \sqrt{3}$. Thus,

$$\frac{1}{\sqrt{3}} \tan\left(\frac{\pi}{3}\right) = \frac{1}{\sqrt{3}} \cdot \sqrt{3} = 1 \text{ (cancelling } \sqrt{3}\text{'s)}$$

(iii) Since $\frac{\pi}{4} = 45^\circ$ we have,

$$\sin\left(\frac{\pi}{4}\right) \cos\left(\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}} \cdot \frac{1}{\sqrt{2}} = \frac{1}{2}$$

6. Since there are 2π radians in each revolution, we multiply by 2π and divide by 60 (60 seconds = 1 min).

$$(i) \frac{2500 \times 2\pi}{60} = 261.8 \text{ rad / s (4 s.f.)}$$

$$(ii) \frac{500 \times 2\pi}{60} = 52.4 \text{ rad / s (3 s.f.)}$$

$$(iii) \frac{6300 \times 2\pi}{60} = 659.7 \text{ rad / s (4 s.f.)}$$