Complete solutions to Exercise 4(f)

In these solutions A =amplitude, T = period and f = frequency.

1. We use (4.29) and (4.30) to find the period and frequency respectively.

(a) $A = 10$. With $\omega = 2$ we have $T = \frac{2\pi}{2} = \pi$, $f = \frac{1}{\pi}$

(b) $A = \frac{1}{10}$. With $\omega = \frac{1}{10}$ we have $T = \frac{2\pi}{(1/10)} = 20\pi$, $f = \frac{1}{20\pi}$

(c) $A = H$. With $\omega = \pi$ we have $T = \frac{2\cancel{\pi}}{\cancel{\pi}} = 2$, $f = \frac{1}{2} = 0.5$

(d) $A = \frac{1}{\pi}$. With $\omega = \pi^2$ we have $T = \frac{2\pi}{\pi^2} = \frac{2}{\pi}$, $f = \frac{\pi}{2}$

(e) $A = 10000$. In this case $\omega = \frac{1}{\pi^2}$, $T = \frac{2\pi}{\left(1/\pi^2\right)} = 2\pi^3$, $f = \frac{1}{2\pi^3}$

(f) $A = \frac{1}{\pi e}$, $T = \frac{2\pi}{(1/\pi e)} = 2\pi^2 e$, $f = \frac{1}{2\pi^2 e}$

(g) $A = 220$, $T = \frac{2\cancel{\pi}}{1000\cancel{\pi}} = 0.002$, $f = 500$

(h) $A = 1$. With $\omega = \frac{1}{(\pi/6)}$ we have:

$$T = \frac{2\pi}{1/(\pi/6)} = 2\pi \div \frac{1}{(\pi/6)} = 2\pi \times \frac{\pi}{6} = \frac{\pi^2}{3}, \quad f = \frac{3}{\pi^2}$$

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2. To find the period we use (4.29) and for the phase we use (4.34).

(a) $A = 4$, $T = \frac{2\pi}{1} = 2\pi$, phase $= \frac{\pi}{3}$ rad lead

(b) $A = 1$, $T = \frac{2\pi}{314.159} = 0.02$, phase $= 0.25$ rad lag

(c) We have $20\sin\left[\left(100t + \frac{1}{5}\right)\pi\right] = 20\sin\left[(100\pi)t + \frac{\pi}{5}\right]$. Thus $\omega = 100\pi$:

$$A = 20, \quad T = \frac{2\pi}{100\pi} = 0.02, \quad \text{phase } = \frac{\pi}{5} \text{ rad lead}$$

(d) $A = 315$, $T = \frac{2\pi}{7}$, phase $= 30°$ lead

(e) Amplitude is the maximum displacement $A = 7 - 5 = 2$ and $\omega = 2\pi$:

$$T = \frac{2\pi}{2\pi} = 1, \quad \text{phase } = \frac{\pi}{2} \text{ rad lag}$$

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3. We use (4.32) and (4.33) with α = phase and ω = angular velocity:

(a) 0 (b) $\frac{\pi}{7}$ lead (c) $\frac{\pi}{4}$ lead

(4.29)	$T = 2\pi/\omega$
(4.30)	$f = \frac{1}{T}$
(4.32) and (4.33)	time displacement= α/ω
(4.34)	$R\sin(\omega t \pm \alpha)$, the phase $= \alpha$

(d) We can rewrite $\cos\left[(100t-1)\pi\right]$ as $\cos(100\pi t-\pi)$. So $\omega=100\pi$ and $\alpha=\pi$ and the time displacement is $\dfrac{\pi}{100\pi}=0.01$ lag . (e) $\dfrac{\pi}{2}$ lag

(f) For $\sin\left(\dfrac{t}{10}-\pi\right)$ we have $\alpha=\pi$ and $\omega=\dfrac{1}{10}$, thus

$$\text{time displacement} = \frac{\pi}{(1/10)}=10\pi \text{ lag}$$

(g) $\dfrac{(\pi/10)}{(1/10)}=\pi$ lag (h) $\dfrac{\left(1/e^2\pi^2\right)}{\left(1/e^2\pi^2\right)}=1$ lead

4. We use (4.32) and (4.33):

(a) $A\sin\left(5t+\dfrac{\pi}{5}\right)$ leads $A\sin(5t)$ by $\dfrac{\pi}{25}$ and $A\sin\left(5t-\dfrac{\pi}{7}\right)$ lags $A\sin(5t)$ by $\dfrac{\pi}{35}$.

$$\text{time displacement} = \frac{\pi}{25}+\frac{\pi}{35}=\frac{12\pi}{175}$$

$A\sin\left(5t+\dfrac{\pi}{5}\right)$ leads $A\sin\left(5t-\dfrac{\pi}{7}\right)$ by $\dfrac{12\pi}{175}$.

(b) $\cos(7t-0.26)$ lags $\cos(7t)$ by $\dfrac{0.26}{7}=0.037$ and $5\cos(7t+\pi)$ leads $\cos(7t)$ by $\pi/7$:

$$\text{time displacement} = \pi/7+0.037=0.487$$

$5\cos(7t+\pi)$ leads $\cos(7t-0.26)$ by 0.487. Notice that the difference in amplitude between the waves plays no part in establishing the time displacement.

(c) $\cos\left(2t+\dfrac{\pi}{2}\right)$ leads $\cos(2t)$ by $\dfrac{\pi}{4}$ and $\cos\left(2t+\dfrac{\pi}{3}\right)$ leads $\cos(2t)$ by $\dfrac{\pi}{6}$

$$\text{time displacement} = \frac{\pi}{4}-\frac{\pi}{6}=\frac{\pi}{12}$$

$\cos\left(2t+\dfrac{\pi}{2}\right)$ leads $\cos\left(2t+\dfrac{\pi}{3}\right)$ by $\dfrac{\pi}{12}$.

5. (a) $A=2,\quad T=\pi$ so $\dfrac{2\pi}{\omega}=\pi$ s transposing gives $\omega=2$ rad/s. We have the cosine graph because it has a maximum at $t=0$. Thus $y=2\cos(2t)$.

(b) $A=7,\quad T=\dfrac{\pi}{4}$ s (There are 4 cycles in π seconds). By (4.29) we have $\dfrac{2\pi}{\omega}=\dfrac{\pi}{4}$ gives $\omega=8$ rad/s. The equation of the waveform is $y=7\sin(8t)$.

(c) $A=10$ and it is the sine graph shifted to the right by π rads. Since the graph completes one cycle in 2π seconds, $T=2\pi$ s:
$2\pi/\omega=2\pi$ gives $\omega=1$ rad/s and the equation of the waveform is $y=10\sin(t-\pi)$.
The answers to question 5 are **not** unique because sine and cosine graphs have the same shape and are out of phase by $\pi/2$ radians. Moreover they repeat the same wave after 2π radians. For example another answer to (c) is $y=10\cos\left(t+\dfrac{\pi}{2}\right)$.

(4.32) and (4.33) $\quad \text{time displacement} = \dfrac{\alpha}{\omega}$

6. $5\cos(\omega t)$ is the cosine graph, cutting the t- axis when $5\cos(\omega t)=0$ which gives $\cos(\omega t)=0$. By examining the cosine graph of Fig 53 in chapter 4 we notice that it cuts the t-axis at $\frac{\pi}{2},\frac{3\pi}{2},\frac{5\pi}{2},\frac{7\pi}{2}$. Thus:

$$\omega t=\frac{\pi}{2},\ \frac{3\pi}{2},\ \frac{5\pi}{2},\ \frac{7\pi}{2}$$

$$t=\frac{\pi}{2\omega},\ \frac{3\pi}{2\omega},\ \frac{5\pi}{2\omega},\ \frac{7\pi}{2\omega}\qquad \text{(Dividing by }\omega\text{)}$$

$5\cos(\omega t)$ has a peak when $\cos(\omega t)=1$. Similarly by looking at the graph of Fig 53:

$$\omega t=0,\ 2\pi,\ 4\pi\qquad \text{(points of peaks)}$$

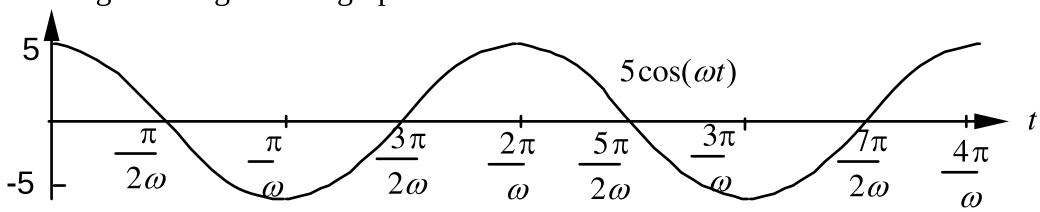
$$t=0,\ \frac{2\pi}{\omega},\ \frac{4\pi}{\omega}\qquad \text{(Dividing by }\omega\text{)}$$

$5\cos(\omega t)$ is minimum when $\cos(\omega t)=-1$, similarly this is where:

$$\omega t=\pi,\ 3\pi$$

$$t=\frac{\pi}{\omega},\ \frac{3\pi}{\omega}$$

Combining all this gives the graph:



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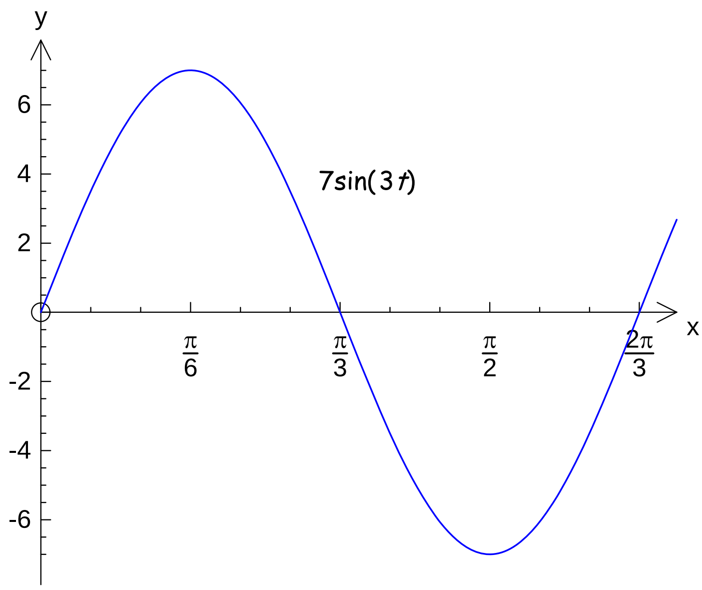
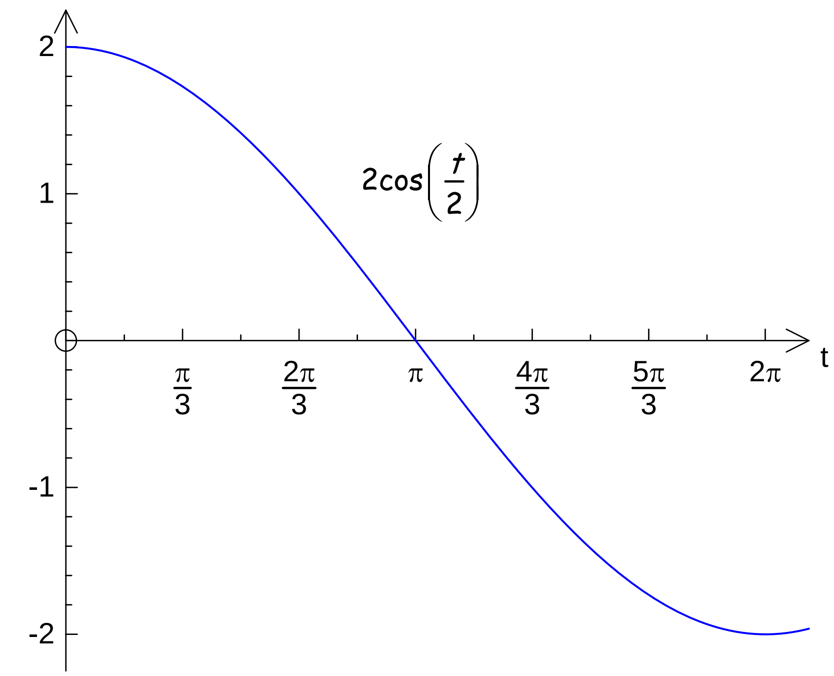
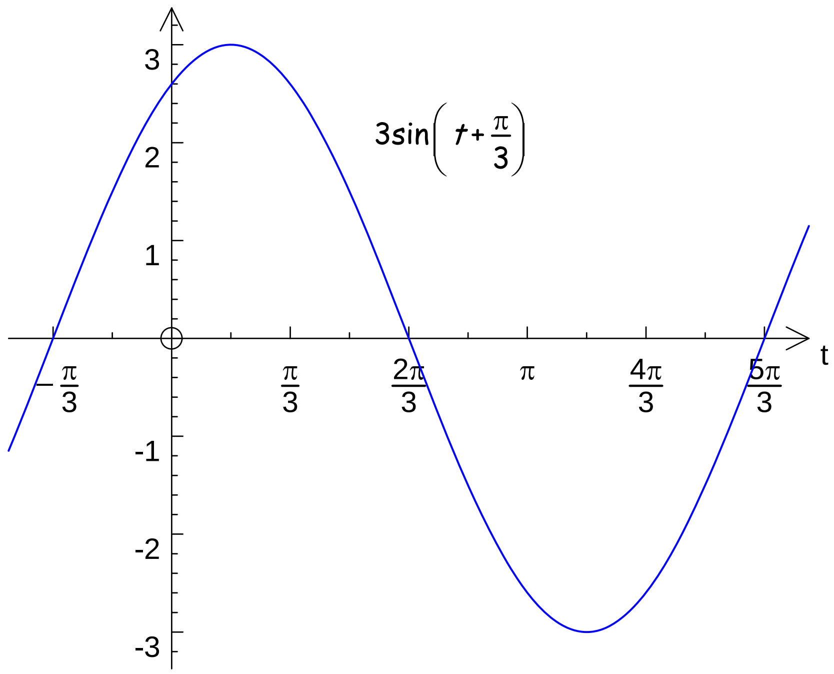
7. (a) $\omega=3$ gives period $T=\frac{2\pi}{3}$s. We have the sine graph with amplitude 7 and period $\frac{2\pi}{3}$ that is the graph completes one cycle over $\frac{2\pi}{3}s$.



(b) $\omega=\frac{1}{2}$, $T=\frac{2\pi}{(1/2)}=4\pi$. The graph of $2\cos(t/2)$ is the cosine graph but stretched so that it makes only one cycle over a period of 4π and has an amplitude of 2:



(c) The graph $3\sin\left(t+\frac{\pi}{3}\right)$ is the graph $3\sin(t)$ shifted to the left by $\frac{\pi}{3}$ *rad* and has an amplitude of 3:



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8. (a) $A=5,\quad \omega=120\pi$ gives $T=\frac{2\pi}{120\pi}=\frac{1}{60}=16.67\text{ ms}$, frequency $f=60$ Hz

$$\text{phase angle} = 0.52\ rad \underbrace{=}_{\text{by }(4.27)} \left(\frac{0.52\times 180}{\pi}\right)^{\circ} = 29.79^{\circ}$$

(b) (i) Substituting $t=0$, into $i=5\sin(120\pi t+0.52)$ gives $i=5\sin(0.52)=2.48$ mA

(ii) Substituting $t=\frac{1}{60}$, $i=5\sin(2\pi+0.52)=2.48\,\text{mA}$

(iii) Substituting $t = \dfrac{1}{30}$, $i = 5\sin(4\pi + 0.52) = 2.48\,\text{mA}$

(iv) Substituting $t = \dfrac{1}{15}$, $i = 5\sin(6\pi + 0.52) = 2.48\,\text{mA}$

Same answer because the sine function has period 2π, that is the sine graph is repeated every 2π seconds.

(c) Maximum occurs when $\sin(120\pi t + 0.52) = 1$ (because the maximum value for the sine function is 1). Taking inverse sin of both sides:

$$120\pi t + 0.52 = \sin^{-1}(1) = \frac{\pi}{2}$$

$$t = \frac{(\pi/2) - 0.52}{120\pi} = 2.79 \times 10^{-3}\,s = 2.79\ \text{ms}$$

(d) The time displacement is obtained by using (4.32) with $\alpha = 0.52$ and $\omega = 120\pi$:

$$\text{time displacement} = \frac{0.52}{120\pi} = 1.38\ \text{ms lead}$$

(e) By using the information obtained we have: