Complete solutions to Exercise 4(h)

1. From the first equation we have $N = \dfrac{mg}{\cos(\theta)}$. Substituting this into the second equation gives:

$$\frac{mg\sin(\theta)}{\cos(\theta)} = \frac{mv^2}{r}$$

$$mg\underbrace{\tan(\theta)}_{\text{by (4.35)}} = \frac{mv^2}{r}$$

Making v^2 the subject:

$$gr\tan(\theta) = v^2$$

Hence taking the square root gives the result $v = \sqrt{gr\tan(\theta)}$.

2. By putting $A = \omega_m t$ and $B = \omega_c t$ into (4.58) we have

$$\cos(\omega_m t)\cos(\omega_c t) = \frac{1}{2}\left[\cos(\omega_m + \omega_c)t + \cos(\omega_m - \omega_c)t\right]$$

So placing this into $v = V_c\cos(\omega_c t) + V_m\cos(\omega_m t)\cos(\omega_c t)$ gives:

$$v = V_c\cos(\omega_c t) + \frac{V_m}{2}\left[\cos(\omega_m + \omega_c)t + \cos(\omega_m - \omega_c)t\right]$$

$$= V_c\left\{\cos(\omega_c t) + \frac{V_m}{2V_c}\left[\cos(\omega_m + \omega_c)t + \cos(\omega_m - \omega_c)t\right]\right\}$$

$$v = V_c\left\{\cos(\omega_c t) + \frac{d}{2}\left[\cos(\omega_m + \omega_c)t + \cos(\omega_m - \omega_c)t\right]\right\} \text{ where } d = V_m/V_c$$

3. We have

$$v = V_c\sin(\omega_c t) + V_m\sin(\omega_c t)\sin(\omega_m t)$$

$$= V_c\sin(\omega_c t) + \underbrace{\frac{V_m}{2}\left[\cos(\omega_c - \omega_m)t - \cos(\omega_c + \omega_m)t\right]}_{\text{by (4.59)}}$$

$$= \frac{V_c}{2}\left\{2\sin(\omega_c t) + \frac{V_m}{V_c}\left[\cos(\omega_c - \omega_m)t - \cos(\omega_c + \omega_m)t\right]\right\}$$

$$v = \frac{V_c}{2}\left[2\sin(\omega_c t) + d\cos(\omega_c - \omega_m)t - d\cos(\omega_c + \omega_m)t\right] \text{ where } d = V_m/V_c$$

4. (a) $$p = IV\sin(\omega t)\sin(\omega t) = IV\sin^2(\omega t)$$

$$p \underbrace{=}_{\text{by (4.68)}} \frac{IV}{2}\left[1 - \cos(2\omega t)\right]$$

(b) $$p = IV\sin(\omega t)\cos(\omega t) \underbrace{=}_{\text{by (4.53)}} \frac{IV}{2}\sin(2\omega t)$$

(4.35)	$\sin(A)/\cos(A) = \tan(A)$
(4.58)	$2\cos(A)\cos(B) = \cos(A+B) + \cos(A-B)$
(4.59)	$2\sin(A)\sin(B) = \cos(A-B) - \cos(A+B)$
(4.68)	$\sin^2(A) = \frac{1}{2}\left[1 - \cos(2A)\right]$

(c) $$p = IV\sin(\omega t)\sin\left(\omega t - \frac{\pi}{2}\right)$$

$$= IV\sin(\omega t)\underbrace{\left[\sin(\omega t)\cos\left(\frac{\pi}{2}\right) - \cos(\omega t)\sin\left(\frac{\pi}{2}\right)\right]}_{\text{by (4.38)}}$$

$$= IV\sin(\omega t)\left[0 - \cos(\omega t)\right] \quad (\text{because} \quad \cos(\pi/2) = 0 \quad \text{and} \quad \sin(\pi/2) = 1)$$

$$= -IV\sin(\omega t)\cos(\omega t)$$

$$p = -\frac{IV}{2}\sin(2\omega t) \qquad (\text{by part (b) above})$$

(d) $$p = IV\cos\left(\omega t + \frac{\pi}{2}\right)\cos(\omega t)$$

$$\underbrace{=}_{\text{by (4.58)}} \frac{IV}{2}\left[\cos\left(\omega t + \frac{\pi}{2} + \omega t\right) + \cos\left(\omega t + \frac{\pi}{2} - \omega t\right)\right]$$

$$= \frac{IV}{2}\left[\cos\left(2\omega t + \frac{\pi}{2}\right) + \cos\left(\frac{\pi}{2}\right)\right]$$

$$p = \frac{IV}{2}\cos\left(2\omega t + \frac{\pi}{2}\right) \qquad (\text{because} \quad \cos(\pi/2) = 0)$$

(e) $$p = IV\sin(\omega t)\sin\left(\omega t - \frac{\pi}{2}\right)$$

$$\underbrace{=}_{\text{by (4.59)}} \frac{IV}{2}\left[\cos\left(\omega t - \left(\omega t - \frac{\pi}{2}\right)\right) - \cos\left(\omega t + \omega t - \frac{\pi}{2}\right)\right]$$

$$= \frac{IV}{2}\left[\cos\left(\frac{\pi}{2}\right) - \cos\left(2\omega t - \frac{\pi}{2}\right)\right]$$

$$p = -\frac{IV}{2}\cos\left(2\omega t - \frac{\pi}{2}\right)$$

(f) $$p = 240\sin(100\pi t) \times 12\sin\left(100\pi t - \frac{\pi}{3}\right)$$

$$\underbrace{=}_{\text{by (4.59)}} \frac{240 \times 12}{2}\left[\cos\left(100\pi t - 100\pi t + \frac{\pi}{3}\right) - \cos\left(100\pi t + 100\pi t - \frac{\pi}{3}\right)\right]$$

$$= 1440\left[\cos\left(\frac{\pi}{3}\right) - \cos\left(200\pi t - \frac{\pi}{3}\right)\right] \quad \left(\text{Remember} \quad \cos\left(\frac{\pi}{3}\right) = \frac{1}{2}\right)$$

$$p = 720 - 1440\cos\left(200\pi t - \frac{\pi}{3}\right)$$

(4.38)	$\sin(A - B) = \sin(A)\cos(B) - \cos(A)\sin(B)$
(4.53)	$\sin(2A) = 2\sin(A)\cos(A)$
(4.58)	$2\cos(A)\cos(B) = \cos(A + B) + \cos(A - B)$
(4.59)	$2\sin(A)\sin(B) = \cos(A - B) - \cos(A + B)$