Complete solutions to Exercise 5(a)

1. Remember $k = 10^3$ so by substituting the given values we have

$$100\times10^3\times(0.3)^{5/3} = 350\times10^3\times(V_2)^{5/3}$$

$$V_2^{5/3} = \frac{(100\times10^3)\times(0.3)^{5/3}}{350\times10^3} = \frac{100\times(0.3)^{5/3}}{350} = 0.038$$

How do we find V_2?

$$V_2 = (0.038)^{\frac{1}{5/3}} = (0.038)^{3/5} = 0.14 \text{ m}^3 \quad \text{(2 s.f.)}$$

2. Very similar to solution 1.

$$100\times10^3\times(0.25)^{1.33} = 450\times10^3\times(V_2)^{1.33}$$

$$(V_2)^{1.33} = \frac{100\times(0.25)^{1.33}}{450} = 0.035$$

$$V_2 = (0.035)^{1/1.33} = 0.081 \text{ m}^3 \quad \text{(2 s.f.)}$$

3. We have $P_1 = 200\times10^3$, $V_1 = 0.1$ and $P_2 = 632\times10^3$. Substituting these into

$$P_1(V_1)^{1.5} = P_2(V_2)^{1.5}$$

gives

$$200\times10^3\times(0.1)^{1.5} = 632\times10^3\times(V_2)^{1.5}$$

So

$$(V_2)^{1.5} = \frac{200}{632}\times(0.1)^{1.5} = 0.01$$

Hence $V_2 = (0.01)^{1/1.5} = 0.046 \text{ m}^3$ (2 s.f.)

4.(i) From $P_1(V_1)^k = P_2(V_2)^k$ we have

$$\frac{P_2}{P_1} = \left(\frac{V_1}{V_2}\right)^k \qquad (\dagger)$$

From the other equation, $\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}$, we have

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)\left(\frac{V_2}{V_1}\right)$$

Using (†) we have

$$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^k\left(\frac{V_2}{V_1}\right) = \left(\frac{V_1}{V_2}\right)^k\left(\frac{V_1}{V_2}\right)^{-1} \underbrace{=}_{\text{by (5.1)}} \left(\frac{V_1}{V_2}\right)^{k-1}$$

(ii) Invert the result of part (i):

$$\frac{T_1}{T_2} = \frac{1}{T_2/T_1}$$

$$\underbrace{=}_{\text{from (i)}} \frac{1}{(V_1/V_2)^{k-1}} = \left(\frac{1}{V_1/V_2}\right)^{k-1} = \left(\frac{V_2}{V_1}\right)^{k-1}$$

(5.1) $a^m a^n = a^{m+n}$

5. Multiplying out the x term inside the brackets gives:

$$x^{\frac{1}{\psi}}y^{\frac{\psi-1}{\psi}} - x^{\frac{1}{\psi}}x^{\frac{\psi-1}{\psi}} = x^{\frac{1}{\psi}}y^{\frac{\psi-1}{\psi}} - \underbrace{x^{\frac{1}{\psi}+1-\frac{1}{\psi}}}_{\text{by (5.1)}}$$

$$= x^{\frac{1}{\psi}}y^{\frac{\psi-1}{\psi}} - x$$

$$= x\left[x^{\frac{1}{\psi}-1}y^{\frac{\psi-1}{\psi}} - 1\right] \qquad (\text{Taking out } x)$$

The index of the x term inside the square bracket can be rewritten as:

$$\frac{1}{\psi} - 1 = \frac{1-\psi}{\psi} = -\left(\frac{\psi-1}{\psi}\right)$$

We have

$$x\left[x^{\frac{1}{\psi}-1}y^{\frac{\psi-1}{\psi}} - 1\right] = x\left[x^{-\left(\frac{\psi-1}{\psi}\right)}y^{\frac{\psi-1}{\psi}} - 1\right]$$

$$= x\left[\frac{y^{\frac{\psi-1}{\psi}}}{\underbrace{x^{\frac{\psi-1}{\psi}}}_{\text{by (5.4)}}} - 1\right] = x\left[\left(\frac{y}{x}\right)^{\frac{\psi-1}{\psi}} - 1\right]$$

6. Remember from chapter 1 the symbol $\sqrt{\ }$ is the square root sign and has an index of $1/2$:

$$v = \frac{r^{\frac{1}{6}}}{\eta}\sqrt{rs} = \frac{r^{\frac{1}{6}}}{\eta}\underbrace{(rs)^{\frac{1}{2}}}_{\text{by (1.1)}} = \frac{r^{\frac{1}{6}}}{\eta}r^{\frac{1}{2}}s^{\frac{1}{2}} \underbrace{=}_{\text{by (5.1)}} \frac{r^{\frac{1}{6}+\frac{1}{2}}s^{\frac{1}{2}}}{\eta} = \frac{r^{\frac{2}{3}}s^{\frac{1}{2}}}{\eta}$$

(1.1) $\sqrt{a} = a^{\frac{1}{2}}$

(5.1) $a^m a^n = a^{m+n}$

(5.4) $\frac{1}{a^n} = a^{-n}$