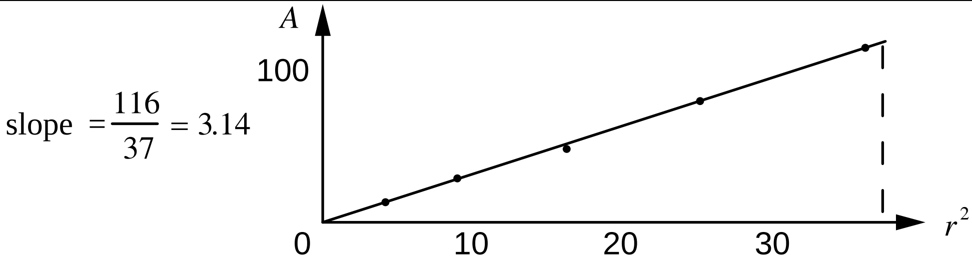
Complete solutions to Exercise 5(d)

1. We plot A against r^2:

r^2	4	9	16	25	36
A	12.57	28.27	50.27	78.54	113.10

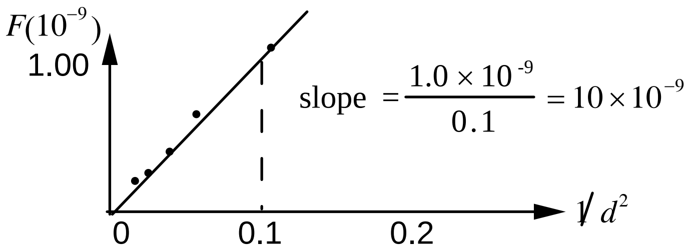


The intercept $= 0$ and gradient $= 3.14$. So $k = 3.14$ and $b = 0$. The results in the table seem to be obtained from the formula for the area of a circle πr^2.

2. Since the rule is $F = \frac{k}{d^2}$ we plot the graph of F against $\frac{1}{d^2}$:

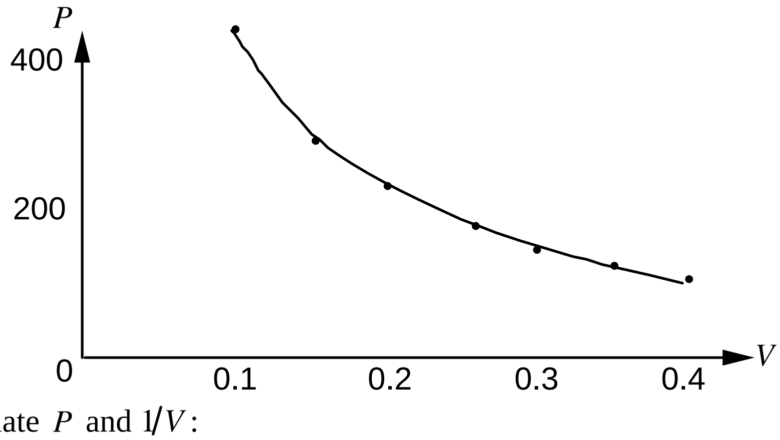
d^2	4	9	16	25	36	49
$1/d^2$	0.250	0.111	0.063	0.040	0.027	0.020
F	2.510	1.110	0.630	0.400	0.280	0.200

We plot $1/d^2$ horizontally against F vertically.



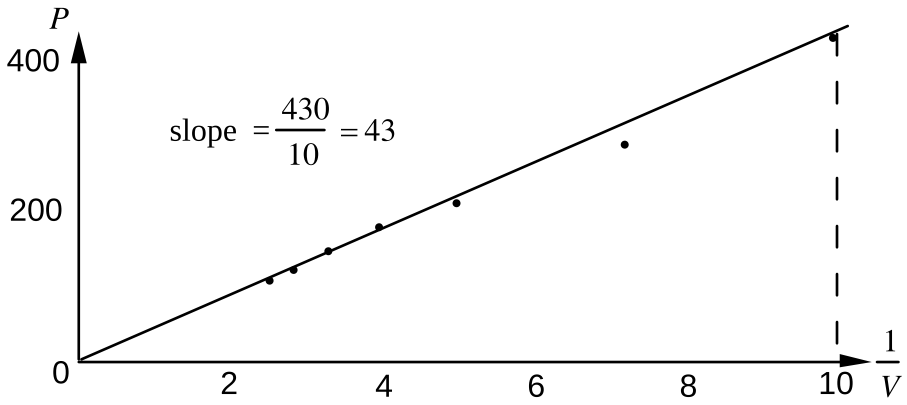
The gradient $= k$, so $k = 10 \times 10^{-9}$. Hence $F = \frac{10 \times 10^{-9}}{d^2}$.

3. (i)



(ii) We tabulate P and $1/V$:

$1/V$	10.00	6.67	5.00	4.00	3.33	2.86	2.50
P	416	277	208	166	139	119	104



(iii) P against $1/V$ is a straight line through the origin, so the intercept is equal to zero, hence $P=\frac{k}{V}$ where k is the gradient. Since $k=43$ we have

$$P=\frac{43}{V}$$

---

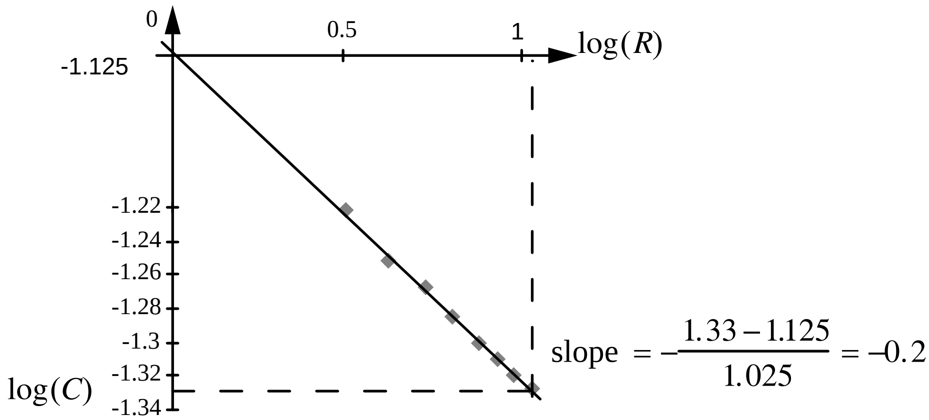
4. Taking $\log s$ of $C=kR^n$ gives:

$$\log(C)=\log(kR^n)\underbrace{=}_{\text{by (5.17)}}\log(R^n)+\log(k)$$

$$\log(C)=\underbrace{n\log(R)}_{\text{by (5.19)}}+\log(k)$$

We plot $\log(C)$ vertically against $\log(R)$ horizontally:

$\log(R)$	0.48	0.60	0.70	0.78	0.85	0.90	0.95	1.00
$\log(C)$	-1.221	-1.252	-1.268	-1.284	-1.301	-1.310	-1.319	-1.328



We have n =gradient=-0.2. What is the value of k ?
Well

$$\log(k)=-1.125 \text{ (vertical intercept)}$$

$$k=10^{-1.125}=0.075$$

Checking the fitted parameters by calculation is useful. As a check the values of C using $0.075R^{-0.2}$ for $R=3,\ 6$ and 10 gives 0.060, 0.052, 0.047 respectively.

(5.17) $\log(AB)=\log(A)+\log(B)$

(5.19) $\log(A^n)=n\log(A)$

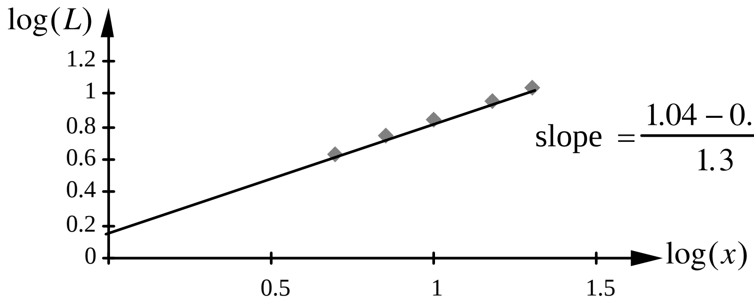
5. Taking $\log s$ of $L = kx^n$ gives:

$$\log(L) = \log(kx^n)$$

$$= n\log(x) + \log(k)$$

Plot $\log(L)$ against $\log(x)$:

$\log(x)$	0.70	0.85	1.00	1.18	1.30
$\log(L)$	0.64	0.74	0.84	0.96	1.04



$$\text{slope} = \frac{1.04 - 0.16}{1.3} = 0.68$$

Since we have a straight line so the results obey

$$L = kx^n$$

where $n = \text{slope} = 0.54$ and

$$\log(k) = 0.164$$

$$k = 10^{0.164} = 1.46$$

Hence $L = 1.46x^{0.68}$.

As a check:

x	5.00	7.00	10.00	15.00	20.00
L (given in question)	4.39	5.49	6.95	9.12	11.05
My L	4.33	5.44	6.92	9.11	11.06