Complete solutions to Exercise 5(e)

1. By using a calculator we have:

$\sinh(10) = 11013.23$, $\sinh(-3) = -10.02$, $\cosh(10) = 11013.23$, $\cosh\left(1\times 10^{-3}\right) = 1$,

$\tanh\left(5\times 10^{7}\right) = 1$ and $\tanh(0.5) = 0.46$

2. (a) See Fig 12. (b) See Fig 13.

3. Replacing x with $-x$ into (5.24) gives:

$$\begin{aligned}\sinh(-x) &= \frac{e^{-x} - e^{-(-x)}}{2} \\ &= \frac{e^{-x} - e^{x}}{2} \\ &= -\left(\frac{e^{x} - e^{-x}}{2}\right) \\ &\underbrace{=}_{\text{by (5.24)}} -\sinh(x)\end{aligned}$$

4. Applying (5.26) gives:

$$\tanh(-x) = \frac{\sinh(-x)}{\cosh(-x)} = \frac{-\sinh(x)}{\cosh(x)} \underbrace{=}_{\text{by (5.26)}} -\tanh(x)$$

5. Applying (1.14):

$$\begin{aligned}\cosh^2(x) - 2\cosh(x)\sinh(x) + \sinh^2(x) &= \left[\cosh(x)\right]^2 - 2\cosh(x)\sinh(x) + \left[\sinh(x)\right]^2 \\ &= \left[\cosh(x) - \sinh(x)\right]^2 \qquad [\text{by (1.14)}] \\ &= \underbrace{\left(e^{-x}\right)^2}_{\text{by (5.30)}} = \underbrace{e^{-2x}}_{\text{by (5.3)}}\end{aligned}$$

6. Similarly

$$\cosh^2(x) + 2\cosh(x)\sinh(x) + \sinh^2(x) = \left[\cosh(x) + \sinh(x)\right]^2 = \left(e^{x}\right)^2 = e^{2x}$$

7. Substituting $x = a\cosh(t)$ and $y = b\sinh(t)$ into $\frac{x^2}{a^2} - \frac{y^2}{b^2}$ gives:

$$\begin{aligned}\frac{x^2}{a^2} - \frac{y^2}{b^2} &= \frac{\left[a\cosh(t)\right]^2}{a^2} - \frac{\left[b\sinh(t)\right]^2}{b^2} \\ &= \frac{a^2\cosh^2(t)}{a^2} - \frac{b^2\sinh^2(t)}{b^2} = \cosh^2(t) - \sinh^2(t) \underbrace{=}_{\text{by (5.31)}} 1\end{aligned}$$

(1.14)	$a^2 - 2ab + b^2 = (a-b)^2$
(5.3)	$\left(a^m\right)^n = a^{mn}$
(5.24)	$\left(e^{x} - e^{-x}\right)/2 = \sinh(x)$
(5.26)	$\tanh(x) = \sinh(x)/\cosh(x)$
(5.30)	$\cosh(x) - \sinh(x) = e^{-x}$
(5.31)	$\cosh^2(x) - \sinh^2(x) = 1$

8. We have:

$$v^2 = 1.8\times 300\times \tanh\left(\frac{6.3\times 35}{300}\right) = 1.8\times 300\times \tanh(0.735) = 338.102$$

$$v = 18.39 \text{ m/s} \qquad (\text{Taking square root})$$

9. Substituting $L = 30\times 10^{-3}$ gives:

$$Q = 15\left[\frac{\sinh(2.56\times 30\times 10^{-3}) + (6\times 10^{-3})\cosh(2.56\times 30\times 10^{-3})}{\cosh(2.56\times 30\times 10^{-3}) + (6\times 10^{-3})\sinh(2.56\times 30\times 10^{-3})}\right]$$

$$= 15\left[\frac{0.077 + (6.018\times 10^{-3})}{1.003 + (4.613\times 10^{-4})}\right] = 1.24W$$

10. Putting $x = L$ gives:

$$I_L = \left(\frac{\frac{v}{z}+I}{2}\right)e^{\gamma L} - \left(\frac{\frac{v}{z}-I}{2}\right)e^{-\gamma L}$$

$$= \underbrace{\frac{v}{z}\left[\frac{e^{\gamma L}-e^{-\gamma L}}{2}\right]}_{\text{collecting the } \frac{v}{z} \text{ terms}} + \underbrace{I\left[\frac{e^{\gamma L}+e^{-\gamma L}}{2}\right]}_{\text{collecting the I terms}}$$

$$= \frac{v}{z}\underbrace{\sinh(\gamma L)}_{\text{by(5.24)}} + I\underbrace{\cosh(\gamma L)}_{\text{by(5.25)}}$$

11. We have:

$$T^2 = H^2\left[1+\sinh^2\left(\frac{wx}{H}\right)\right] \underbrace{=}_{\text{by (5.31)}} H^2\cosh^2\left(\frac{wx}{H}\right)$$

Hence

$$T = \sqrt{H^2\cosh^2\left(\frac{wx}{H}\right)} = H\cosh\left(\frac{wx}{H}\right)$$

12. We have:

$$I_x = \frac{Ie^{-\gamma L}}{Z_0+Z}\left[(Z_0+Z)e^{\gamma x} + (Z_0-Z)e^{-\gamma x}\right]$$

$$= \frac{Ie^{-\gamma L}}{Z_0+Z}\left[Z_0\left(e^{\gamma x}+e^{-\gamma x}\right) + Z\left(e^{\gamma x}-e^{-\gamma x}\right)\right]$$

$$= \frac{Ie^{-\gamma L}}{Z_0+Z}\left[2Z_0\left(\frac{e^{\gamma x}+e^{-\gamma x}}{2}\right) + 2Z\left(\frac{e^{\gamma x}-e^{-\gamma x}}{2}\right)\right]$$

$$= \frac{2Ie^{-\gamma L}}{Z_0+Z}\left[Z_0\underbrace{\cosh(\gamma x)}_{\text{by (5.25)}} + Z\underbrace{\sinh(\gamma x)}_{\text{by (5.24)}}\right]$$

(5.24) $\left(e^x - e^{-x}\right)/2 = \sinh(x)$

(5.25) $\left(e^x + e^{-x}\right)/2 = \cosh(x)$

(5.31) $\cosh^2(x) - \sinh^2(x) = 1$

13. Similar to solution 11. Multiplying numerator and denominator by $\left(Z_0+Z\right)$ gives:

$$Z_x=Z_0\left[\frac{\left(Z_0+Z\right)e^{\gamma x}+\left(Z-Z_0\right)e^{-\gamma x}}{\left(Z_0+Z\right)e^{\gamma x}+\left(Z_0-Z\right)e^{-\gamma x}}\right]$$

$$=Z_0\left[\frac{Z_0\left(e^{\gamma x}-e^{-\gamma x}\right)+Z\left(e^{\gamma x}+e^{-\gamma x}\right)}{Z_0\left(e^{\gamma x}+e^{-\gamma x}\right)+Z\left(e^{\gamma x}-e^{-\gamma x}\right)}\right]$$

$$=Z_0\left[\frac{2Z_0\sinh\left(\gamma x\right)+2Z\cosh\left(\gamma x\right)}{2Z_0\cosh\left(\gamma x\right)+2Z\sinh\left(\gamma x\right)}\right]$$

$$Z_x=Z_0\left[\frac{Z_0\sinh\left(\gamma x\right)+Z\cosh\left(\gamma x\right)}{Z_0\cosh\left(\gamma x\right)+Z\sinh\left(\gamma x\right)}\right] \quad \text{(Cancelling 2's)}$$

14.(i) From $A+B=\theta_0$ we have $B=\theta_0-A$. Substituting this into $\theta_L=Ae^{kL}+Be^{-kL}$ gives:

$$\theta_L=Ae^{kL}+\left(\theta_0-A\right)e^{-kL}$$

$$=A\left(e^{kL}-e^{-kL}\right)+\theta_0e^{-kL}$$

Rearranging: $$A\left(e^{kL}-e^{-kL}\right)=\theta_L-\theta_0e^{-kL}$$

$$A=\frac{\theta_L-\theta_0e^{-kL}}{e^{kL}-e^{-kL}}$$

(ii) $$B=\theta_0-A\underbrace{=}_{\text{from part (i)}}\theta_0-\frac{\theta_L-\theta_0e^{-kL}}{e^{kL}-e^{-kL}}$$

$$=\frac{\theta_0\left(e^{kL}-e^{-kL}\right)-\theta_L+\theta_0e^{-kL}}{e^{kL}-e^{-kL}}$$

$$=\frac{\theta_0e^{kL}-\theta_0e^{-kL}-\theta_L+\theta_0e^{-kL}}{e^{kL}-e^{-kL}}$$

$$B=\frac{\theta_0e^{kL}-\theta_L}{e^{kL}-e^{-kL}}$$

(iii) Substituting for A and B from parts (i) and (ii) gives:

$$\theta(x)=Ae^{kx}+Be^{-kx}=\frac{\left(\theta_L-\theta_0e^{-kL}\right)e^{kx}+\left(\theta_0e^{kL}-\theta_L\right)e^{-kx}}{e^{kL}-e^{-kL}}$$

$$=\frac{\theta_Le^{kx}-\theta_0e^{-kL+kx}+\theta_0e^{kL-kx}-\theta_Le^{-kx}}{e^{kL}-e^{-kL}}$$

$$=\frac{\theta_L\left(e^{kx}-e^{-kx}\right)+\theta_0\left[e^{k(L-x)}-e^{-k(L-x)}\right]}{e^{kL}-e^{-kL}}$$

$$\theta\left(x\right)\underbrace{=}_{\text{by (5.24)}}\frac{\theta_L\sinh\left(kx\right)+\theta_0\sinh\left[k\left(L-x\right)\right]}{\sinh\left(kL\right)}$$

(5.24) $$\left(e^x-e^{-x}\right)/2=\sinh(x)$$