Complete solutions to Exercise 6(b)

1. (a) We have

$$y = x^3$$

$$\frac{dy}{dx} \underbrace{=}_{\text{by (6.2)}} 3x^2$$

(b) $y = 3x^5 + x^3$. We have

$$\frac{dy}{dx} = \underbrace{(5\times 3)x^{5-1}}_{\text{by (6.3)}} + \underbrace{3x^2}_{\text{by part (a)}}$$

$$= 15x^4 + 3x^2 = 3x^2\left(5x^2+1\right) \quad \text{(Factorizing)}$$

(c) $y = \sqrt{x} + \sin(x)$. Rewrite this as $y = x^{1/2} + \sin(x)$

$$\frac{dy}{dx} = \underbrace{\frac{1}{2}x^{1/2-1}}_{\text{by (6.2)}} + \underbrace{\cos(x)}_{\text{by (6.6)}} = \frac{1}{2}x^{-1/2} + \cos(x)$$

(d) $y = \frac{1}{\sqrt[3]{x}} + e^x + \cos(x)$. Note that $\sqrt[n]{a} = a^{1/n}$

$$y = \frac{1}{x^{1/3}} + e^x + \cos(x) = x^{-1/3} + e^x + \cos(x)$$

$$\frac{dy}{dx} = -\frac{1}{3}x^{-1/3-1} + e^x + \underbrace{\left(-\sin(x)\right)}_{\text{by (6.7)}}$$

$$= -\frac{1}{3}x^{-4/3} + e^x - \sin(x)$$

(e) $y = \frac{3}{\sqrt{x}} + \frac{5}{\sqrt[3]{x}}$. Rewrite this as $y = \frac{3}{x^{1/2}} + \frac{5}{x^{1/3}} = 3x^{-1/2} + 5x^{-1/3}$.

Differentiating by (6.3) gives:

$$\frac{dy}{dx} = 3\left(-\frac{1}{2}\right)x^{-1/2-1} + 5\left(-\frac{1}{3}\right)x^{-1/3-1}$$

$$= -\frac{3}{2}x^{-3/2} - \frac{5}{3}x^{-4/3}$$

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2. (i) $v = \frac{ds}{dt} = 75 - 3t^2$

(ii) Hence substituting $v = 0$ into part (i):

$$75 - 3t^2 = 0$$

$$3t^2 = 75$$

$$t^2 = 25 \text{ gives } t = 5s$$

(6.2) $\left(x^n\right)' = nx^{n-1}$

(6.3) $\left(kx^n\right)' = nkx^{n-1}$

(6.6) $\left(\sin(x)\right)' = \cos(x)$

(6.7) $\left(\cos(x)\right)' = -\sin(x)$

3. We have $V = \frac{1}{\rho} = \rho^{-1}$. Differentiating gives $\frac{dV}{d\rho} = -\rho^{-2} = -\frac{1}{\rho^2}$.

4. Using TABLE 2 we have

(a) $\frac{d}{dx}\left[\cos(3x)\right] = -3\sin(3x)$ (b) $\frac{d}{dx}\left[\sin(\pi x)\right] = \pi\cos(\pi x)$

(c) $\frac{d}{dx}\left(e^{11x}\right) = 11e^{11x}$

5. Since $\phi = K\sin(2\pi ft)$ we have

$$\begin{aligned} E &= -N\frac{d}{dt}\left[K\sin(2\pi ft)\right] \\ &= -KN\frac{d}{dt}\left[\sin(2\pi ft)\right] \\ &\underbrace{=}_{\text{by (6.12)}} -2\pi fKN\cos(2\pi ft) \end{aligned}$$

6. We use the chain rule, (6.10), for each case:

(a) Let $u = x^2$ then $\frac{du}{dx} = 2x$ and $y = \sin(u)$, $\frac{dy}{du} = \cos(u)$

$$\frac{dy}{dx} = 2x\cos(u) = 2x\cos\left(x^2\right)$$

(b) Let $u = \sin(x)$ then $\frac{du}{dx} = \cos(x)$ and $y = e^u$, $\frac{dy}{du} = e^u$

$$\frac{dy}{dx} = e^u\cos(x) = e^{\sin(x)}\cos(x)$$

(c) Let $u = x^2$, $\frac{du}{dx} = 2x$ and $y = \ln(u)$, $\frac{dy}{du} \underbrace{=}_{\text{by (6.9)}} \frac{1}{u}$

$$\begin{aligned} \frac{dy}{dx} &= \frac{1}{u}2x \\ &= \frac{1}{x^2}2x = \frac{2}{x} \end{aligned}$$

Alternatively: $\ln\left(x^2\right) = 2\ln(x)$

$$\frac{dy}{dx} = 2\frac{d}{dx}\left[\ln(x)\right] = \frac{2}{x}$$

(d) Let $u = 2x^3 - 3x$, $\frac{du}{dx} = 6x^2 - 3$ and $y = \cos(u)$, $\frac{dy}{du} = -\sin(u)$

$$\begin{aligned} \frac{dy}{dx} &= \left(6x^2 - 3\right)\left[-\sin(u)\right] \\ &= -\left(6x^2 - 3\right)\left[\sin\left(2x^3 - 3x\right)\right] \\ &= \left(3 - 6x^2\right)\left[\sin\left(2x^3 - 3x\right)\right] \end{aligned}$$

(6.9) $$\frac{d}{dx}\left[\ln(x)\right] = \frac{1}{x}$$

(6.10) $$\frac{dy}{dx} = \frac{dy}{du}\cdot\frac{du}{dx}$$

(6.12) $$\frac{d}{dt}\left[\sin(kt)\right] = k\cos(kt)$$

(e) Let $u = x^2 + 1$, $\frac{du}{dx} = 2x$ and $y = u^{10}$, $\frac{dy}{du} = 10u^9$

$$\frac{dy}{dx} = 10u^9(2x) = 10(2x)\left(x^2+1\right)^9 = 20x\left(x^2+1\right)^9$$

(f) Let $u = 5x^7 + 3x^4$, $\frac{du}{dx} = 35x^6 + 12x^3$ and $y = \tan(u)$, $\frac{dy}{du} = \sec^2(u)$

$$\frac{dy}{dx} = \left(35x^6 + 12x^3\right)\sec^2\left(5x^7 + 3x^4\right)$$