Complete solutions to Exercise 6(c)

1. (a) Applying (6.14) with $u = x^2 + 2$ gives:

$$\frac{dy}{dx} = 3\left(x^2+2\right)^2 \frac{d}{dx}\left(x^2+2\right)$$
$$= 3\left(x^2+2\right)^2 2x$$
$$= 6x\left(x^2+2\right)^2$$

(b) Using (6.19) with $u = x^2$:

$$\frac{dy}{dx} = \cos\left(x^2\right)\frac{d}{dx}\left(x^2\right)$$
$$= \cos\left(x^2\right).2x = 2x\cos\left(x^2\right)$$

(c) Similarly:

$$\frac{dy}{dx} = -\sin\left(x^2+2x\right).\frac{d}{dx}\left(x^2+2x\right)$$
$$= -\sin\left(x^2+2x\right).\left(2x+2\right)$$
$$= -\left(2x+2\right)\sin\left(x^2+2x\right)$$

(d) Using (6.18) with $u = x^3 + x$:

$$\frac{dy}{dx} = \frac{1}{x^3+x}\frac{d}{dx}\left(x^3+x\right)$$
$$= \frac{1}{x^3+x}\left(3x^2+1\right) = \frac{3x^2+1}{x^3+x}$$

(e) Finding the relevant formula from **TABLE 3**:

$$\frac{dy}{dx} = \cosh\left(3x^5+x^3\right).\frac{d}{dx}\left(3x^5+x^3\right)$$
$$= \cosh\left(3x^5+x^3\right).\left(15x^4+3x^2\right)$$
$$= \left(15x^4+3x^2\right)\cosh\left(3x^5+x^3\right)$$

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2. (a) Using (6.18) with $u = \sin(x)$:

$$\frac{dy}{dx} = \frac{1}{\sin(x)}\frac{d}{dx}\left[\sin(x)\right]$$
$$= \frac{1}{\sin(x)}\left[\cos(x)\right]$$
$$= \frac{\cos(x)}{\sin(x)} = \cot(x)$$

(6.14) $\left(u^n\right)' = nu^{n-1}\frac{du}{dx}$

(6.18) $\left(\ln[u]\right)' = \frac{1}{u}\frac{du}{dx}$

(6.19) $\left[\sin(u)\right]' = \cos(u)\frac{du}{dx}$

(b) Applying (6.18) again with $u = \cosh(x)$

$$\begin{aligned}\frac{dy}{dx} &= \frac{1}{\cosh(x)}\frac{d}{dx}\left[\cosh(x)\right]\\ &= \frac{\sinh(x)}{\cosh(x)}\\ &= \tanh(x)\end{aligned}$$

(c) Using (6.21) with $u = \cos(x)$:

$$\begin{aligned}\frac{dy}{dx} &= \sec^2\left[\cos(x)\right]\frac{d}{dx}\left[\cos(x)\right]\\ &= -\sin(x)\sec^2\left[\cos(x)\right]\end{aligned}$$

(d) By (6.17) with $u = x^2$ and $a = 10$:

$$\begin{aligned}\frac{dy}{dx} &= 10^{x^2}\ln(10)\frac{d}{dx}\left(x^2\right)\\ &= 10^{x^2}\ln(10).2x\\ &= 2x(10)^{x^2}\ln(10)\end{aligned}$$

(e) Using (6.18) and (6.19):

$$\begin{aligned}\frac{d}{dx}\left[\sin\left[\ln\left(x^2\right)\right]\right] &= \cos\left[\ln\left(x^2\right)\right].\frac{1}{x^2}2x\\ &= \frac{2\cos\left[\ln\left(x^2\right)\right]}{x}\end{aligned}$$

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3. By using (6.16) with $u = -(1\times 10^3)t$ and $\frac{du}{dt} = -(1\times 10^3)$:

$$\frac{di}{dt} = \left(5\times10^{-3}\right)\left[\left(1\times10^3\right)e^{-\left(1\times10^3\right)t}\right]$$

$$v = \left(3\times10^{-3}\right)\left(5\times10^{-3}\right)\left(1\times10^3\right)e^{-\left(1\times10^3\right)t} = \left(15\times10^{-3}\right)e^{-\left(1\times10^3\right)t}$$

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4. To differentiate the exponential function we use (6.16) with $u = -t/RC$:

$$\begin{aligned}i &= C\frac{d}{dt}\left[Ee^{-t/RC}\right]\\ &= \cancel{C}.E\left(-\frac{1}{R\cancel{C}}\right)e^{-t/RC}\\ &= -\frac{E}{R}e^{-t/RC}\end{aligned}$$

(6.17) $\left(a^u\right)' = a^u\ln(a)\frac{du}{dx}$

(6.18) $\left(\ln[u]\right)' = \frac{1}{u}\frac{du}{dx}$

(6.21) $\left(\tan(u)\right)' = \sec^2(u)\frac{du}{dx}$

5. We have $i = I_s\left(e^{11600V/\eta T} - 1\right)$. By using (6.16) with $u = 11600V/\eta T$:

$$\frac{di}{dV} = \frac{11600}{\eta T} I_s e^{11600V/\eta T}$$

Substituting into $r = \dfrac{1}{di/dV}$ gives:

$$r = \frac{1}{\dfrac{11600}{\eta T} I_s e^{11600V/\eta T}} = \frac{\eta T e^{-11600V/\eta T}}{11600 I_s}$$

6. $q = \tau I_s e^{\alpha v/\eta}$. By (6.16) with $u = \alpha v/\eta$ and so $\dfrac{du}{dv} = \dfrac{\alpha}{\eta}$:

$$C_D = \frac{dq}{dv} = \frac{\alpha}{\eta} \tau I_s e^{\alpha v/\eta}$$

7. We have $\dfrac{d}{dt}\left[1 - e^{-\sqrt{t}}\right] = -\dfrac{d}{dt}\left[e^{-t^{1/2}}\right]$ because the derivative of 1 is 0.

Let $u = -t^{1/2}$, then $\dfrac{du}{dt} = -\dfrac{1}{2}t^{-1/2}$. By (6.16) we have

$$\frac{d}{dt}\left[e^{-t^{1/2}}\right] = -\frac{1}{2}t^{-1/2}e^{-t^{1/2}}$$

$$= -\frac{e^{-t^{1/2}}}{2\sqrt{t}}$$

Hence $\dfrac{d}{dt}\left[1 - e^{-\sqrt{t}}\right] = \dfrac{e^{-t^{1/2}}}{2\sqrt{t}} = \dfrac{1}{2\sqrt{t}e^{\sqrt{t}}}$.

8. We have $F = 4 - 20(5 + t/3)^{-1}$. Differentiating F by using (6.14) with $u = 5 + \dfrac{t}{3}$:

$$\frac{dF}{dt} = 0 - \left(20 \times (-1)\right)(5 + t/3)^{-2} \frac{d}{dt}(5 + t/3)$$

$$= 20(5 + t/3)^{-2}\frac{1}{3}$$

$$= \frac{20}{3(5 + t/3)^2}$$

(6.14) $\quad \left(u^n\right)' = nu^{n-1}\dfrac{du}{dx}$

(6.16) $\quad \left(e^u\right)' = e^u\dfrac{du}{dx}$