Complete solutions to Exercise 7(d)

1 (i) For $s = 0$ we have:

$$s = t^2 - 4t + 3 = 0$$
$$(t-3)(t-1) = 0 \qquad [\text{Factorizing}]$$
$$t = 1\text{ s},\ t = 3\text{ s gives zero displacement}$$

(ii) $s = t^2 - 4t + 3,\ v = \dfrac{ds}{dt} = 2t - 4,\ a = \dfrac{d^2s}{dt^2} = 2\text{ m / s}^2$

2.
$$h = 25t - 4.9t^2 \qquad (\dagger)$$
$$v = \frac{dh}{dt} = 25 - 9.8t$$
$$a = \frac{d^2h}{dt^2} = -9.8\text{ m/s}^2$$

Note constant acceleration. The maximum height is reached by the particle when $\dfrac{dh}{dt} = 0$ and $\dfrac{d^2h}{dt^2} < 0$. We already have $\dfrac{d^2h}{dt^2} = -9.8 < 0$.

$$25 - 9.8t = 0 \text{ gives } t = \frac{25}{9.8} = 2.55\text{s}$$

By (7.2), when $t = 2.55s$ the maximum height is reached. So the maximum height can be found by substituting $t = 2.55$ into $(\dagger)$:

$$h = (25 \times 2.55) - (4.9 \times 2.55^2) = 31.9\text{ m}$$

3. We have $s = 3t^3 - 9t^2 + 10$:

$$v = \frac{ds}{dt} = 9t^2 - 18t$$
$$9t^2 - 18t = 0,\ t(9t - 18) = 0 \quad \text{gives} \quad t = 0\text{ s or } t = 2\text{ s}$$
$$a = \frac{dv}{dt} = \frac{d}{dt}(9t^2 - 18t) = 18t - 18$$
$$18t - 18 = 0 \quad \text{gives} \quad t = 1\text{ s}$$

4. We have:

$$s = 3t^3 - 10t^2 + t - 10$$
$$v = \frac{ds}{dt} = 9t^2 - 20t + 1$$
$$a = \frac{d^2s}{dt^2} = 18t - 20$$

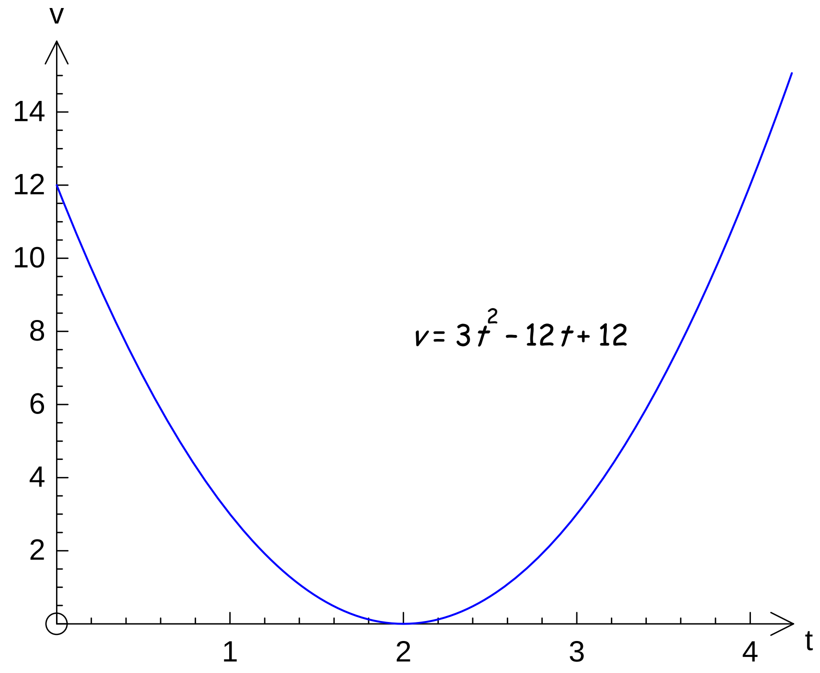
5. From $s = t^3 - 6t^2 + 12t$ we have

$$v = \frac{ds}{dt} = 3t^2 - 12t + 12$$
$$= 3(t^2 - 4t + 4) \qquad [\text{Factorizing}]$$
$$= 3(t-2)^2$$

By using the section on completing the square of chapter 2, we know the minimum velocity occurs at $t = 2$ and has a value of $v = 0\text{ m / s}$. Also when

(7.2) $\quad h' = 0,\ h'' < 0$ maximum

$t=0,\ \ v=3(0-2)^2=12\ \text{m}\ /\ \text{s}$. We have:



6. Differentiating $s=t^3-12t^2+18t$ gives:

$$v=\frac{ds}{dt}=3t^2-24t+18=3\left(t^2-8t+6\right) \qquad (*)$$

To find the max, min points of v, we can complete the square on (*).

$$v=3\left(t^2-8t+6\right)=3(t-4)^2-30$$

We have a min at $t=4$ with a velocity $=-30\ \text{m}\ /\ \text{s}$.

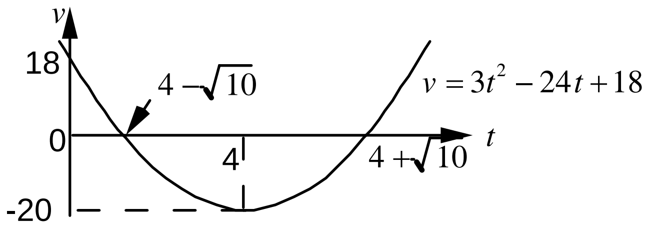
When $t=0,\ \ v=18$ m/s. Also we can find t when $v=0$. For (*) to be zero we have

$$3(t-4)^2-30=0$$

$$(t-4)^2=\frac{30}{3}=10$$

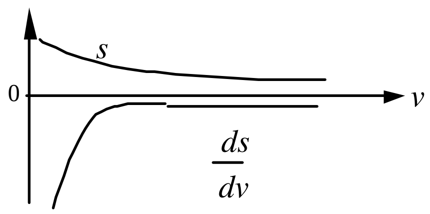
$$t=4\pm\sqrt{10}$$

Hence $v=0$ at $t=4+\sqrt{10},\quad t=4-\sqrt{10}$. Combining the above:



7. We have

$$s=v^{-1},\ \ \frac{ds}{dv}=-v^{-2}=-\frac{1}{v^2}:$$



$-\frac{1}{v^2}$ varies much more rapidly than $\frac{1}{v}$

8. We have

$$\theta = t^3 - 80\pi t$$

$$\omega = \frac{d\theta}{dt} = 3t^2 - 80\pi \qquad \text{[Differentiating]}$$

$$\alpha = \frac{d^2\theta}{dt^2} = 6t \qquad \text{[Differentiating]}$$

At $t = 1$, $\omega = 3.(1)^2 - 80\pi = -248$ rad/s and $\alpha = 6$ rad/s^2.

9. Similar to **EXAMPLE 14**. (i) By (7.12), we need to differentiate θ twice to obtain α.

$$\theta = \frac{1}{4}\sin\left(2t + \frac{\pi}{6}\right),\ \frac{d\theta}{dt} = \frac{2}{4}\cos\left(2t + \frac{\pi}{6}\right) = \frac{1}{2}\cos\left(2t + \frac{\pi}{6}\right),\ \frac{d^2\theta}{dt^2} = -\frac{2}{2}\sin\left(2t + \frac{\pi}{6}\right)$$

(ii) For $\alpha = 0$ we have

$$-\sin\left(2t + \frac{\pi}{6}\right) = 0$$

$$\sin\left(2t + \frac{\pi}{6}\right) = 0$$

$$2t + \frac{\pi}{6} = \sin^{-1}(0) = \pi$$

$$2t = \pi - \frac{\pi}{6} = \frac{5\pi}{6},\ t = \frac{5\pi}{12} s$$

10. The MAPLE output is on Web site.

(7.12) $\qquad \alpha = \theta''$