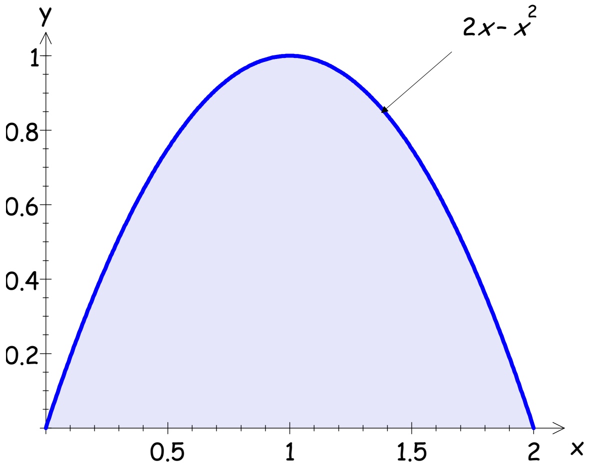
Complete solutions to Exercise 8(d)

1. Need to find the area of glass needed.



$$\text{Area} = \int_0^2 \left(2x - x^2\right) dx$$

$$= \left[x^2 - \frac{x^3}{3}\right]_0^2 = \left[2^2 - \frac{2^3}{3}\right] = \frac{4}{3} \text{ units}^2$$

2.

$$s = \int_0^2 \left(t^2 - 2t\right) dt$$

$$\underbrace{=}_{\text{by (8.1)}} \left[\frac{t^3}{3} - t^2\right]_0^2 = \frac{2^3}{3} - 2^2 = -\frac{4}{3} \text{ m}$$

3. Taking out the 100 gives:

$$W = 100\int_0^{0.5} x\,d\,x$$

$$= 100\left[\frac{x^2}{2}\right]_0^{0.5} = 100 \times \frac{0.5^2}{2} = 12.5 \text{ J}$$

4.

$$W = \int_0^{0.3} \left(1000\,x + 50\,x^3\right) d\,x = \left[\frac{1000\,x^2}{2} + \frac{50\,x^4}{4}\right]_0^{0.3} = 45.1 \text{ J}$$

5. Taking out the 20 gives:

$$s = 20\int_0^2 \left(1 - e^{-t}\right) d\,t$$

$$= 20\left[t + e^{-t}\right]_0^2 = 20\left[\left(2 + e^{-2}\right) - \left(0 + e^0\right)\right] = 22.71 \text{ m}$$

(8.1) $$\int u^n du = \frac{u^{n+1}}{n+1}$$

6.

$$s = 10\int_5^6 e^{0.4t}\,d\,t$$

$$= \underbrace{10\left[\frac{e^{0.4t}}{0.4}\right]_5^6}_{\text{by (8.41)}} = \frac{10}{0.4}\left[e^{(0.4\times 6)} - e^{(0.4\times 5)}\right] = 90.85\text{ m}$$

7.

$$P = \int_0^6\left(800 + \frac{x^3}{2}\right)d\,x = \left[800x + \frac{x^4}{8}\right]_0^6 \qquad (\text{Integrating})$$

$$= (800\times 6) + \frac{6^4}{8} = 4962 = 4.96\text{ kN (3 s.f.)}$$

$$R = \int_0^6\left(800 + \frac{x^3}{2}\right)xd\,x = \int_0^6\left(800x + \frac{x^4}{2}\right)dx \qquad (\text{Multiplying by } x)$$

$$= \left[400x^2 + \frac{x^5}{10}\right]_0^6 = (400\times 36) + \frac{6^5}{10} = 15177.6$$

$$= 15.2\text{ kNm (3 s.f.)}$$

8. We have

$$\Delta h = \frac{1.5}{1300}\int_{150}^{500}(3000 + T - 100)\,d\,T$$

$$= \frac{1.5}{1300}\int_{150}^{500}(2900 + T)\,d\,T$$

$$= \frac{1.5}{1300}\left[2900T + \frac{T^2}{2}\right]_{150}^{500} = 1.3\times 10^3\text{ J or }1.3\text{ kJ}$$

9. Differentiating $600 - 3v$ with respect to v gives -3. So

$$t = \int_{80}^{120}\frac{dv}{600-3v} = -\frac{1}{3}\int_{80}^{120}\frac{-3dv}{600-3v}$$

$$\underbrace{=}_{\text{by (8.42)}} -\frac{1}{3}\Big[\ln|600-3v|\Big]_{80}^{120}$$

$$= -\frac{1}{3}\Big[\ln|600-(3\times 120)| - \ln|600-(3\times 80)|\Big]$$

$$= -\frac{1}{3}\underbrace{\ln\left(\frac{600-360}{600-240}\right)}_{\text{by (5.12)}} = 0.135\text{hr} \approx 8.1\text{min}$$

(8.41) $\int e^{kt+m}dt = \dfrac{e^{kt+m}}{k}$

(5.12) $\ln(A) - \ln(B) = \ln(A/B)$

10. From $PV^{1.25} = 1789$ we have

$$P = \frac{1789}{V^{1.25}} = 1789V^{-1.25}$$

Substituting for $P = 1789V^{-1.25}$ into the integral gives

$$W = 1789\int_{0.01}^{0.1} V^{-1.25}\, dV = 1789\left[\underbrace{\frac{V^{-0.25}}{-0.25}}_{\text{by (8.1)}}\right]_{0.01}^{0.1}$$

$$= -\frac{1789}{0.25}\left[0.1^{-0.25} - 0.01^{-0.25}\right] = 9903.95 = 9.9 \text{ kJ}$$

If W is positive then this indicates expansion.

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11. Taking out the constant $\frac{2m}{r^2}$ gives:

$$I = \frac{2m}{r^2}\int_0^r x^3\, dx = \frac{2m}{r^2}\left[\frac{x^4}{4}\right]_0^r \qquad \text{(Integrating)}$$

$$= \frac{2m}{r^2}\left(\frac{r^4}{4}\right) = \frac{mr^2}{2} \qquad \text{(Cancelling)}$$

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12. Taking out the constant $\frac{2m}{b^2 - a^2}$ gives:

$$I = \frac{2m}{b^2 - a^2}\int_a^b x^3\, dx$$

$$= \frac{2m}{b^2 - a^2}\left[\frac{x^4}{4}\right]_a^b \qquad \text{(Integrating)}$$

$$= \frac{2m}{b^2 - a^2}\left[\frac{b^4 - a^4}{4}\right] \qquad \text{(Substituting)}$$

$$= \frac{m}{b^2 - a^2}\,\frac{\left(b^2 - a^2\right)\left(b^2 + a^2\right)}{2}$$

$$I = \frac{m\left(b^2 + a^2\right)}{2} \qquad \left(\text{Cancelling } b^2 - a^2\right)$$

Note that $b^4 - a^4$ can be written as the difference between two squares:

$$b^4 - a^4 = \left(b^2 - a^2\right)\left(b^2 + a^2\right)$$

13.

$$I = \frac{m}{2r}\int_0^{2r} x^2\, dx = \frac{m}{2r}\left[\frac{x^3}{3}\right]_0^{2r} = \frac{m}{2r}\left(\frac{8r^3}{3}\right) = \frac{4mr^3}{r3} = \frac{4mr^2}{3} \qquad \text{(Cancelling the } r\text{'s)}$$

(8.1) $$\int u^n du = \frac{u^{n+1}}{n+1}$$

14. Taking out $\dfrac{m\sin^2(\theta)}{l}$ as a constant.

$$I = \frac{m\sin^2(\theta)}{l}\int_0^l x^2\,d\,x$$

$$= \frac{m\sin^2(\theta)}{l}\underbrace{\left[\frac{x^3}{3}\right]_0^l}_{\text{by (8.1)}} = \frac{m\sin^2(\theta)}{l}\left(\frac{l^3}{3}\right) = \frac{m\,l^2\sin^2(\theta)}{3} \qquad [\text{Cancelling } l\text{'s}]$$

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15. Same as solution 11: $J = \dfrac{mr^2}{2}$

16.

$$M = k\int_0^r \left(rx^{2/7} - x^{9/7}\right)d\,x$$

$$\underbrace{=}_{\text{by (8.1)}} k\left[\frac{rx^{9/7}}{\frac{9}{7}} - \frac{x^{16/7}}{\frac{16}{7}}\right]_0^r \qquad (\text{Integrating})$$

$$= k\left(\frac{7rr^{9/7}}{9} - \frac{7r^{16/7}}{16}\right) \qquad (\text{Substituting})$$

$$= 7k\left(\frac{r^{16/7}}{9} - \frac{r^{16/7}}{16}\right) = 7kr^{16/7}\left(\frac{1}{9} - \frac{1}{16}\right) = 7kr^{16/7}\left(\frac{7}{144}\right) = \frac{49kr^{16/7}}{144}$$

17.

$$Q = 2\pi u\int_0^R r\,d\,r = 2\pi u\left[\frac{r^2}{2}\right]_0^R = 2\pi u\frac{R^2}{2} = \pi u R^2 \qquad \text{(Cancelling 2's)}$$

18.

$$v_{avg} = \frac{V}{T}\int_0^T \cos(\omega t + \theta)\,d\,t = \frac{V}{T}\underbrace{\left[\frac{\sin(\omega t + \theta)}{\omega}\right]_0^T}_{\text{by}(8.38)} = \frac{V}{\omega T}\left[\sin\left(\omega T + (\theta)\right) - \sin(\theta)\right]$$

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19. Very similar to solution 18:

$$\overline{x} = \frac{a}{T}\int_0^T \cos(\omega t + \phi)\,d\,t = \frac{a}{T}\underbrace{\left[\frac{\sin(\omega t + \phi)}{\omega}\right]_0^T}_{\text{by}(8.38)} = \frac{a}{\omega T}\left[\sin(\omega T + \phi) - \sin(\phi)\right]$$

(8.1) $\displaystyle\int u^n du = \frac{u^{n+1}}{n+1}$

(8.38) $\displaystyle\int \cos(kt + m)dt = \sin(kt + m)/k$

20.

$$I_{AV}=\frac{\omega}{\pi}\int_0^{\frac{\pi}{\omega}} I\sin(\omega t)\,dt$$

$$=\frac{\omega I}{\pi}\int_0^{\frac{\pi}{\omega}}\sin(\omega t)\,dt=\frac{\omega I}{\pi}\underbrace{\left[\frac{-\cos(\omega t)}{\omega}\right]_0^{\frac{\pi}{\omega}}}_{\text{by}(8.39)}\qquad\text{(Integrating)}$$

$$=-\frac{\omega I}{\pi\omega}\left\{\underbrace{\left[\cos\left[\omega\left(\frac{\pi}{\omega}\right)\right]\right]}_{\text{substituting } t=\frac{\pi}{\omega}}-\underbrace{\left[\cos(0)\right]}_{\text{substituting } t=0}\right\}$$

$$=-\frac{I}{\pi}(\cos(\pi)-1)=-\frac{I}{\pi}(-1-1)=-\frac{I}{\pi}(-2)$$

$$I_{AV}=\frac{2}{\pi}I$$

21. Taking out the C gives:

$$W=C\int_0^V V\,dV=C\left[\frac{V^2}{2}\right]_0^V=\frac{CV^2}{2}$$

22.

$$R=\frac{\rho}{2\pi}\int_a^b\frac{dr}{r}=\frac{\rho}{2\pi}\underbrace{\left[\ln(r)\right]_a^b}_{\text{by (8.2)}}=\frac{\rho}{2\pi}\left[\ln(b)-\ln(a)\right]=\frac{\rho}{2\pi}\underbrace{\ln\left(\frac{b}{a}\right)}_{\text{by (5.12)}}$$

23. By taking out 100 we have

$$v=\frac{100}{10\times10^{-6}}\int_0^t e^{-(5\times10^3)t}\,dt$$

$$=(10\times10^6)\underbrace{\left[\frac{e^{-(5\times10^3)t}}{-(5\times10^3)}\right]_0^t}_{\text{by (8.41)}}$$

$$=-\left(\frac{10\times10^6}{5\times10^3}\right)\left[e^{-(5\times10^3)t}-\underbrace{1}_{\text{because } e^0=1}\right]$$

$$v=-(2\times10^3)\left[e^{-(5\times10^3)t}-1\right]$$

(8.2) $\int\frac{du}{u}=\ln|u|$

(5.12) $\ln(A)-\ln(B)=\ln(A/B)$

(8.39) $\int\sin(kt+m)dt=-\cos(kt+m)/k$

(8.41) $\int e^{kt+m}dt=e^{kt+m}/k$

24. We can take out most of the integrand, $\dfrac{Ir\sin(\beta)}{4\pi(d^2+r^2)}$:

$$F=\frac{Ir\sin(\beta)}{4\pi\left(d^2+r^2\right)}\int_0^{2\pi}d\phi=\frac{Ir\sin(\beta)}{4\pi\left(d^2+r^2\right)}[\phi]_0^{2\pi}$$

$$=\frac{Ir\sin(\beta)}{4\pi\left(d^2+r^2\right)}(2\pi-0)=\frac{Ir\sin(\beta)}{4\pi\left(d^2+r^2\right)}2\pi=\frac{Ir\sin(\beta)}{2\left(d^2+r^2\right)}$$

25.

$$F=\int_0^{\pi}\frac{I\sin(\theta)}{4\pi r}d\theta$$

$$=\frac{I}{4\pi r}\int_0^{\pi}\sin(\theta)\,d\theta$$

$$=-\frac{I}{4\pi r}\left[\cos(\theta)\right]_0^{\pi} \qquad \left(\text{Integrating by }\int\sin(\theta)\,d\theta=-\cos(\theta)\right)$$

$$=-\frac{I}{4\pi r}\Bigg(\underbrace{\cos(\pi)}_{=-1}-\underbrace{\cos(0)}_{=1}\Bigg) \qquad (\text{Substituting})$$

$$=-\frac{I}{4\pi r}[-1-1]=-\frac{I}{4\pi r}(-2)$$

$$F=\frac{I}{2\pi r}$$

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26. Taking out $\dfrac{Q}{4\pi\varepsilon_0}$ gives:

$$V=\frac{Q}{4\pi\varepsilon_0}\int_a^b\frac{dr}{r^2}=\frac{Q}{4\pi\varepsilon_0}\int_a^b r^{-2}dr=\frac{Q}{4\pi\varepsilon_0}\left[\frac{r^{-1}}{-1}\right]_a^b=-\frac{Q}{4\pi\varepsilon_0}\left[\frac{1}{r}\right]_a^b=-\frac{Q}{4\pi\varepsilon_0}\left[\frac{1}{b}-\frac{1}{a}\right]$$

Taking the negative sign into the square brackets gives:

$$V=\frac{Q}{4\pi\varepsilon_0}\left[\frac{1}{a}-\frac{1}{b}\right]$$

27. Since $h(x)$ is a function of x, we replace the t with x:

$$\int_0^t h(x)\,dx=\int_0^t\left(x^2-4x+9\right)dx=\left[\frac{x^3}{3}-\frac{4x^2}{2}+9x\right]_0^t=\frac{t^3}{3}-2t^2+9t$$

Substituting into $f(t)$ gives:

$$f(t)=\left(t^2-4t+9\right)\exp\left[-\left(\frac{t^3}{3}-2t^2+9t\right)\right]$$

28. We find $h(x)$ by replacing t with x in $h(t)$. Thus $h(x)=3-x$.

$$\int_0^t h(x)\,dx=\int_0^t(3-x)\,dx=\left[3x-\frac{x^2}{2}\right]_0^t=3t-\frac{t^2}{2}$$

Substituting into $f(t)$ gives:

$$f(t)=(3-t)\exp\left[-\left(3t-\frac{t^2}{2}\right)\right]=(3-t)\exp\underbrace{\left[\frac{t^2}{2}-3t\right]}_{\text{taking in the negative sign}}$$

29. We have $f(x) = 0.02(10 - x)$ replacing t with x. Considering the integration:

$$\int_0^t f(x)\,dx = \int_0^t 0.02(10-x)\,dx = 0.02\int_0^t (10-x)\,dx$$

$$= 0.02\left[10x - \frac{x^2}{2}\right]_0^t = 0.02\left[10t - \frac{t^2}{2}\right]$$

$$= 0.2t - 0.01t^2$$

Thus

$$R(t) = 1 - \left(0.2t - 0.01t^2\right) = 1 - 0.2t + 0.01t^2$$

Substituting this into $h(t)$ gives

$$h(t) = \frac{f(t)}{R(t)} = \frac{0.02(10-t)}{0.01t^2 - 0.2t + 1}$$

30. Replacing t with x gives $h(x) = \left(2\times 10^{-3}\right)x^{-1/2}$:

$$\int_0^t h(x)\,dx = \left(2\times10^{-3}\right)\int_0^t x^{-1/2}\,dx$$

$$= \left(2\times10^{-3}\right)\left[\frac{x^{1/2}}{\frac{1}{2}}\right]_0^t = \left(4\times10^{-3}\right)t^{1/2}$$

Hence $R(t) = e^{-\left(4\times10^{-3}\right)t^{1/2}}$.

31.

$$W = C\int_{v_1}^{v_2}\frac{dV}{V} = C\underbrace{\left[\ln(V)\right]_{v_1}^{v_2}}_{\text{by (8.2)}}$$

$$= C\left[\ln(v_2) - \ln(v_1)\right] \underbrace{=}_{\text{by (5.12)}} C\ln\left(\frac{v_2}{v_1}\right)$$

32.

$$W \underbrace{=}_{\text{by (8.1)}} C\left[\frac{P^{-\frac{1}{n}+1}}{-\frac{1}{n}+1}\right]_{p_1}^{p_2} = \frac{nC}{n-1}\left[p_2^{1-\frac{1}{n}} - p_1^{1-\frac{1}{n}}\right]$$

33.

$$W = C\int_{V_1}^{V_2} V^{-k}\,dV$$

$$\underbrace{=}_{\text{by (8.1)}} C\left[\frac{V^{-k+1}}{-k+1}\right]_{V_1}^{V_2} \qquad \text{(Integrating)}$$

$$= \frac{C}{1-k}\left(V_2^{-k+1} - V_1^{-k+1}\right) \qquad \text{(Substituting)}$$

(5.12) $\ln(A) - \ln(B) = \ln(A/B)$ (8.2) $\int \frac{du}{u} = \ln|u|$

(8.1) $\int u^n\,du = \frac{u^{n+1}}{n+1}$

$$= \frac{CV_2^{-k}V_2 - CV_1^{-k}V_1}{1-k} = \frac{P_2V_2^kV_2^{-k}V_2 - P_1V_1^kV_1^{-k}V_1}{1-k}$$

$$= \frac{P_2V_2 - P_1V_1}{1-k} \underbrace{=}_{\substack{\text{multiplying top} \\ \text{and bottom by -1}}} \frac{P_1V_1 - P_2V_2}{k-1}$$

34. Rearranging gives $P = CV^{-1.5}$. Therefore

$$W = \int_{V_1}^{V_2} PdV = \int_{V_1}^{V_2} CV^{-1.5}dV = \frac{C}{-0.5}\left(V_2^{-0.5} - V_1^{-0.5}\right)$$

$$= -2C\left(\frac{1}{V_2^{0.5}} - \frac{1}{V_1^{0.5}}\right)$$

$$= 2C\left(\frac{1}{V_1^{0.5}} - \frac{1}{V_2^{0.5}}\right)$$

$$= 2C\left(\frac{1}{\sqrt{V_1}} - \frac{1}{\sqrt{V_2}}\right)$$

35. Our limits of integration are $\frac{x}{L} = 1$ and $\frac{x}{L} = 0$:

$$C_F = k\int_0^1 \left(\frac{x}{L}\right)^{-1/2} d\left(\frac{x}{L}\right) = k \underbrace{\left[\frac{\left(\frac{x}{L}\right)^{1/2}}{1/2}\right]_0^1}_{\text{by (8.1) with } u=x/L}$$

$$= 2k\left[\left(\frac{x}{L}\right)^{1/2}\right]_0^1 = 2k\left[1-0\right] = 2k$$

(8.1) $\int u^n du = \frac{u^{n+1}}{n+1}$