Complete solutions to Miscellaneous Intro

1. What does $1mm^3$ mean?

$$1mm^3 = 1mm \times 1mm \times 1mm$$

Since $1000mm = 1m$ so $1mm = 0.001m$

$$1mm^3 = 1mm \times 1mm \times 1mm = 0.001m \times 0.001m \times 0.001m$$
$$= 1 \times 10^{-9} m^3$$

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2. (a) $5^3 = 5 \times 5 \times 5 = 125$

(b) $\sqrt{100} = 10$

(c) We can either use a calculator to evaluate $\sqrt[3]{-8}$ or we know $2 \times 2 \times 2 = 8$ and so $(-2)\times(-2)\times(-2) = -8$. Thus

$$\sqrt[3]{-8} = -2$$

(d) $\sqrt{\dfrac{196}{49}} = \dfrac{\sqrt{196}}{\sqrt{49}} = \pm\dfrac{14}{7} = (14 \div 7) = 2$

(e) In the case $\sqrt{12^2 + 5^2}$ we cannot take the square root of each individual term. We first evaluate $12^2 + 5^2$ and then take the square root.

$$\sqrt{12^2 + 5^2} = \sqrt{144 + 25} = \sqrt{169} = 13$$

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3. (a) To add fractions $\dfrac{2}{3} + \dfrac{3}{5}$ we have to find the lowest common multiple of the denominators 3 and 5. Since both these numbers are prime the lowest common multiple is $3 \times 5 = 15$.

$$\frac{2}{3} = \frac{2 \times 5}{3 \times 5} = \frac{10}{15}$$

$$\frac{3}{5} = \frac{3 \times 3}{5 \times 3} = \frac{9}{15}$$

Replacing the original fractions, $\dfrac{2}{3}$ and $\dfrac{3}{5}$, with equivalent fractions, $\dfrac{10}{15}$ and $\dfrac{9}{15}$ respectively, gives:

$$\frac{2}{3} + \frac{3}{5} = \frac{10}{15} + \frac{9}{15} = \frac{10+9}{15} = \frac{19}{15}$$

Of course the numbers 19 and 15 have no whole number which is a common factor so it cannot be reduced any further.

(b) How do we multiply fractions?

Multiply the numerators and multiply the denominators:

$$\frac{2}{3} \times \frac{4}{5} = \frac{2 \times 4}{3 \times 5} = \frac{8}{15}$$

(c) How do we divide fractions?

We turn the second fraction upside down and multiply:

$$\frac{2}{3} \div \frac{3}{5} = \frac{2}{3} \times \frac{5}{3} = \frac{2 \times 5}{3 \times 3} = \frac{10}{9}$$

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4. (a) (i) $3.1415926 = 3.142$ (3 d.p.) (ii) $3.1415926 = 3.14$ (3 s.f.)

(b) We first evaluate $\sqrt{10}$ on a calculator:

$$\sqrt{10} = 3.162278 \text{ (6 d.p.)}$$

(i) $\sqrt{10} = 3.16$ (2 d.p.) (ii) $\sqrt{10} = 3.162$ (3 d.p.)

(c) (i) $1.6449 = 1.64$ (2 d.p.) (ii) $1.6449 = 1.645$ (3 d.p.)

(d) First we calculate 16^5 on a calculator:

$$16^5 = 1048576$$

(i) $16^5 = 1000000$ (1 s.f.) (ii) 1050000 (3 s.f.)

(e) How do we evaluate $2^{2^4}+1$ on a calculator?

Using a calculator, $2^4 = 16$. We have

$$\begin{aligned} 2^{2^4}+1 &= 2^{16}+1 \\ &= 65536+1 \\ &= 65537 \end{aligned}$$

(i) $2^{2^4}+1 = 65500$ (3 s.f.) (ii) $2^{2^4}+1 = 66000$ (2 s.f.)

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5. For this question we use BROIDMAS and a calculator: (All solutions are correct to 2 d.p.).

(a) $\dfrac{\pi}{4}+1 = (\pi \div 4)+1 = 1.79$

(b) We have

$$\begin{aligned} \frac{-(-7)\pm\sqrt{(-7)^2-(4\times1\times12)}}{2} &= \frac{7\pm\sqrt{49-48}}{2} \\ &= \frac{7\pm\sqrt{1}}{2} \\ &= \frac{7\pm1}{2} \\ &= \frac{7+1}{2} \text{ or } \frac{7-1}{2} \\ &= \frac{8}{2} \text{ or } \frac{6}{2} \\ &= 4 \text{ or } 3 = 4.00 \text{ or } 3.00 \end{aligned}$$

(c) We first evaluate the square root of $(30\times5)+1$ and then multiply the result by $\dfrac{2}{3}$:

$$\begin{aligned} \frac{2}{3}\sqrt{(30\times5)+1} &= \frac{2}{3}\sqrt{151} = \frac{2}{3}\times\sqrt{151} \\ &= \frac{2}{3}\times 12.288 \\ &= 8.19 \end{aligned}$$

(d) We first evaluate the brackets (<u>B</u>ROIDMAS) and then take the result to the power of 5. To get the final answer we multiply by $\sqrt{10\pi}$:

$$\begin{aligned} \left(\frac{5}{2.718}\right)^5\sqrt{10\pi} &= (1.840)^5\sqrt{10\pi} \\ &= 21.067\times\sqrt{10\times\pi} \\ &= 21.067\times\sqrt{31.416} \\ &= 21.067\times5.605 \\ &= 118.08 \end{aligned}$$

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6. Need to use the Exp or EE button on your calculator for this question. The Exp or EE button gives 10^{index}:

(a) $\dfrac{1}{2\pi\times 50\times 3\times 10^{-6}}=\dfrac{1}{9.425\times 10^{-4}}$ then press $\dfrac{1}{x}$ or x^{-1} button to give the result 1061.03 (2 d.p.)

(b) $\dfrac{\left(5\times 10^{6}\right)\pm\sqrt{\left(5\times 10^{6}\right)^{2}-\left(16\times 10^{12}\right)}}{2}=\dfrac{\left(5\times 10^{6}\right)\pm\sqrt{\left(25\times 10^{12}\right)-\left(16\times 10^{12}\right)}}{2}$

$$=\frac{\left(5\times 10^{6}\right)\pm\sqrt{9\times 10^{12}}}{2}$$

$$=\frac{\left(5\times 10^{6}\right)\pm\left(3\times 10^{6}\right)}{2}$$

$$=\frac{\left(5\times 10^{6}\right)-\left(3\times 10^{6}\right)}{2}\text{ or }\frac{\left(5\times 10^{6}\right)+\left(3\times 10^{6}\right)}{2}$$

$$=\frac{2\times 10^{6}}{2}\text{ or }\frac{8\times 10^{6}}{2}$$

$$=1\times 10^{6}\text{ or }4\times 10^{6}$$

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7. (a) $378000V=0.378\times 10^{6}\,V=0.378MV$ $\left(\text{because } M=mega=10^{6}\right)$

(b) $0.00001A=10\times 10^{-6}\,A=10\mu A$ $\left(\text{because } \mu=micro=10^{-6}\right)$

(c) $1300\Omega=1.3\times 10^{3}\,\Omega=1.3k\Omega$ $\left(\text{because } k=kilo=10^{3}\right)$

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8. (a) We first find 5% of 100Ω:

$$5\%\text{ of }100\Omega=\frac{5}{1\cancel{0}\cancel{0}}\times 1\cancel{0}\cancel{0}\Omega=5\Omega$$

Thus $100\Omega\pm 5\%=100\Omega\pm 5\Omega=(100-5)\Omega$ to $(100+5)\Omega=95\Omega$ to 105Ω.

(b) What is $5k\Omega$ equal to?

$$5k\Omega=5\times 10^{3}\Omega=5000\Omega$$

How do we find 2.5% of 5000Ω?

$$2.5\%\text{ of }5000\Omega=\frac{2.5}{100}\times 5000\Omega$$

$$=125\Omega$$

Thus

$$5k\Omega\pm 2.5\%=5000\Omega\pm 125\Omega$$

$$=5000\Omega-125\Omega\text{ to }5000\Omega+125\Omega$$

$$=4875\Omega\text{ to }5125\Omega$$

$$=4.875k\Omega\text{ to }5.125k\Omega$$

(c) What is $13M\Omega$?

$$13M\Omega=13\times 10^{6}\,\Omega=13000000\Omega$$

Similarly

$$0.1\%\text{ of }13000000\Omega=\frac{0.1}{1\cancel{0}\cancel{0}}\times 130000\cancel{0}\cancel{0}\Omega$$

$$=13000\Omega$$

Also

$$13M\Omega \pm 0.1\% = 13000000\Omega \pm 13000\Omega$$
$$= 13000000\Omega - 13000\Omega \text{ to } 13000000\Omega + 13000\Omega$$
$$= 12987000\Omega \text{ to } 13013000\Omega$$
$$= 12.987M\Omega \text{ to } 13.013M\Omega$$

9. What is the easiest way to simplify the ratio $1\frac{1}{2} : 10\frac{3}{4}$?

Use your calculator $\left(a\, {}^{b}\!/_{c} \text{ button}\right)$:

$$1\frac{1}{2} : 10\frac{3}{4} = 1\frac{1}{2} \div 10\frac{3}{4}$$
$$= \frac{6}{43} \qquad \text{(by calculator)}$$

Hence $1\frac{1}{2} : 10\frac{3}{4} = 6:43$.

10. We first simplify $0.5 \times 10^{-3} \times 50 \times 10^{3}$, how?

$10^{-3} = \frac{1}{10^3}$, we have

$$0.5 \times 10^{-3} \times 50 \times 10^{3} = 0.5 \times \frac{1}{10^3} \times 50 \times 10^3$$
$$= 0.5 \times 1 \times 50 \qquad \left(\text{cancelling } 10^3\text{'}s\right)$$
$$= 25$$

We need to find 15% of 25.

$$15\% \text{ of } 25 = \frac{15}{100} \times 25$$
$$= \frac{15}{4}$$

Hence 15% of $0.5 \times 10^{-3} \times 50 \times 10^{3}$ is $\frac{15}{4}$.

11. Similar to question 10, what can we cancel this time?

10^6's because they are common between the numerator and denominator:

$$\text{thermal efficiency} = \frac{3.5 \times 10^6}{24 \times 10^6} = \frac{3.5}{24}$$

How do we write $\frac{3.5}{24}$ as a percentage?

Multiply by 100: $\frac{3.5}{24} \times 100 = 14.58\%$ (2 d.p.)

12. What is $20kW$ equal to ?

Remember k denotes kilo which is 10^3:

$$20kW = 20 \times 10^3\, W = 20000W$$

Thus the loss is:

$$6\% \text{ of } 20000W = \frac{6}{10\not{0}\not{0}} \times 200\not{0}\not{0}W = 1200W = 1.2kW$$

13. Use BROIDMAS and a calculator. (All solutions are correct to 2 d.p.)

(a) For 10^{index} we use Exp or EE button on our calculator.

$$\frac{20 \times 10^{11}}{1.5 \times 10^{6}} = 1333333.33$$

(b) Applying BROIDMAS gives:

$$\frac{115\times10^3}{(15+1.8)^2}=\frac{115\times10^3}{16.8^2}$$

$$=\frac{115\times10^3}{282.24}$$

$$=\left(115\times10^3\right)\div282.24$$

$$=407.45$$

(c) Use BROIDMAS:

$$500+\frac{100^2-157^2}{3\times10^3}-160=500+\frac{10000-24649}{3\times10^3}-160$$

$$=500+\frac{-14649}{3\times10^3}-160$$

$$=500-\frac{14649}{3\times10^3}-160$$

$$=500-4.883-160=335.12$$

(d) Similarly:

$$\frac{\left(2\times10^6\times0.015\right)-\left(2\times10^5\times0.075\right)}{1.44-1}=\frac{30000-15000}{0.44}$$

$$=\frac{15000}{0.44}\quad(=15000\div0.44)$$

$$=34090.91$$

(e) We have $\dfrac{\left(20\times10^9\right)\left(1\times10^3\right)\left[\left(1.5\times10^3\right)^2-\left(0.5\times10^3\right)^2\right]^{\frac{3}{2}}}{25\times10^{21}}$ (†)

We first evaluate the terms inside the square brackets:

$$\left[\left(1.5\times10^3\right)^2-\left(0.5\times10^3\right)^2\right]^{\frac{3}{2}}=\left[2250000-250000\right]^{\frac{3}{2}}$$

$$=\left(2\times10^6\right)^{\frac{3}{2}}$$

$$=2.828\times10^9$$

Substituting this into (†) gives

$$\frac{\left(20\times10^9\right)\left(1\times10^3\right)\left(2.828\times10^9\right)}{25\times10^{21}}=\frac{5.656\times10^{22}}{25\times10^{21}}$$

$$=2.26$$