Complete Solutions to Intro(c)

1. In this question we list the multiples.

(a) The multiples of 5 are $5, 10, 15, 20, 25, 30, 35, 40, \ \ldots$

The multiples of 6 are $6, 12, 18, 24, 30, \ \ldots$

What number lies in both lists?

30. Hence the LCM of 5 and 6 is 30.

(b) What are the multiples of 6 and 14?

$6, 12, 18, 24, 30, 42, 48, 54, \ \ldots$

$14, 28, 42, \ \ldots$ we can stop at 42 because this number (42) also lies in the list of multiples of 6.

Thus the LCM of 6 and 14 is 42.

(c) We have the multiples of 6 in (b) so we only need to examine the multiples of 27: $27, 54, \ \ldots$ we can stop at 54, why?

Because 54 is a multiple of 6. Therefore the LCM of 6 and 27 is 54.

(d) Since 3 divides exactly into 42 and 7 divides exactly into 42, so 42 is a multiple of 3 and 7. So the lowest common multiple of $3, 7$ and 42 must be 42.

---

2. (a) We know $90 = 9 \times 10$. However 9 and 10 are not prime. What are the prime factors of 9 and 10?

$$9 = 3 \times 3 = 3^2$$
$$10 = 5 \times 2$$

Hence

$$90 = 10 \times 9 = 5 \times 2 \times 3^2 = 2 \times 3^2 \times 5$$

(b) Since 144 is an even number we know 2 divides into 144. We have

$$144 \div 2 = 72$$

Again 72 is an even number, so

$$72 \div 2 = 36$$

From our 4 times table, 36 can be written as:

$$\begin{aligned} 36 &= 9 \times 4 \\ &= (3 \times 3) \times (2 \times 2) \end{aligned}$$

How can we write the prime factors of 144?

We know

$$144 = 36 \times 2 \times 2$$

because 36×2 gives 72 and $72 \times 2 = 144$. We have

$$\begin{aligned} 144 &= 36 \times 2 \times 2 \\ &= \underbrace{(3 \times 3) \times (2 \times 2)}_{\text{from above}} \times 2 \times 2 \\ &= \underbrace{(3 \times 3)}_{\text{2 copies}} \times \underbrace{(2 \times 2 \times 2 \times 2)}_{\text{4 copies}} \end{aligned}$$

$$144 = 3^2 \times 2^4$$

Hence $144 = 2^4 \times 3^2$.

(c) 94 is an even number, so 2 divides into 94:

$$94 \div 2 = 47$$

47 is a prime number. Hence

$$94 = 2 \times 47$$

(d) Since the last digit in 495 is 5 so we know 5 divides into 495. By using a calculator, or otherwise, we have

$$495 \div 5 = 99$$

Is 99 a prime number?
No, because from our 11 times table we have $99 = 11 \times 9$.
Hence

$$\begin{aligned} 495 &= 99 \times 5 \\ &= 11 \times 9 \times 5 \end{aligned}$$

Remember 9 is not a prime number so it cannot be a prime factor.
$9 = 3 \times 3$, so $9 = 3^2$. Thus

$$\begin{aligned} 495 &= 11 \times 3^2 \times 5 \\ &= 3^2 \times 5 \times 11 \end{aligned}$$

---

3. For this question we first write down the prime decompositions of each number.
(a) We have

$$\begin{aligned} 24 &= 12 \times 2 \\ &= (3 \times 4) \times 2 \\ &= (3 \times \underbrace{2 \times 2}_{=4}) \times 2 \\ &= 3 \times \underbrace{2 \times 2 \times 2}_{\text{3 copies}} \\ 24 &= 2^3 \times 3 \end{aligned}$$

Similarly

$$\begin{aligned} 54 &= 9 \times 6 \\ &= (3 \times 3) \times (3 \times 2) \\ &= 3^3 \times 2 \end{aligned}$$

Highest power of prime factors are 3^3 and 2^3. Hence the LCM of 24 and 54 can be found by multiplying 2^3 and 3^3:

$$2^3 \times 3^3 = 216$$

LCM of 24 and 54 is 216.
(b) We have

$$\begin{aligned} 8 &= 4 \times 2 \\ &= 2 \times 2 \times 2 \\ 8 &= 2^3 \end{aligned}$$

Similarly

$$\begin{aligned} 27 &= 9 \times 3 \\ &= (3 \times 3) \times 3 \\ 27 &= 3^3 \end{aligned}$$

Also

$$\begin{aligned} 64 &= 8 \times 8 \\ &= (4 \times 2) \times (4 \times 2) \\ &= 2 \times 2 \times 4 \times 4 \end{aligned}$$

But 4 is not a prime number because $2 \times 2 = 4$. We have

$$\begin{aligned} 64 &= \underbrace{2 \times 2 \times (2 \times 2) \times (2 \times 2)}_{\text{6 copies}} \\ &= 2^6 \end{aligned}$$

Highest index of each prime factor is 2^6 and 3^3 (the 2^6 overides the 2^3 for 8). Hence the lowest common multiple of $8, 27$ and 64 is

$$2^6 \times 3^3 = 1728$$

(c) We know from our 11 times table that $11 \times 11 = 11^2 = 121$.
Since 5 is the last digit in 125, we know 5 divides into 125:

$$125 \div 5 = 25$$

Furthermore $25 = 5 \times 5$. Hence

$$\begin{aligned} 125 &= 25 \times 5 \\ &= (5 \times 5) \times 5 \\ 125 &= 5^3 \end{aligned}$$

By solution to question 2 (b) we have

$$144 = 2^4 \times 3^2$$

The highest index of each prime factor is $2^4, 3^2, 5^3$ and 11^2. So the lowest common multiple of $121, 125$ and 144 is

$$2^4 \times 3^2 \times 5^3 \times 11^2 = 2178000$$