Complete solutions to Intro(e)

1.(a) We first have to find the lowest common multiples (LCM) of 2 and 3.
The multiples of 2 and 3 are
$2,4,6,8,10,\ldots$
$3,6,9,12,\ldots$

LCM of 2 and 3 is 6. How do we write $\frac{1}{2}$ with denominator of 6?

Multiply numerator and denominator by 3:

$$\frac{1}{2}=\frac{1\times 3}{2\times 3}=\frac{3}{6}$$

Similarly multiply numerator and denominator of $\frac{1}{3}$ by 2:

$$\frac{1}{3}=\frac{1\times 2}{3\times 2}=\frac{2}{6}$$

Adding the two fractions gives:

$$\frac{1}{2}+\frac{1}{3}=\frac{3}{6}+\frac{2}{6}$$
$$=\frac{3+2}{6}$$
$$=\frac{5}{6}$$

(b) The multiples of 4 and 9 are :
$4,8,12,16,20,24,28,32,\underline{36},40,48,\ldots$
$9,18,27,\underline{36},45,\ldots$

The LCM of 4 and 9 is 36. Multiplying the numerator and denominator of $\frac{1}{4}$ by 9:

$$\frac{1}{4}=\frac{1\times 9}{4\times 9}=\frac{9}{36}$$

Multiplying the numerator and denominator of $\frac{1}{9}$ by 4:

$$\frac{1}{9}=\frac{1\times 4}{9\times 4}=\frac{4}{36}$$

Hence

$$\frac{1}{4}+\frac{1}{9}=\frac{9}{36}+\frac{4}{36}$$
$$=\frac{9+4}{36}$$
$$=\frac{13}{36}$$

(c) Since $6\times 10=60$ we can write $\frac{1}{6}$ with denominator 60, how?

Multiply numerator and denominator by 10:

$$\frac{1}{6}=\frac{1\times 10}{6\times 10}=\frac{10}{60}$$

So

$$\frac{1}{6}+\frac{1}{60}=\frac{10}{60}+\frac{1}{60}$$
$$=\frac{10+1}{60}$$
$$=\frac{11}{60}$$

Similarly for (d) $\frac{12}{35}$ and (e) $\frac{17}{12}$

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2. (a) From solution to question 1(a) we know $\frac{1}{2}=\frac{3}{6}$ and $\frac{1}{3}=\frac{2}{6}$.

Substituting these gives:

$$\frac{1}{2}-\frac{1}{3}=\frac{3}{6}-\frac{2}{6}$$
$$=\frac{3-2}{6}$$
$$=\frac{1}{6}$$

(b) We list the multiplies of 7 and 9:
$7,14,21,28,35,49,56,63,70,\ldots$
$9,18,27,36,45,54,63,\ldots$

Since $7\times 9=63$, we multiply the numerator and denominator of $\frac{22}{7}$ by 9:

$$\frac{22}{7}=\frac{22\times 9}{7\times 9}=\frac{198}{63}$$

Similarly

$$\frac{16}{7}=\frac{16\times 7}{9\times 7}=\frac{112}{63}$$

We have

$$\frac{22}{7}-\frac{16}{9}=\frac{198}{63}-\frac{112}{63}$$
$$=\frac{198-112}{63}$$
$$=\frac{86}{63}$$

$\frac{86}{63}$ is a top-heavy fraction which we can write as a mixed fraction:

$$\frac{86}{63}=86\div 63=1 \text{ remainder } 23$$

So $\frac{86}{63}=1\frac{23}{63}$. Hence $\frac{22}{7}-\frac{16}{9}=1\frac{23}{63}$.

(c) Since $29\times 5=145$, we can express $\frac{41}{29}$ with denominator 145, how ?

Multiply numerator and denominator by 5:

$$\frac{41}{29}=\frac{41\times 5}{29\times 5}=\frac{205}{145}$$

Substituting this gives:

$$\frac{41}{29}-\frac{204}{145}=\frac{205}{145}-\frac{204}{145}$$
$$=\frac{205-204}{145}$$
$$=\frac{1}{145}$$

(d) Similarly $\frac{61}{1271}$. (Note that 31 and 41 are prime numbers).

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3. (a) We know from solution 1(a) that $\frac{1}{2}+\frac{1}{3}=\frac{5}{6}$ so

$$\underbrace{\frac{1}{2}+\frac{1}{3}}_{=5/6}+\frac{1}{6}=\frac{5}{6}+\frac{1}{6}$$
$$=\frac{5+1}{6}$$
$$=\frac{6}{6}=1$$

(b) Again $\frac{1}{2}+\frac{1}{3}=\frac{5}{6}$ so we need to evaluate

$$\frac{5}{6}-\frac{1}{4}+\frac{1}{5} \qquad (*)$$

How can we calculate this fraction?
We have to find the LCM of $4,5$ and 6. The multiplies of $4,5$ and 6 are:
$4,8,12,16,20,24,28,32,36,40,44,48,52,56,\underline{60},64,68,\ldots$
$5,10,15,20,25,30,35,40,45,50,55,\underline{60},65,\ldots$
$6,12,18,24,30,36,42,48,54,\underline{60},\ldots$
(We could also use prime factorization method). By examining these three lists we see that the LCM of $4,5$ and 6 is 60. Since $4\times 15=60$, we have:

$$\frac{1}{4}=\frac{1\times 15}{4\times 15}=\frac{15}{60}$$

Also $5\times 12=60$ so we have:

$$\frac{1}{5}=\frac{1\times 12}{5\times 12}=\frac{12}{60}$$

Similarly $6\times 10=60$:

$$\frac{5}{6}=\frac{5\times 10}{6\times 10}=\frac{50}{60}$$

Replacing with denominator of 60 for each fraction into (*) we have

$$\frac{5}{6}-\frac{1}{4}+\frac{1}{5}=\frac{50}{60}-\frac{15}{60}+\frac{12}{60}$$
$$=\frac{50-15+12}{60}$$
$$=\frac{47}{60}$$

(c) $\frac{5}{84}$

4. (a) $\frac{1}{2}\times\frac{1}{3}=\frac{1\times 1}{2\times 3}=\frac{1}{6}$

(b) 12 and 84 have a common factor because $12\times 7=84$. So we can cancel 12's on numerator and denominator:

$$\frac{17}{12}\times\frac{84}{60}=\frac{17}{12}\times\frac{12\times 7}{60}=\frac{17\times 7}{60}=\frac{119}{60}=1\frac{59}{60}$$

(c) 235 and 5 have a common factor of 5. Also 38 and 19 have a common factor of 19. Before multiplication we can cancel out:

$$\frac{235}{19}\times\frac{38}{5}=\frac{47\times 5}{19}\times\frac{19\times 2}{5}=\frac{47}{1}\times\frac{2}{1}=47\times 2=94$$

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5. For division we turn the second fraction upside down and then multiply the fractions.

(a) $\frac{1}{2}\div\frac{1}{3}=\frac{1}{2}\times\frac{3}{1}=\frac{1\times 3}{2\times 1}=\frac{3}{2}=1\frac{1}{2}$

(b) $\frac{99}{70}\div\frac{22}{7}=\frac{99}{70}\times\frac{7}{22}$. How can we evaluate this?

Since 70 and 7 have a common factor of 7, we can perform some cancelling. Also 99 and 22 have a common factor of 11. Hence

$$\begin{aligned}\frac{99}{70}\times\frac{7}{22}&=\frac{9}{10}\times\frac{1}{2}\\&=\frac{9\times 1}{10\times 2}\\&=\frac{9}{20}\end{aligned}$$

So $\frac{99}{70}\div\frac{22}{7}=\frac{9}{20}$. Remember cancel down first (if possible) and then multiply.

(c) $\frac{235}{19}\div\frac{5}{38}=\frac{235}{19}\times\frac{38}{5}$. What do you notice?

This is same as question 4(c), so $\frac{235}{19}\div\frac{5}{38}=94$.

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6. Use your calculator as described in the text.

(a) $1\frac{497}{1200}$ (b) $\frac{2}{97}$ (c) $\frac{1}{39780}$ (d) $\frac{1}{6360}$ (e) $2\frac{1}{288}$ (f) $2\frac{1}{144}$

(g) $\frac{2870}{2871}$ (h) $\frac{2639}{2640}$