Complete solutions to Intro(g)

1.(a) The number between 1 and 10 is 1.86. How many places do we need to shift the decimal point?

$1\underbrace{86000}_{5\text{ places}}$,5 places to the left. Hence

$$186\,000 = 1.86 \times 10^5$$

(b) Similarly $1\,392\,000 = 1.392 \times 10^6$.

(c) $136\,000 = 1.36 \times 10^5$

(d) The number between 1 and 10 is 3.439, we need to shift the decimal point:

$$0.\underbrace{000\,000\,03}_{8\text{ places}}439 = 3.439 \times 10^{-8}$$

Negative index because we are moving the decimal point to the right (it's a small number).

(e) Similarly $0.\underbrace{000\,000\,09}_{8\text{ places}}51 = 9.51 \times 10^{-8}$

(f) $0.009\,29 = 9.29 \times 10^{-3}$

(g) $0.000\,025\,8 = 2.58 \times 10^{-5}$

(h) 14.96×10^6 is not in standard form, why not?

Because 14.96 is not between 1 and 10, remember the first number needs to lie between 1 and 10. How can we rewrite this number?

$$14.96 = 1.496 \times 10$$

Substituting this into the original number gives:

$$\begin{aligned} 14.96 \times 10^6 &= \underbrace{1.496 \times 10}_{=14.96} \times 10^6 \\ &= 1.496 \times 10^7 \end{aligned}$$

(i) $273.15 = 2.7315 \times 10^2$

(j) This number is already in standard form.

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2. Write them in conventional form means write out the whole number without a power of 10.

(a) $6.4 \times 10^6 = 6.400000 \times 10^6$, multiplying by 10^6 moves the decimal point 6 places to the right:

$$6.4 \times 10^6 = 6\,400\,000$$

(b) We can place as many zeros as we want in front of a number without changing the number:

$$3.3 \times 10^{-9} = 0\,000\,000\,003.3 \times 10^{-9}$$

The index, -9, shifts the decimal point 9 places to the left. Hence

$$3.3 \times 10^{-9} = 0.000\,000\,003\,3$$

(c) Similarly:

$$\begin{aligned} 7.292 \times 10^{-5} &= 000\,007.292 \times 10^{-5} \\ &= 0.000\,072\,92 \end{aligned}$$

(d) Also

$$\begin{aligned} 3 \times 10^8 &= \underbrace{3.000\,000\,00}_{=3} \times 10^8 \\ &= 300\,000\,000 \end{aligned}$$

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3. (a) Writing the middle numbers in conventional form gives:

$$12.75 \times 10^{2} = 1275$$
$$12.75 \times 10^{-3} = 0012.75 \times 10^{-3} = 0.01275$$

We have $12750, 1275, 0.01275$ and 12.75. Putting this in order with smallest first gives $0.01275, 12.75, 1275$ and 12750. Hence this is:

$$12.75 \times 10^{-3}, 12.75, 12.75 \times 10^{2} \text{ and } 12750$$

(b) Note that $3.14 \div 10^{3} = 3.14 \div \frac{10^{3}}{1} = 3.14 \times \frac{1}{\underbrace{10^{3}}_{=10^{-3}}} = 3.14 \times 10^{-3}$

The numbers are 3.14×10^{3}, 3.14×10^{-3} and 3.14×10^{-2}, which one is smallest? The more negative an index the smaller the number, so 3.14×10^{-3} is smaller than 3.14×10^{-2}. We have

3.14×10^{-3}, 3.14×10^{-2} and 3.14×10^{3} or $3.14 \div 10^{3}$, 3.14×10^{-2} and 3.14×10^{3}

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4. Use your calculator for this question. To enter a number with 10^{3} use EXP, EE or E button on the calculator.

(a) To evaluate $\dfrac{1.25 \times 10^{3} \times 0.15 \times 348}{15 \times 10^{5}}$ on a calculator, PRESS;

[(] [1.25] [EXP] [3] [x] [0.15] [x] [348] [)] [÷] [(] [15] [EXP] [5] [)] [=] shows 0.0435=0.04 (2 d.p.).

(b) Similarly by using our calculator we have 1.58.

(c) By using a calculator we have 0.49.

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5. Need to write each to the power of 10 and which is a multiple of 3:

(a) 100×10^{-12} farads $= 100pF$ because p is the symbol for pico $= 10^{-12}$

(b) $30000 \text{ ohms} = 30 \times \underbrace{10^{3}}_{=\text{kilo}} \Omega = 30k\Omega$

(c) $0.0003 \text{ amps} = 0.3 \times \underbrace{10^{-3}}_{=\text{m(milli)}} A = 0.3mA$

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6. (a) $8536N = 8.536 \times 10^{3} N = 8.536kN$

(b) $75000000W = 75 \times 10^{6} W = 75MW$

(c) There is no 10^{12} given in TABLE 2 so we use 10^{9}, how can we write 0.2×10^{12} to the power of 9?

$$0.2 \times 10^{12} = 0.200 \times 10^{12} = 200 \times 10^{-3} \times 10^{12} \qquad (\dagger)$$

Let's examine
$$10^{-3} \times 10^{12} = \frac{1}{10^{3}} \times 10^{12}$$
$$= \frac{1}{10 \times 10 \times 10} \times \underbrace{(10 \times 10 \times 10 \times \ldots \times 10)}_{12 \text{ copies}}$$
$$= \underbrace{(10 \times 10 \times \ldots \times 10)}_{9 \text{ copies}} \text{ cancelling } 10 \times 10 \times 10$$
$$= 10^{9}$$

Substituting this into the Right Hand Side of $(\dagger)$ gives:

$$200 \times 10^{-3} \times 10^{12} = 200 \times 10^{9}$$

Hence $0.2 \times 10^{12} Pa = 200 \times 10^{9} Pa = 200GPa$ (G is giga $= 10^{9}$)

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7. Use TABLE 2 and TABLE 3 to see what the symbols represent.

(a) $3000mm = 3000$ millimeters $= 3000 \times 10^{-3} m$, this is now in the units of metres but we can simplify this further by writing 3000 as 3×10^{3}. We have

$$3000\times10^{-3}=3\times10^{3}\times10^{-3}=3\times10^{3}\times\frac{1}{10^{3}}=3\ \text{(cancelling } 10^{3}\text{)}$$

Hence $3000mm=3m$.

(b) $573kN=573\times10^{3}\,N$

(c) $25MJ=25\times10^{6}\,J$

(d) $12\,ps=12\times10^{-12}\,s$

(e) $25mW=25\times10^{-3}\,W$

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8. (a) The top-heavy fraction $\frac{22}{7}$ can be written as:

$$\frac{22}{7}\approx\frac{21}{7}=3$$

(b) We can write $\frac{333}{106}\approx\frac{300}{100}=3$, is a close approximation.

(c) $99\times99\approx100\times100=10000$

(d) Rounding 714 to 700, 0.63 to 0.6 and 14.45 to 14 gives

$$\frac{714\times0.63}{14.45}\approx\frac{700\times0.6}{14}$$

Now $700\times0.6=700\times\frac{6}{10}=70\times6$. Therefore

$$\frac{700\times0.6}{14}=\frac{70\times6}{14}$$

$$=\frac{420}{14}$$

$$=30\ \text{(because } 42\div14=3\text{)}$$

$$\frac{714\times0.63}{14.45}\approx30$$