Complete solutions to Intro(i)

We use BROIDMAS throughout this exercise.

1. (a) What do we evaluate first?

Brackets has priority, so $1+3+5=9$ and $1\times 3\times 5=15$. We have

$$(1+3+5)\times(1\times 3\times 5)=9\times 15=135$$

(b) Same approach as (a), 144.

(c) Again evaluate within the brackets and then add the results:

$$\underbrace{(49\times 1)}_{=49}+\underbrace{(49\times 3)}_{=147}+\underbrace{(49\times 5)}_{=245}=49+147+245=441$$

(d) BRO<u>I</u>DM<u>A</u>S reveals that indices have priority over addition. So

$$1+\underbrace{7^2}_{=49}+\underbrace{5^3}_{=125}=1+49+125=175$$

(e) Similarly

$$\begin{aligned}(2\times\underbrace{4^3}_{=64})+(4\times\underbrace{3^3}_{=27})+(3\times 1)&=(2\times 64)+(4\times 27)+(3\times 1)\\&=128+108+3\\&=239\end{aligned}$$

(f) Evaluate the multiplication within the brackets first:

$$\underbrace{(2\times 3\times 5\times 7\times 11)}_{=2310}+1=2310+1=2311$$

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2. (a) BRO<u>I</u>DMA<u>S</u> discloses that indices have priority over subtraction, therefore

$$2^{11}-1=\underbrace{2048}_{=2^{11}}-1=2047$$

(b) We can write $\dfrac{1-5}{2}$ as $\dfrac{(1-5)}{2}=\dfrac{-4}{2}=-4\div 2=-2$.

(c) By applying BRO<u>I</u>D<u>M</u>AS, we know indices have preference over multiplication:

$$\frac{1}{2}-\frac{1}{3\times\underbrace{2^3}_{=8}}+\frac{1}{5\times\underbrace{2^5}_{=32}}=\frac{1}{2}-\frac{1}{3\times 8}+\frac{1}{5\times 32}$$

Which operation do we implement next?

Multiplication

$$\frac{1}{2}-\frac{1}{3\times 8}+\frac{1}{5\times 32}=\frac{1}{2}-\frac{1}{24}+\frac{1}{160}=\frac{223}{480}$$

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3. (a) Since we have the same denominator we can collect the numerators together:

$$\frac{1+\sqrt{5}}{2}+\frac{1-\sqrt{5}}{2}=\frac{1+\sqrt{5}+1-\sqrt{5}}{2}=\frac{(1+1)+\left(\sqrt{5}-\sqrt{5}\right)}{2}=\frac{2+0}{2}=\frac{2}{2}=1$$

(b) B<u>R</u>OI<u>D</u>M<u>A</u>S tell us that the square root has precedence over addition or division:

$$\frac{5+\sqrt{25-24}}{2}=\frac{5+\sqrt{1}}{2}=\frac{5+1}{2}=\frac{6}{2}=3$$

Remember when we have $\dfrac{5+1}{2}$ we can write this as $\dfrac{(5+1)}{2}$ or $(5+1)\div 2$, that is why the addition is carried out before the division.

For (c), (d) and (e) we have a plus or minus symbol, so we result in 2 answers in each case.

(c) Priority arrangement is similar to (b):

$$\frac{5 \pm \sqrt{(-5)^2 - (4 \times 1 \times 6)}}{2} = \frac{5 \pm \sqrt{25 - 24}}{2}$$

$$= \frac{5 \pm \sqrt{1}}{2}$$

$$\underbrace{=}_{\substack{\text{using the plus} \\ \text{or minus sign}}} \frac{5+1}{2} \text{ or } \frac{5-1}{2}$$

$$= \frac{6}{2} \text{ or } \frac{4}{2}$$

$$= 3 \text{ or } 2$$

(d) This is very similar to (c):

$$\frac{-4 \pm \sqrt{(4)^2 - 4 \times (-5)}}{2} = \frac{-4 \pm \sqrt{16 - (-20)}}{2}$$

$$= \frac{-4 \pm \sqrt{16 + 20}}{2} \quad (\text{minus} \times \text{minus} = \text{plus})$$

$$= \frac{-4 \pm \sqrt{36}}{2}$$

$$= \frac{-4+6}{2} \text{ or } \frac{-4-6}{2}$$

$$= \frac{2}{2} \text{ or } \frac{-10}{2}$$

$$\frac{-4 \pm \sqrt{(4)^2 - 4 \times (-5)}}{2} = 1 \text{ or } -5$$

(e) Priority of operations similar to (c) and (d):

$$\frac{-65 \pm \sqrt{(-65)^2 + (4 \times 14 \times 25)}}{28} = \frac{-65 \pm \sqrt{4225 + 1400}}{28}$$

$$= \frac{-65 \pm \sqrt{5625}}{28}$$

$$= \frac{-65 \pm 75}{28}$$

$$= \frac{-65+75}{28} \text{ or } \frac{-65-75}{28}$$

$$= \frac{10}{28} \text{ or } \frac{-140}{28}$$

$$\frac{-65 \pm \sqrt{(-65)^2 + (4 \times 14 \times 25)}}{28} = \frac{5}{14} \text{ or } -5$$

4. (All solutions are given correct to 2 d.p.).
(a) Index has priority, $7^2 = 49$:

$$\pi \times 7^2 \times 3 = \pi \times 49 \times 3 = 461.81$$

(b) Again we evaluate 4^2 first, $4^2 = 16$, therefore

$$\frac{\pi}{3} \times 4^2 \times 5 = \frac{\pi}{3} \times 16 \times 5$$

Since multiplication and division have the same priority it does not make any difference if you divide π by 3 first and then carry out the multiplication or you evaluate $\pi \times 16 \times 5$ first and then divide your result by 3:

$$\begin{aligned}\frac{\pi}{3} \times 16 \times 5 &= (\pi \div 3) \times 16 \times 5 \\ &= 83.78\end{aligned}$$

(c) Similarly, $5^3 = 125$, so we have

$$\begin{aligned}\frac{4\pi \times 5^3}{3} &= \frac{4\pi \times 125}{3} \\ &= (4 \times \pi \times 125) \div 3 \\ &= (1570.796) \div 3 \\ &= 523.60\end{aligned}$$

(d) <u>B</u>ROIDMAS tell us that the brackets are evaluated first:

$$2\pi \times 7(5+7) = 2\pi \times \underbrace{7(12)}_{=7\times 12} = 2\pi \times 84 = 527.79$$

(e) B<u>RO</u>IDMAS reveals that square root has priority, but how do we evaluate $\sqrt{\dfrac{3^2+4^2}{2}}$?

First we calculate the indices, 3^2 and 4^2, and then add our results:

$$\sqrt{\frac{3^2+4^2}{2}} = \sqrt{\frac{9+16}{2}} = \sqrt{\frac{25}{2}}$$

What can we do next?
Remember the square root sign can be taken inside the division:

$$\sqrt{\frac{25}{2}} = \frac{\sqrt{25}}{\sqrt{2}} = \frac{5}{\sqrt{2}}$$

Hence replacing $\sqrt{\dfrac{3^2+4^2}{2}} = \dfrac{5}{\sqrt{2}}$ into the original problem gives:

$$\begin{aligned}2\pi\sqrt{\frac{3^2+4^2}{2}} &= 2\pi\frac{5}{\sqrt{2}} = \frac{2\pi \times 5}{\sqrt{2}} \\ &= (2 \times \pi \times 5) \div \left(\sqrt{2}\right) \\ &= 22.21\end{aligned}$$

5. (a) BROIDMAS reveals that brackets have preference:

$$\left(2^5 - 1\right) = 32 - 1 = 31$$

Hence replacing $2^5 - 1$ with 31 gives

$$2^4\left(2^5 - 1\right) = 2^4(31) = 31 \times 2^4$$

How do we evaluate 31×2^4?
First we calculate 2^4 which is 16 and then multiply by 31:

$$31 \times 2^4 = 31 \times 16 = 496$$

We have $2^4\left(2^5 - 1\right) = 496$.

(b) How do we evaluate $2^{2^3} + 1$?
The first base has index 2^3 so we evaluate this first.

$$2^3 = 8$$

Hence we have

$$2^{2^3} + 1 = 2^8 + 1$$

What do we compute next?
BROIDMAS tell us that indices have priority over addition, so we evaluate 2^8 first and then add 1 to the result:

$$\begin{aligned} 2^{2^3} + 1 &= 2^8 + 1 \\ &= 256 + 1 \\ &= 257 \end{aligned}$$