Complete solutions to Intro(j)

1. Multiply each figure by 100:

(a) $\frac{1}{10\!\!\!/}\times 100\!\!\!/ = \frac{10}{1} = 10\%$

Use the fraction button on your calculator to write the top-heavy fraction as a mixed fraction:

(b) $\frac{1}{12}\times 100 = \frac{100}{12} = 8\frac{1}{3}\%$

(c) $\frac{7}{8}\times 100 = \frac{700}{8} = 87.5\%$

(d) $\frac{3}{13}\times 100 = \frac{300}{13} = 23\frac{1}{13}\%$

For (e) and (f) move the decimal point two places to the right:

(e) $0.167\times 100 = 16.7\%$

(f) $2.583\times 100 = 258.3\%$

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2. Remember % means out of 100, so we write each figure out of 100. Again use the fraction button on your calculator:

(a) $4\% = \frac{4}{100} = \frac{1}{25}$

(b) $9\% = \frac{9}{100}$ (This cannot be simplified any further because 9 and 100 have no factors in common).

(c) $17.5\% = \frac{17.5}{100}$. Using the fraction button on most calculators for $\frac{17.5}{100}$ displays 0.175, but we want to write $\frac{17.5}{100}$ as a simplified fraction, how?

We can multiply the numerator and denominator by 10 without changing the fraction:

$$\frac{17.5}{100} = \frac{17.5\times 10}{100\times 10} = \frac{175}{1000}$$

Enter $\frac{175}{1000}$ into your calculator, the display should show $\frac{7}{40}$.

Hence $17.5\% = \frac{7}{40}$.

(d) Very similar to (c):

$$2.5\% = \frac{2.5}{100} = \frac{25}{1000} = \frac{1}{40}$$

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3. (a) We first evaluate 10% of 900.

$$\begin{aligned} 10\% \text{ of } 900 &= \frac{10}{1\not{0}\not{0}}\times 9\not{0}\not{0} \\ &= \frac{10\times 9}{1} \\ &= 90 \end{aligned}$$

How do we calculate $900\pm 10\%$?

Since 10% is 90Ω so we plus and minus this to the 900Ω. Hence

$$\begin{aligned} 900\pm 10\% &= 900\pm 90 \\ &= 900-90,\ 900+90 \\ &= 810,\ 990 \end{aligned}$$

The range of the resistor is 810Ω to 990Ω.

(b) What is 5% of 1200?

$$5\% \text{ of } 1200 = \frac{5}{100} \times 1200 = \frac{5 \times 12}{1} = 60$$

Thus $1200 \pm 5\% = 1200 \pm 60 = 1200 - 60, 1200 + 60 = 1140, 1260$.

The range of the resistor is 1140Ω to 1260Ω.

(c) $27\ \text{k}\Omega = 27000\Omega$ because k represent kilo $(10^3 = 1000)$.

$$1\% \text{ of } 27000 = \frac{1}{100} \times 27000 = 270$$

The range is $27000 - 270$ to $27000 + 270$. Hence 26730Ω to 27270Ω.

(d) Again $19\ \text{k}\Omega = 19000\Omega$.

$$3\% \text{ of } 19000 = \frac{3}{100} \times 19000 = \frac{3 \times 190}{1} = 570$$

The resistor value lies between $19000 - 570$ to $19000 + 570$. Thus the range is 18430Ω to 19570Ω.

(e) $5\ \text{M}\Omega = 5000000\Omega$ because $\text{M} = \text{mega} = 10^6 = 1\,000\,000$.

$$0.15\% \text{ of } 5000000 = \frac{0.15}{100} \times 5000000 = 0.15 \times 50000 = 7500$$

Range is $5000000 - 7500$ to $5000000 + 7500$. Hence 4992500Ω to 5007500Ω.

4.

$$4.5\% \text{ of } 120 = \frac{4.5}{100} \times 120 = \frac{4.5 \times 12}{10} = 5.4$$

The range of speed is $120 \pm 5.4 = 120 - 5.4,\ 120 + 5.4$
$= 114.6,\ \ 125.4$

Hence range is $114.6\ \text{km/h}$ to $125.4\ \text{km/h}$.

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5. We have to evaluate 0.15% of 3.567:

$$0.15\% \text{ of } 3.567 = \frac{0.15}{100} \times 3.567$$
$$= \frac{0.15 \times 3.567}{100}$$
$$= 5.35 \times 10^{-3}$$

Since the bar expands we add 5.35×10^{-3} to the original length 3.567, hence the new length of the bar is $3.567 + \left(5.35 \times 10^{-3}\right) = 3.572 \text{ m}$ (4 s.f.)

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6. Putting exact value = 342, experimental value = 345 into (0.1) gives:

$$\% \text{ error } = \frac{345 - 342}{342} \times 100$$
$$= \frac{3}{342} \times 100$$
$$= 0.9\% \ (1 \text{ s.f.})$$

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7. Use a calculator. We can write $0.005 = 5 \times 10^{-3}$. Subtract 0.57% of this to obtain the new volume. On a calculator, PRESS;
[5] [EXP] [(-)] [3] [×] [0.57] [SHIFT] [=] [-] shows 4.9715^{-03}.
Rounding this gives the new volume as $4.97 \times 10^{-3} m^3$ (3 s.f.)

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8. The expansion is given by the difference, $0.38 - 0.35 = 0.03$. We need to write $0.03m$ as a percentage of the original length, hence

$$\frac{0.03}{0.35} \times 100 = 8.6\% \ (2 \text{ s.f.})$$

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9. Remember $25mA = 25 \times 10^{-3} A$ $\left(\text{because } m = \text{milli } = 10^{-3}\right)$. We need to evaluate 4.3% of 25×10^{-3}:

$$\frac{4.3}{100} \times \left(25 \times 10^{-3}\right) = 1.075 \times 10^{-3}$$

The highest value is evaluated by adding the original value, $25mA$, and 1.075×10^{-3}:

$$\text{Highest value } = \left(25 \times 10^{-3}\right) + \left(1.075 \times 10^{-3}\right)$$
$$= 26.075 \times 10^{-3}$$
$$= 26.1 \, m\text{A}$$

(0.1) $$\% \text{ error } = \frac{\text{experimental value} - \text{exact value}}{\text{exact value}} \times 100$$