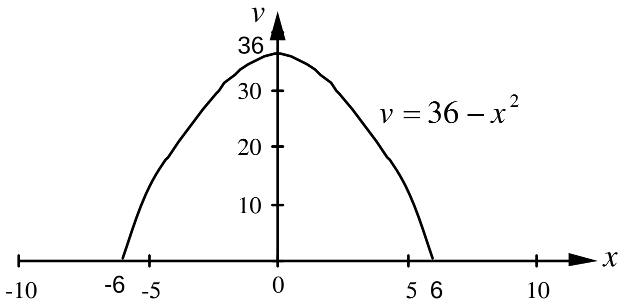
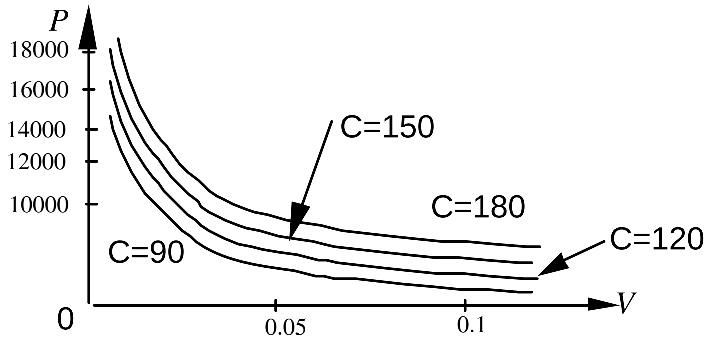
Complete solutions to Miscellaneous Exercise 2

1. Same as Fig 14 of chapter 2 with (x, y) axes labelled as (t, s) respectively.

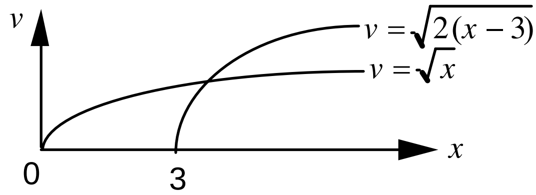
2. The graph of $v = 36 - x^2$ is given by:



3. Similar in shape to the graph of $y = \frac{1}{x}$:



4.



5. We have $v = 5\sqrt{1 - \frac{x^2}{9}}$ and at $x = 0$:

$$v = 5\sqrt{1 - 0^2/9} = 5$$

So the graph crosses the v axis at 5. Where does the graph cut the x axis? At v=0:

$$5\sqrt{1 - x^2/9} = 0$$

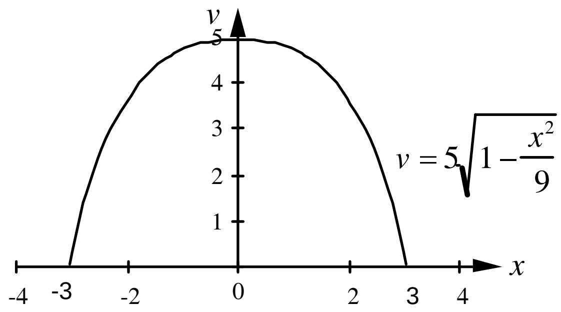
$$1 - x^2/9 = 0$$

$$x^2/9 = 1$$

$$x^2 = 9$$

$$x = -3,\ 3$$

The graph cuts the x axis at -3 and 3. Combining the data gives the graph:



6. For $0 \le t \le 20$, gradient$=\frac{10}{20} = \frac{1}{2}$ and $v-$intercept $= 0$. By (2.1)

$$v = \frac{1}{2}t$$

For $20 < t \le 35$, gradient $= \frac{12-10}{35-20} = \frac{2}{15}$

$$v = \frac{2}{15}t + c \text{ where } c \text{ is the } v-\text{intercept}$$

We know when $t = 35, v = 12$. Substituting these values:

$$12 = \left(\frac{2}{15} \times 35\right) + c \text{ gives } c = \frac{110}{15}$$

$$v = \frac{2}{15}t + \frac{110}{15}$$

$$= \frac{(2t+110)}{15} = \frac{2(t+55)}{15}$$

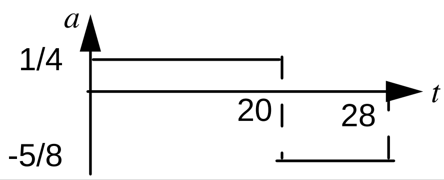
$$v = \frac{2}{15}(t+55)$$

Collecting these together:

$$v = \begin{cases} \frac{1}{2}t & 0 \le t \le 20 \\ \frac{2}{15}(t+55) & 20 < t \le 35 \end{cases}$$

7. For $0 \le t \le 20$, gradient$=\frac{5}{20} = \frac{1}{4}$.

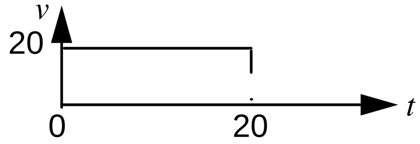
For $20 < t \le 28$, gradient$=-\frac{5}{8}$. Hence $a-t$ graph is:



8. The gradient of the straight line$=\frac{405-5}{20} = 20$. Using (2.1) with gradient 20 and intercept 5 we have, $s = 20t + 5$.

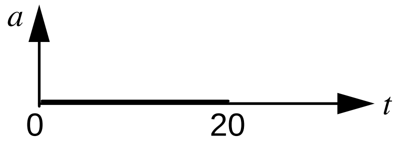
(2.1) $y = mx + c$ where m is the gradient, c is intercept

Since the $v-t$ is determined by the gradient of the $s-t$, we have:



Gradient $= 0$ (Zero)

Similarly the $a-t$ graph is the gradient of the $v-t$ graph:



9. Expanding $\left(x+y\sqrt{-2}\right)^3$ by (2.6) gives:

$$\left(x+y\sqrt{-2}\right)^3 = x^3+3x^2\left(y\sqrt{-2}\right)+3x\left(y\sqrt{-2}\right)^2+\left(y\sqrt{-2}\right)^3$$

$$= x^3+3x^2y\sqrt{-2}+3xy^2\left(\sqrt{-2}\right)^2+y^3\left(\sqrt{-2}\right)^3 \qquad (*)$$

By using the rules of indices we have

$$\left(\sqrt{-2}\right)^2 = (-2)^1 = -2 \qquad (\dagger)$$

$$\left(\sqrt{-2}\right)^3 = \left(\sqrt{-2}\right)^2\left(\sqrt{-2}\right) = \underbrace{-2}_{\text{by } (\dagger)}\sqrt{-2}$$

Substituting $\left(\sqrt{-2}\right)^2 = -2$ and $\left(\sqrt{-2}\right)^3 = -2\sqrt{-2}$ into (*) gives:

$$\left(x+y\sqrt{-2}\right)^3 = x^3+3x^2y\sqrt{-2}-6xy^2-2y^3\sqrt{-2}$$

Using the given equation we have:

$$z+\sqrt{-2} = \left(x+y\sqrt{-2}\right)^3 = x^3+3x^2y\sqrt{-2}-6xy^2-2y^3\sqrt{-2}$$

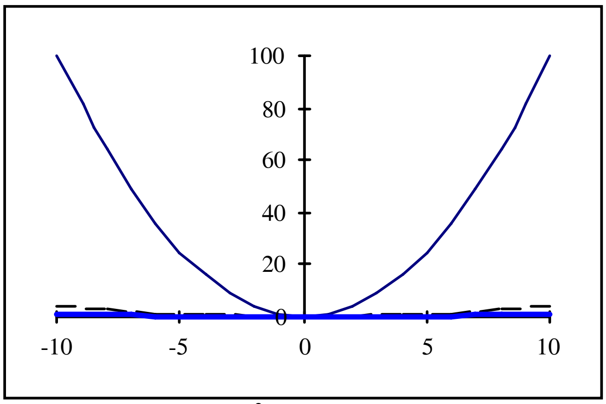
$$z+\sqrt{-2} = \left(3x^2y-2y^3\right)\sqrt{-2}+\left(x^3-6xy^2\right)$$

Therefore equating coefficients of $\sqrt{-2}$ in the above gives:

$$1 = 3x^2y-2y^3$$

$$= y\left(3x^2-2y^2\right)$$

10.



As a increases the quadratic $y=\dfrac{x^2}{a^2}$ becomes less steep.

(2.9) $$(a+b)^n = C_na^n+C_{n-1}a^{n-1}b+C_{n-2}a^{n-2}b^2+\ldots+C_0b^n$$

11. For $0 < t \le 5$; Gradient $\frac{2}{5} = 0.4$, intercept $= 0$

$$Q = 0.4t$$

For $t > 5$; $Q = 2$. So we have:

$$Q = \begin{cases} 0.4t & 0 < t \le 5 \\ 2 & t > 5 \end{cases}$$

12. Each of the lines is of the form $h = mt + c$ where m is the gradient and c is the intercept.

For $0 < t \le 2$; The gradient $= -\frac{2}{2} = -1$ and intercept $= 3$. We have

$$h = 3 - t$$

For $2 < t \le 5$ we have a horizontal line:

$$h = 1$$

For $t > 5$; The gradient $\frac{2}{1} = 2$. To find the intercept, c, we substitute the given values and solve for c. We know $h = 1$ at $t = 5$:

$$1 = (2 \times 5) + c \text{ gives } c = -9$$

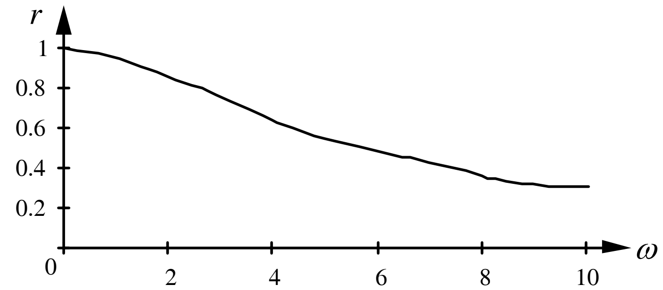
$$h = 2t - 9$$

By collecting the above terms we have:

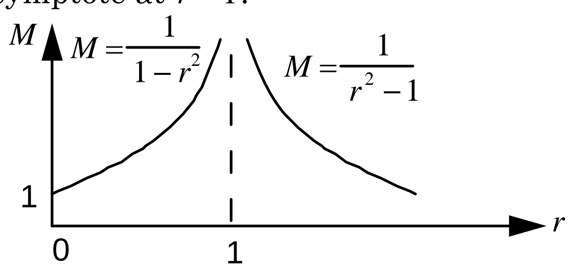
$$h = \begin{cases} 3 - t & 0 < t \le 2 \\ 1 & 2 < t \le 5 \\ 2t - 9 & t > 5 \end{cases}$$

13. We can establish a table of values for $r = 1\big/\sqrt{1 + \omega^2/9}$ and take whole numbers for ω between 0 and 10:

ω	0	1	2	3	4	5	6	7	8	9	10
r	1	0.95	0.83	0.71	0.60	0.51	0.45	0.39	0.35	0.32	0.29



14. The following graph is the out put from a graphical calculator. Notice that there is an asymptote at $r = 1$:



The remaining solutions are the output from MAPLE:

15.

```
> P:=100/V;
```

$$P := \frac{100}{V}$$

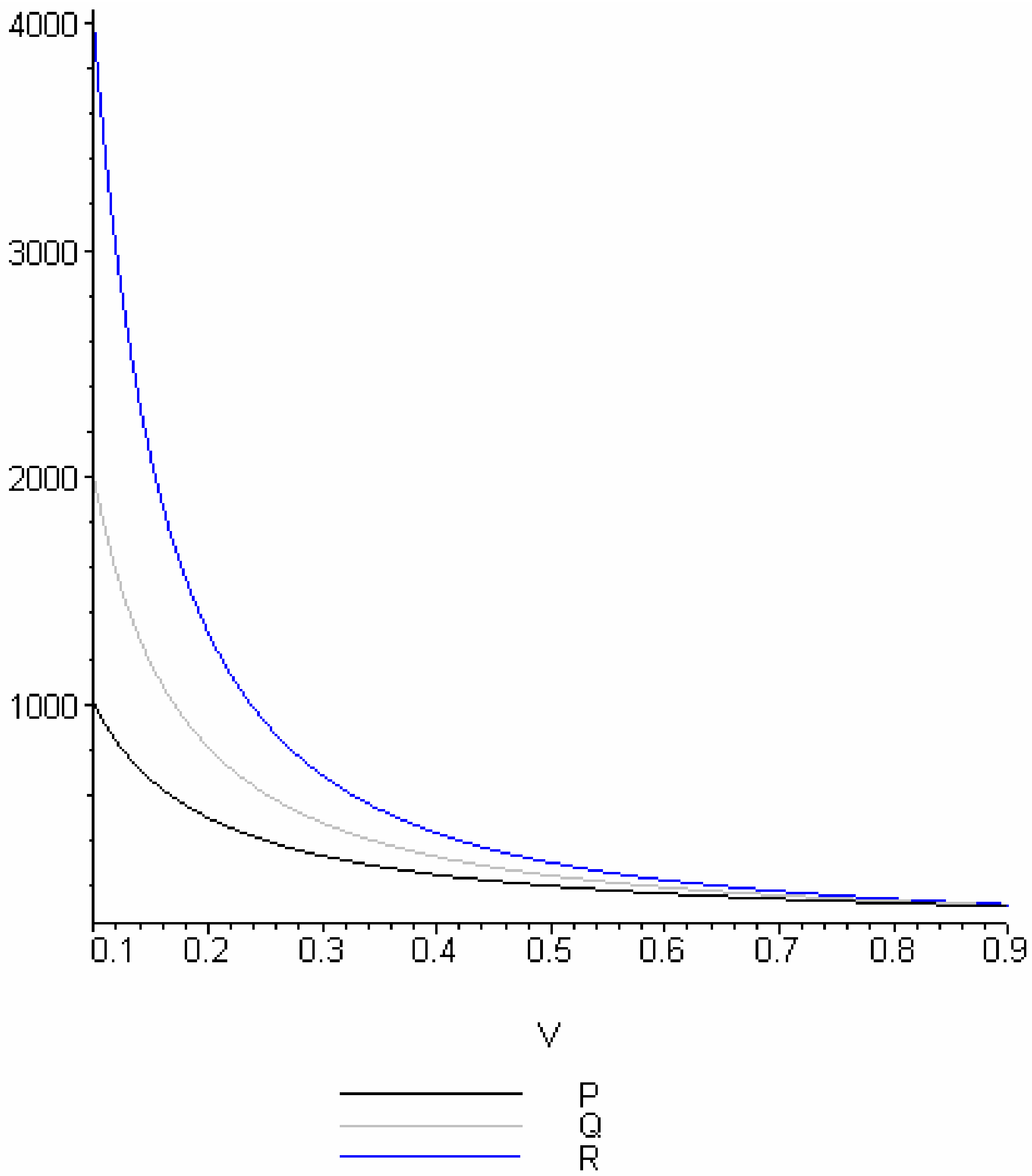
```
> Q:=100/V^1.3;
```

$$Q := \frac{100}{V^{1.3}}$$

```
> R:=100/V^1.6;
```

$$R := \frac{100}{V^{1.6}}$$

```
> plot({P,Q,R},V=0.1..0.9
,color=[black,gray,blue]);
```



16. Similarly we have:

```
> P:=1000/V^1.2;
```

$$P := \frac{1000}{V^{1.2}}$$

```
> Q:=1000/V^1.3;
```

$$Q := \frac{1000}{V^{1.3}}$$

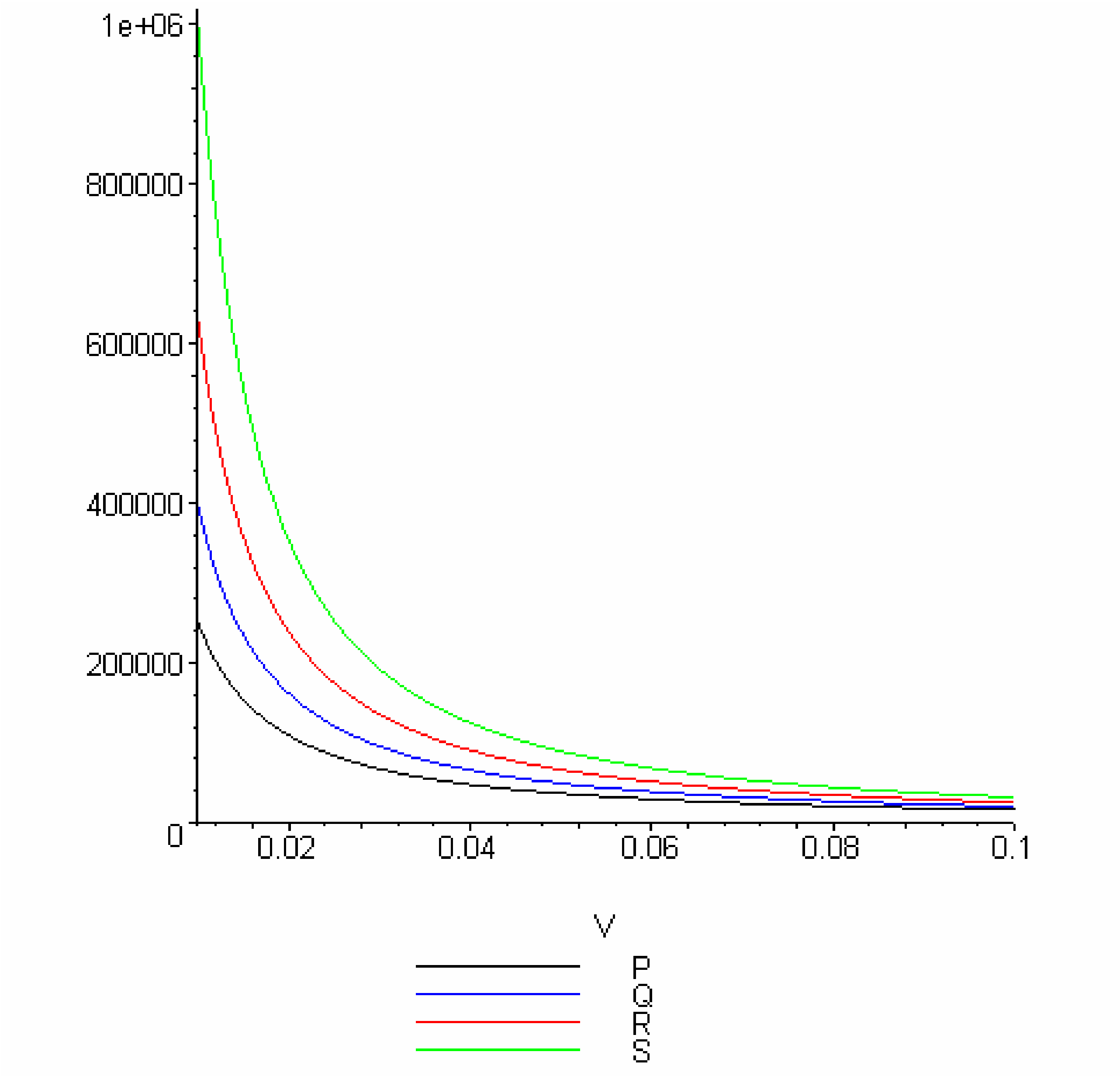
```
> R:=1000/V^1.4;
```

$$R := \frac{1000}{V^{1.4}}$$

```
> S:=1000/V^1.5;
```

$$S := \frac{1000}{V^{1.5}}$$

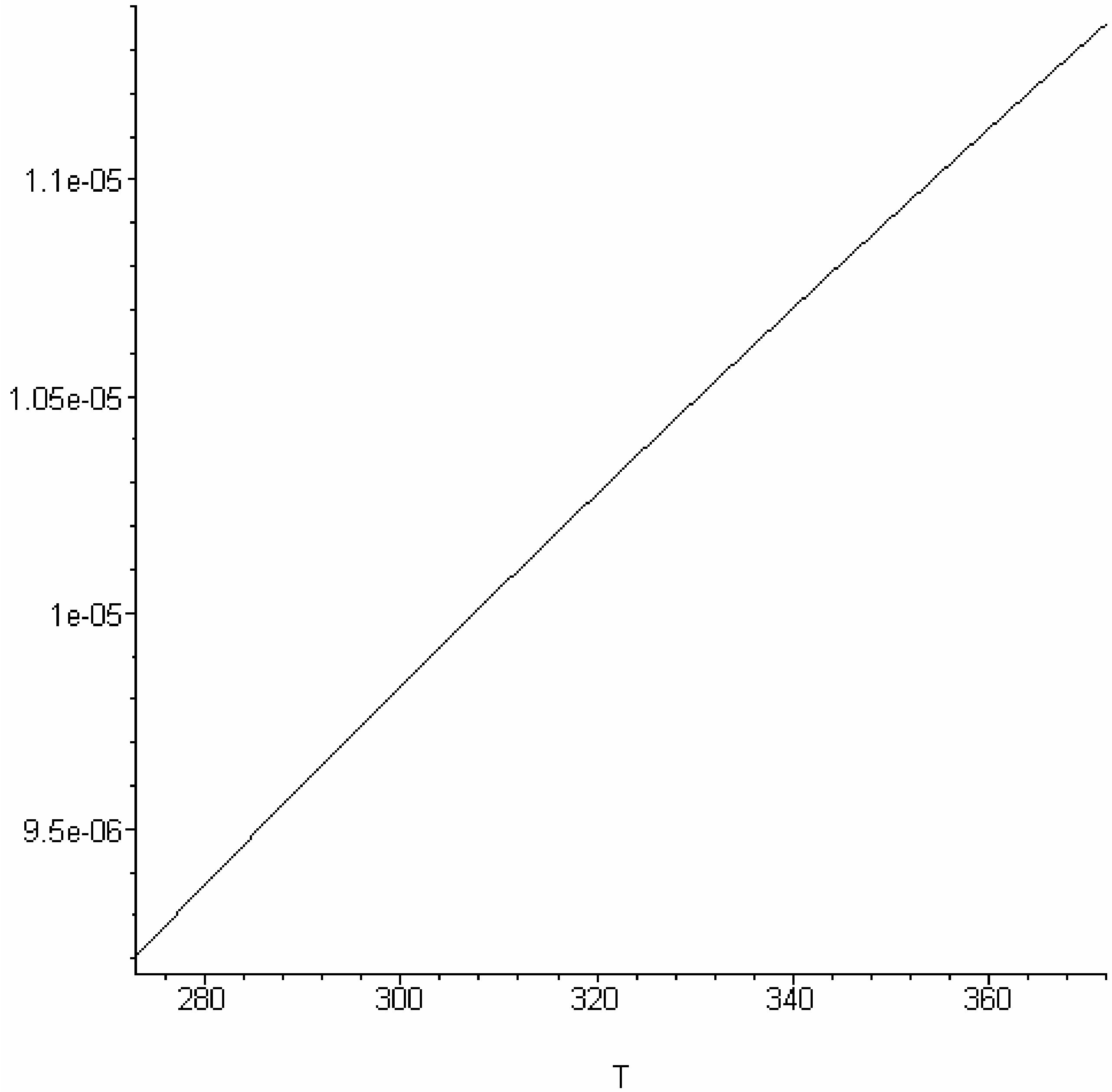
```
>
plot({P,Q,R,S},V=0.01..0.1,color=[black,blue,red,green]);
```




```
> eta:=(0.7e-6)*(T^1.5/(T+70));
```

$$\eta := \frac{0.7\ 10^{-6}\ T^{1.5}}{T+70}$$

```
> plot(eta,T=273..372,color=black);
```



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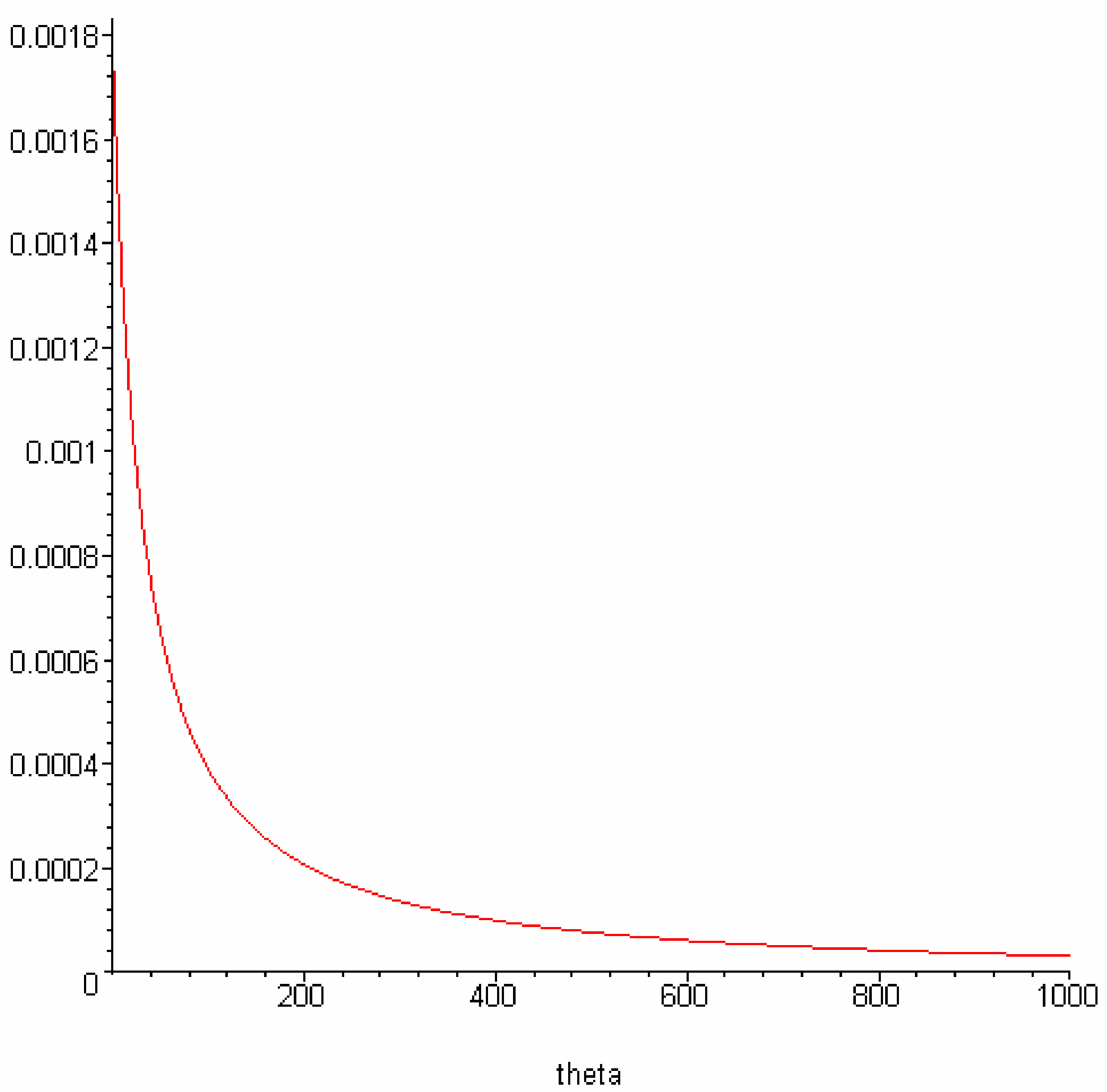
```
> eta:=1.8e-3/(1+34E-3*theta+22E-6*theta^2);;
```

$$\eta := \frac{0.0018}{1 + 0.034\,\theta + 0.000022\,\theta^2}$$

```
> plot(eta,theta=0..1000);
```



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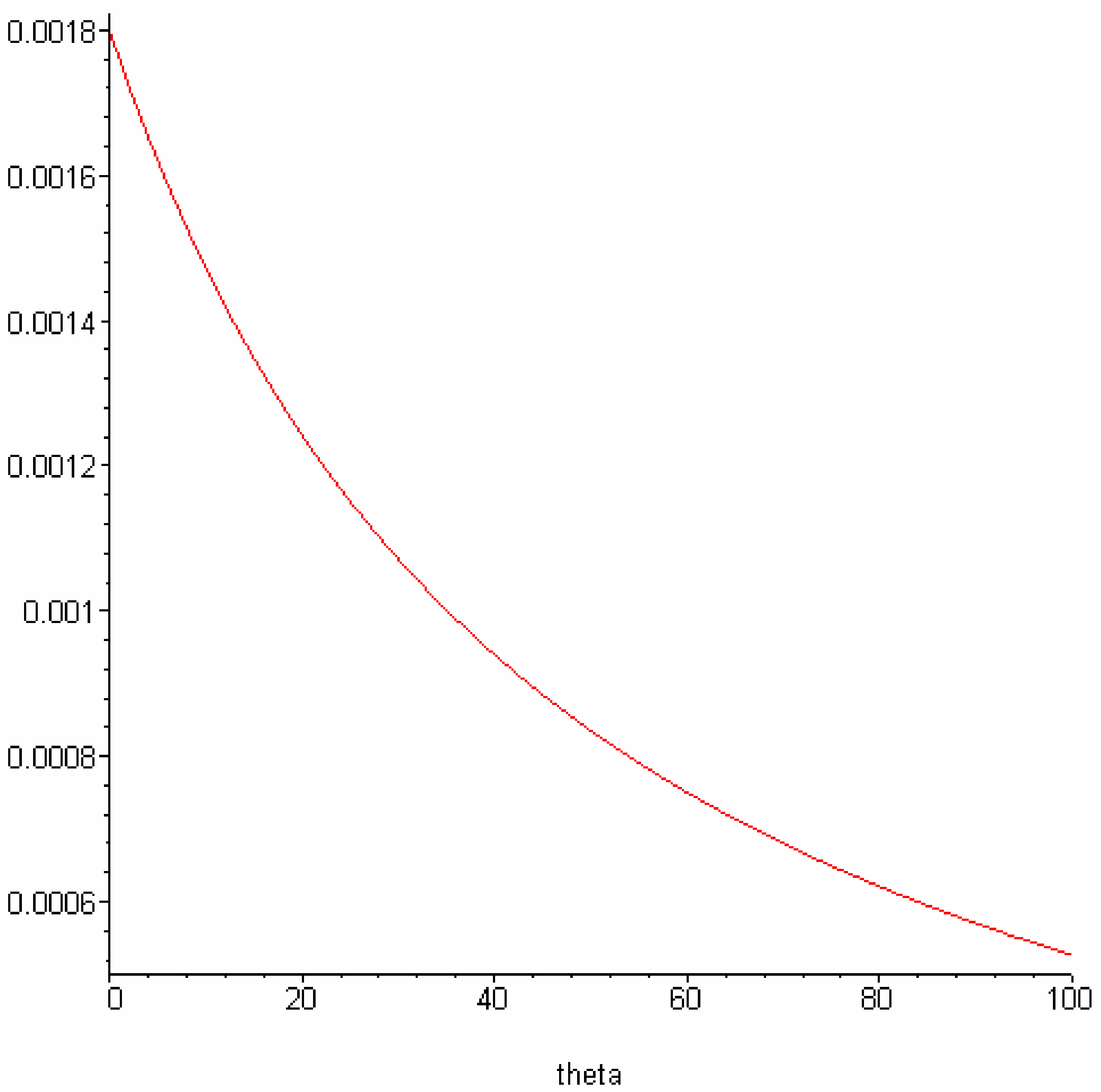
```
> eta:=1.8e-3/(1+22e-3*theta+22e-6*theta^2);;
```

$$\eta := \frac{0.0018}{1 + 0.022\,\theta + 0.000022\,\theta^2}$$

```
> plot(eta,theta=0..100);
```


0.0018
0.0016
0.0014
0.0012
0.001
0.0008
0.0006
0
20
40
60
80
100
theta