Complete solutions to Exercise 8(b)

1. By using the table on pages 408-410 we have:

(a) $\int \sin(t)\, dt = -\cos(t) + C$

(b) $\int \cos(t)\, dt = \sin(t) + C$

(c) $\int \tan(t)\, dt = \ln\left|\sec(t)\right| + C$

(d) $\int e^t dt = e^t + C$

(e) $\int \cosh(t) = \sinh(t) + C$

(f) $\int 9.81\, dt = 9.81t + C$

(g) $\int 25\, dx = 25x + C$

(h) $\int \frac{1}{2} dx = \frac{1}{2}x + C$

2. Selecting the appropriate formula from TABLE 1 we have

(a) $\int \cos(\omega t) d(\omega t) = \sin(\omega t) + C \qquad \left[\text{Using } \int \cos(u)\, du = \sin(u)\right]$

(b) $\int 10^x dx = \frac{10^x}{\ln(10)} + C \qquad \left[\text{Using } \int a^u du = \frac{a^u}{\ln(a)} \text{ with } a = 10\right]$

(c) $\int \sinh(t) dt = \cosh(t) + C$

(d) $\int \sec(x) dx = \ln\left|\sec(x) + \tan(x)\right| + C$

3. (a) $\int PV^{1.35} dV = P\int V^{1.35} dV \underbrace{=}_{\text{by (8.1)}} P\left(\frac{V^{1.35+1}}{1.35+1}\right) + C = \frac{PV^{2.35}}{2.35} + C$

(b) $\int PV^{1.61} dV \underbrace{=}_{\text{by (8.1)}} P\left(\frac{V^{1.61+1}}{1.61+1}\right) + C = \frac{PV^{2.61}}{2.61} + C$

4. (a) $\int \left(x^2 + 2x\right) dx = \int x^2 dx + 2\int x dx \underbrace{=}_{\text{by (8.1)}} \frac{x^{2+1}}{2+1} + 2\left(\frac{x^2}{2}\right) + C = \frac{x^3}{3} + x^2 + C$

(b) We can write $\frac{1}{\sqrt{x}} = \frac{1}{x^{1/2}} = x^{-1/2}$, so we have

$$\int\left(\frac{1}{\sqrt{x}} + \tan(x)\right) dx = \int x^{-1/2} dx + \int \tan(x)\, dx$$

$$= \frac{x^{-1/2+1}}{-1/2+1} + \underbrace{\ln\left|\sec(x)\right|}_{\text{by (8.9)}} + C = \frac{x^{1/2}}{1/2} + \ln\left|\sec(x)\right| + C = 2x^{1/2} + \ln\left|\sec(x)\right| + C$$

(c) Splitting the integrand and then using the table for each component gives:

$$\int \left(\sec^2(x) - 5e^x + 1\right) dx = \int \sec^2(x) dx - 5\int e^x dx + \int 1 dx = \tan(x) - 5e^x + x + C$$

(8.9) $\int \tan(x) dx = \ln\left|\sec(x)\right|$

(d) Similarly

$$\int\left(\sin(x)+2\sqrt{x}-1\right)dx=\int\sin(x)dx+2\int\sqrt{x}dx-\int 1dx$$

$$=\underbrace{-\cos(x)}_{\text{by (8.7)}}+\frac{2x^{1/2+1}}{1/2+1}-x+C$$

$$=-\cos(x)+\frac{2x^{3/2}}{3/2}-x+C$$

$$=-\cos(x)+\frac{(2)2(x)^{3/2}}{3}-x+C$$

$$\int\left[\sin(x)+2\sqrt{x}-1\right]dx=\frac{4x^{3/2}}{3}-\cos(x)-x+C$$

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5. (i) We need to use the quotient rule (6.32) with

$$u=3-4x \qquad v=1+x^2$$

$$u'=-4 \qquad v'=2x$$

$$\frac{d}{dx}\left(\frac{3-4x}{1+x^2}\right)=\frac{-4\left(1+x^2\right)-\left(3-4x\right)2x}{\left(1+x^2\right)^2}$$

$$=\frac{-4-4x^2-6x+8x^2}{\left(1+x^2\right)^2}=\frac{4x^2-6x-4}{\left(1+x^2\right)^2}$$

(ii) $\int\frac{4x^2-6x-4}{\left(1+x^2\right)^2}=\frac{3-4x}{1+x^2}+C$ by part (i) because integration is the inverse process of differentiation.

(6.32) $$\frac{d}{dx}\left(\frac{u}{v}\right)=\frac{u'v-uv'}{v^2}$$

(8.7) $$\int\sin(x)dx=-\cos(x)$$