Complete solutions to Exercise 8(e)

1. (a)

$$\int 2xe^x\,dx = 2\int xe^x\,dx = 2\underbrace{\left[e^x\left(x-1\right)\right]}_{\text{by EXAMPLE 19}} + C$$

(b) $\int t\sin(t)dt$. Using the priority list we have

$$u = t \qquad v' = \sin(t)$$

$$u' = 1 \qquad v = \int \sin(t)dt \underbrace{=}_{\text{by (8.7)}} -\cos(t)$$

Substituting these into (8.45) gives

$$\begin{aligned}\int t\sin(t)dt &= t\left[-\cos(t)\right] - \int\left[-\cos(t)\right]dt \\ &= -t\cos(t) + \int \cos(t)dt \\ &= -t\cos(t) + \underbrace{\sin(t)}_{\text{by (8.8)}} + C = \sin(t) - t\cos(t) + C\end{aligned}$$

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2. (a) $\int q\cos(3q)dq$. Let

$$u = q \qquad v' = \cos(3q)$$

$$u' = 1 \qquad v = \int \cos(3q)\,dq = \frac{\sin(3q)}{3}$$

Applying (8.45):

$$\begin{aligned}\int q\cos(3q)\,dq &= uv - \int u'v\,dq \\ &= q\frac{\sin(3q)}{3} - \int \frac{\sin(3q)}{3}dq \\ &= q\frac{\sin(3q)}{3} - \frac{1}{3}\int \sin(3q)dq \\ &= q\frac{\sin(3q)}{3} - \frac{1}{3}\left(-\frac{\cos(3q)}{3}\right) + C \\ &= q\frac{\sin(3q)}{3} + \frac{\cos(3q)}{9} + C = \frac{1}{9}\left[3q\sin(3q) + \cos(3q)\right] + C\end{aligned}$$

(b) Apply (8.45):

$$u = \ln(s) \qquad v' = s^2$$

$$u' = \frac{1}{s} \qquad v = \int s^2\,ds = \frac{s^3}{3}$$

(8.7) $\int \sin(t)dt = -\cos(t)$

(8.8) $\int \cos(t)dt = \sin(t)$

(8.45) $\int uv'\,dx = uv - \int u'v\,dx$

$$\int s^2 \ln(s)\,ds = uv - \int u'v\,ds$$
$$= \frac{s^3}{3}\ln(s) - \int\left[\frac{1}{s}\frac{s^3}{3}\right]ds$$
$$= \frac{s^3}{3}\ln(s) - \frac{1}{3}\int s^2 ds \qquad \text{(Simplifying)}$$
$$= \frac{s^3}{3}\ln(s) - \frac{1}{3}\left(\frac{s^3}{3}\right) + C$$
$$= \frac{s^3}{9}\left(3\ln(s) - 1\right) + C$$

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3. Let

$$u = t \qquad v' = e^{2t}$$
$$u' = 1 \qquad v = \int e^{2t}dt \underbrace{=}_{\text{by (8.41)}} \frac{e^{2t}}{2}$$

Using (8.45)

$$\int te^{2t}\,dt = uv - \int u'v\,dt$$
$$= (t)\frac{e^{2t}}{2} - \int (1)\frac{e^{2t}}{2}dt$$
$$= \frac{te^{2t}}{2} - \underbrace{\frac{e^{2t}}{4}}_{\text{by (8.41)}} + C$$
$$= \frac{e^{2t}}{4}(2t - 1) + C \qquad \text{[Factorizing]}$$

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4. Let

$$u = p \qquad v' = \sqrt{1+p} = (1+p)^{1/2}$$
$$u' = 1 \qquad v = \int (1+p)^{1/2}\,dp$$
$$\underbrace{=}_{\text{by (8.1)}} \frac{(1+p)^{3/2}}{3/2} = \frac{2(1+p)^{3/2}}{3}$$

(8.1) $\int u^n du = \dfrac{u^{n+1}}{n+1}$

(8.41) $\int e^{kt+m}dt = e^{kt+m}/k$

(8.45) $\int uv\,dx = uv - \int u'v\,dx$

Applying (8.45) gives

$$\int p\sqrt{1+p}\,dp = p\left[\frac{2(1+p)^{3/2}}{3}\right] - \int \frac{2(1+p)^{3/2}}{3}dp$$

$$= \frac{2}{3}\left[p(1+p)^{3/2} - \int (1+p)^{3/2}\,dp\right] \qquad (*)$$

We still need to find $\int (1+p)^{3/2}\,dp$.

$$\int (1+p)^{3/2}\,dp = \frac{(1+p)^{5/2}}{5/2} = \frac{2(1+p)^{5/2}}{5}$$

Substituting this into (*) and adding a constant C gives

$$\int p\sqrt{1+p}\,dp = \frac{2}{3}\left[p(1+p)^{3/2} - \frac{2}{5}(1+p)^{5/2}\right] + C$$

$$= \frac{2}{3}\underbrace{(1+p)^{3/2}}_{\substack{\text{taking out a common} \\ \text{factor of } (1+p)^{3/2}}}\left[p - \frac{2}{5}(1+p)\right] + C$$

$$= \frac{2}{3}(1+p)^{3/2}\left[p - \frac{2}{5} - \frac{2}{5}p\right] + C$$

$$= \frac{2}{3}(1+p)^{3/2}\left[\frac{3}{5}p - \frac{2}{5}\right] + C = \frac{2}{\underbrace{15}_{=3\times 5}}(1+p)^{3/2}\left[3p-2\right] + C$$

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5. $\int \ln(x)\,dx = \int (1\times \ln(x))\,dx$. By using our list we have

$$u = \ln(x) \qquad v' = 1$$

$$u' = \frac{1}{x} \qquad v = \int 1\,dx = x$$

Substituting these into (8.45) gives

$$\int \ln(x)\,dx = x\ln(x) - \int\left[x.\frac{1}{x}\right]dx$$

$$= x\ln(x) - \int 1\,dx$$

$$= x\ln(x) - x + C = x\left[\ln(x) - 1\right] + C$$

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6. We have

$$i = \frac{1}{10\times 10^{-3}}\int_0^t 5te^{-t}\,dt$$

$$= \underbrace{500}_{=\frac{5}{10\times 10^{-3}}}\int_0^t te^{-t}\,dt \qquad (*)$$

How do we integrate te^{-t}?

Use the integration by parts formula (8.45):

$$u = t \qquad v' = e^{-t}$$

$$u' = 1 \qquad v = \int e^{-t}\,dt = -e^{-t}$$

(8.45) $\int uv\,dx = uv - \int u'v\,dx$

$$\int_0^t te^{-t}dt = \left[-te^{-t}\right]_0^t + \int_0^t e^{-t}dt$$
$$= \left[-te^{-t}\right]_0^t - \left[e^{-t}\right]_0^t$$
$$= -te^{-t} - e^{-t} + 1 \qquad \text{(Substituting)}$$

Putting this into (*) gives

$$i = 500\left(1 - e^{-t} - te^{-t}\right)$$

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7. Substituting $L = 10 \times 10^{-3}$ and $v = t\cos(t)$ into the given formula:

$$w = \frac{1}{20\times 10^{-3}}\left(\int_0^t t\cos(t)\,dt\right)^2 \qquad (*)$$

How do we integrate within the brackets?
Use (8.45) (integration by parts).

$$u = t \qquad v' = \cos(t)$$
$$u' = 1 \qquad v = \int \cos(t)dt = \sin(t)$$

Substituting into (8.45) gives

$$\int_0^t t\cos(t)\,dt = \left[t\sin(t)\right]_0^t - \int_0^t \sin(t)\,dt$$
$$= t\sin(t) + \left[\cos(t)\right]_0^t = t\sin(t) + \cos(t) - 1$$

Substituting this into (*) gives

$$w = \frac{1}{20\times 10^{-3}}\left[t\sin(t) + \cos(t) - 1\right]^2 = 50\left[t\sin(t) + \cos(t) - 1\right]^2$$

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8. We have

$$i = \frac{1}{1\times 10^{-3}}\int_0^1 t^2 e^{-t}dt \qquad (*)$$

Consider $\int_0^1 t^2 e^{-t}dt$. Let

$$u = t^2 \qquad v' = e^{-t}$$
$$u' = 2t \qquad v = \int e^{-t}dt = -e^{-t}$$

Substituting these into (8.45) gives:

$$\int_0^1 t^2 e^{-t}dt = \left[t^2\left(-e^{-t}\right)\right]_0^1 + \int_0^1 2t\left(e^{-t}\right)dt$$
$$= -\left(1e^{-1}\right) + 2\int_0^1 te^{-t}dt$$
$$= -e^{-1} + 2\left\{\left[t\left(-e^{-t}\right)\right]_0^1 - \int_0^1 1.\left(-e^{-t}\right)dt\right\} \qquad \text{(applying (8.45) again)}$$
$$= -e^{-1} + 2\left\{\left[-te^{-t}\right]_0^1 + \left[-e^{-t}\right]_0^1\right\}$$
$$= -e^{-1} + 2\left\{\left(-1.e^{-1}\right) - \left[e^{-1} - 1\right]\right\} = -e^{-1} - 4e^{-1} + 2 = 2 - 5e^{-1}$$

Putting this into (*) gives:

$$i = \frac{1}{1\times 10^{-3}}\left(2 - 5e^{-1}\right) = 160.60 = 161\text{ A}$$

(8.45) $$\int uv'\,dt = uv - \int u'v\,dt$$