Complete solutions to Exercise 16(h)

1. Very similar to EXAMPLES 45 and 46.
(a) 0.9332 (b) 0.95 (c) 0.975 (d) 0.05 (e) 0.05
(f) 0.0228 (g) 0.9 (h) 0.1191 (i) 0.1662

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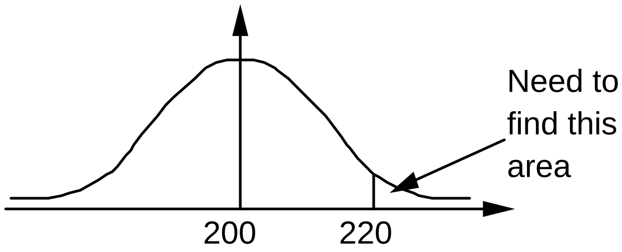
2. Putting $x = 540$, $\mu = 500$ and $\sigma = 22$ into (16.48) gives

$$z = \frac{540 - 500}{22} = 1.818$$

From the tables for $z = 1.818$ we have 0.9655.

$$P(\text{bulb lasting} < 540 \text{ hours}) = 0.9655$$

3.



Using (16.48) with $x = 220$, $\mu = 200$ and $\sigma = 10$ we have

$$z = \frac{220 - 200}{10} = 2$$

Using the table with $z = 2$ gives 0.9772
Hence

$$P(\text{wearout time} > 220 \text{ hours}) = 1 - 0.9772 = 0.0228$$

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4. (a) Substituting $x = 660$, $\mu = 600$ and $\sigma = 20$ into (16.48) gives

$$z = \frac{660 - 600}{20} = 3$$

The table for $z = 3$ gives 0.99865. The probability of a product having a wearout time of more than 660 hours is

$$1 - 0.99865 = 0.00135$$

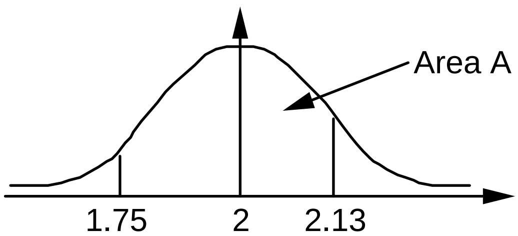
(b) For $x = 550$

$$z = \frac{550 - 600}{20} = -2.5$$

The table gives 0.99379

$$P(\text{a product with a wearout time} < 550) = 1 - 0.99379 = 0.00621$$

5.



We need to find the area A. Substituting $\mu = 2$, $\sigma = 0.15$ and $x = 2.13$ into

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(16.48) $$z = \frac{x - \mu}{\sigma}$$

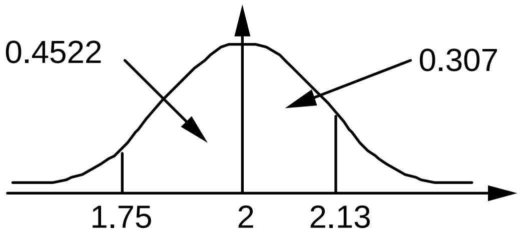
(16.48) gives

$$z=\frac{2.13-2}{0.15}=0.867$$

The area below $z=0.867$ from the tables is 0.807. For $x=1.75$

$$z=\frac{1.75-2}{0.15}=-1.667$$

The corresponding value in the table is 0.9522. Since 0.5 of the area lies in each half so we have



The probability of a fuse blowing between 1.75 and 2.13 is

$$0.4522+0.307=0.7592$$

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6. (a) Putting $\mu=1.7,\ \sigma=0.15$ and $x=1.8$ into $z=\dfrac{x-\mu}{\sigma}$ gives

$$z=\frac{1.8-1.7}{0.15}=0.667$$

From the tables we have 0.7477 for $z=0.667$. The probability corresponding to a z value >0.667 is

$$1-0.7477=0.2523$$

$$\text{Number of students }=500\times 0.2523$$
$$=126.15$$

126 students have a height of more than $1.8m$.

(b) Substituting $\mu=1.7,\ \sigma=0.15$ and $x=1.5$ into (16.48) gives

$$z=\frac{1.5-1.7}{0.15}=-1.333$$

The area from the tables corresponding to $z=1.333$ is 0.9087.

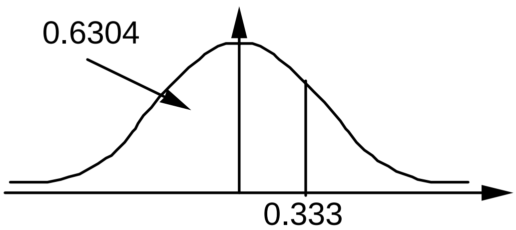
$$\text{Number of students }=500\times(1-0.9087)$$
$$=45.65$$

45 students have a height of less than 1.5 m.

(c) Using (16.48) with $\mu=1.7,\ \sigma=0.15$ and $x=1.65$ gives

$$z_1=\frac{1.65-1.7}{0.15}=-0.333$$

From the tables we have 0.6304

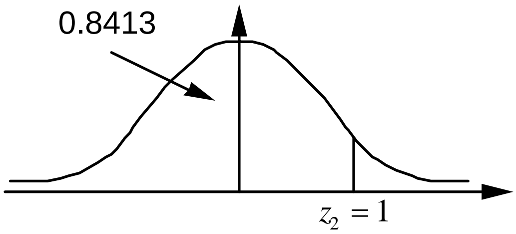


(16.48)
$$z=\frac{x-\mu}{\sigma}$$

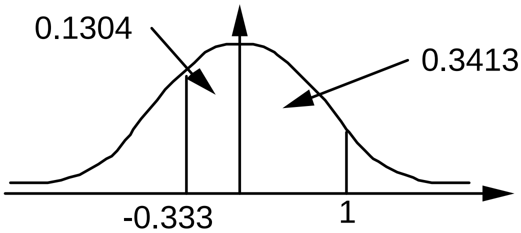
For $x = 1.85$

$$z_2 = \frac{1.85 - 1.7}{0.15} = 1$$

From the tables we have 0.8413



Remember z_1 is to the left of the mean and we take off 0.5 from each area because one half is not included.



The corresponding area $= 0.1304 + 0.3413 = 0.4717$

$$\text{Number of students } = 500 \times 0.4717$$

$$= 235.85$$

235 students have heights between 1.65m and 1.85m.

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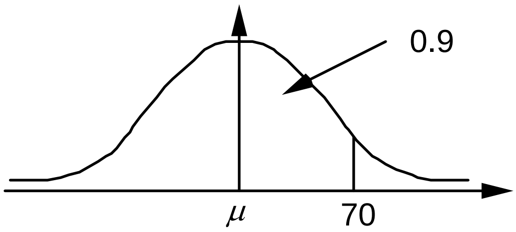
7. Since 90% of the students have a mass less than $70kg$ we have



The z value for an area of 0.9 is 1.281. Using (16.48) with $\sigma = 1.2$ gives

$$\frac{70 - \mu}{1.2} = 1.281$$

Transposing to make μ the subject gives

$$\mu = 70 - (1.2 \times 1.281) = 68.4628$$

The mean is $68.463kg$ (3 d.p.)

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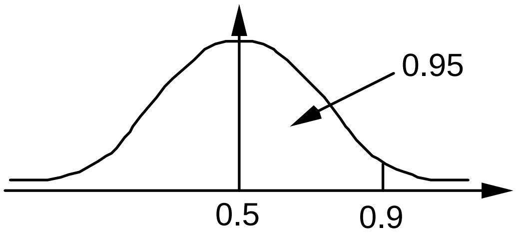
8. We have

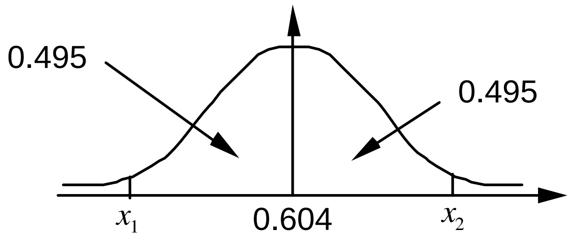


(16.48) $$z = \frac{x - \mu}{\sigma}$$

By finding 0.95 in the table we get a z value of 1.645. Using (16.48) with $x = 0.9$, $\mu = 0.5$ and $z = 1.645$ we have

$$\frac{0.9 - 0.5}{\sigma} = 1.645 \text{ gives } \sigma = 0.243 \text{ m (3 d.p.)}$$

9. Half of 99% is 49.5%. So we examine an area of 0.495 in each half.



The area less than x_2 is $0.5 + 0.495 = 0.995$. From the table the z value corresponding to 0.995 is 2.575. Using (16.48) with $z = 2.575$, $\mu = 0.604$ and $\sigma = 0.01$ gives

$$\frac{x_2 - 0.604}{0.01} = 2.575$$

$$x_2 = 0.62975$$

Similarly

$$\frac{x_1 - 0.604}{0.01} = -2.575$$

$$\text{gives} \quad x_1 = 0.57825$$

99% of the cylinders have diameters between 0.57825 m to 0.62975 m.

(16.48) $$z = \frac{x - \mu}{\sigma}$$