Complete Solutions to Exercise 1(c)

1. Straightforward application of the rules of indices.

(a) $x^5x^2 = x^{5+2} = x^7$

(b) $x^{\frac{1}{5}}x^{\frac{1}{2}} = x^{\frac{1}{5}+\frac{1}{2}} = x^{\frac{2}{10}+\frac{5}{10}} = x^{\frac{7}{10}}$

(c) $\dfrac{x^3}{x^3} = 1$

(d) $\dfrac{x^7}{x^9} = x^{7-9} = x^{-2} = \dfrac{1}{x^2}$

(e) $\left(\sqrt[5]{x}\right)^2 \sqrt[3]{x} = x^{\frac{2}{5}}x^{\frac{1}{3}} = x^{\frac{2}{5}+\frac{1}{3}} = x^{\frac{6}{15}+\frac{5}{15}} = x^{\frac{11}{15}}$

(f) $\sqrt{x^2}\,\sqrt[3]{x^3} = xx = x^2$

(g) $\sqrt[7]{x^3x^4} = \sqrt[7]{x^{3+4}} = \sqrt[7]{x^7} = x$

2. (a) $(1+y)^2(1+y) = (1+y)^{2+1} = (1+y)^3$ (Take $a = 1+y$ in formula (1.5)).

(b) $\dfrac{\left(1+x^2\right)^5}{\left(1+x^2\right)^3} \underbrace{=}_{\text{by (1.6)}} \left(1+x^2\right)^{5-3} = \left(1+x^2\right)^2$

(c) The expression cannot be simplified any further because the terms under the cube root are different. However we can rewrite it as:

$$\left(\sqrt[3]{x^2+x+1}\right)^5\left(\sqrt[3]{x^2+x}\right) = \underbrace{\left(x^2+x+1\right)^{\frac{5}{3}}}_{\text{by (1.2) and (1.7)}}\underbrace{\left(x^2+x\right)^{\frac{1}{3}}}_{\text{by (1.2)}}$$

(d) In this case we have the same terms so

$$\left(\sqrt[3]{x^2+x+1}\right)^5\left(\sqrt[3]{x^2+x+1}\right) = \left(x^2+x+1\right)^{\frac{5}{3}}\left(x^2+x+1\right)^{\frac{1}{3}}$$

$$\underbrace{=}_{\substack{\text{by (1.5) with}\\ a=x^2+x+1}} \left(x^2+x+1\right)^{\frac{5}{3}+\frac{1}{3}}$$

$$= \left(x^2+x+1\right)^2$$

(e) We **cannot** simplify the given expression any further because we have **different** expressions under the square root signs - $\sqrt{x^2+2x}\sqrt{x^2+x}$.

3. Similar to **EXAMPLE 12**. $V_2 = 0.734\text{m}^3$.

4. Substitute for C into W and apply the rules of indices.

5. Rearranging $\dfrac{P_1V_1}{T_1} = \dfrac{P_2V_2}{T_2}$ we have

$$\frac{T_1}{T_2} = \frac{P_1V_1}{P_2V_2} = \frac{P_1}{P_2}.\frac{V_1}{V_2} \qquad (*)$$

From the other formula, $P_1V_1^{\,n} = P_2V_2^{\,n}$, we have $\dfrac{V_1^{\,n}}{V_2^{\,n}} = \dfrac{P_2}{P_1}$ and so

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(1.2) $\sqrt[n]{a} = a^{\frac{1}{n}}$ (1.5) $a^ma^n = a^{m+n}$ (1.6) $\dfrac{a^m}{a^n} = a^{m-n}$ (1.7) $\left(a^m\right)^n = a^{m\times n}$

$$\left(\frac{V_1}{V_2}\right)^n = \frac{P_2}{P_1}$$

Taking the nth root gives

$$\left(\frac{V_1}{V_2}\right) = \left(\frac{P_2}{P_1}\right)^{\frac{1}{n}}$$

$$\frac{T_1}{T_2} = \frac{P_1}{P_2}\left(\frac{P_2}{P_1}\right)^{\frac{1}{n}}$$

$$= \frac{P_1}{P_2}\underbrace{\left(\frac{P_1}{P_2}\right)^{-\frac{1}{n}}}_{\text{by (1.4)}} = \underbrace{\left(\frac{P_1}{P_2}\right)^{1-\frac{1}{n}}}_{\text{by (1.5)}}$$

6. We have

$$\frac{\rho_2}{\rho_1} = \left(\frac{T_2}{T_1}\right)^{\frac{g}{LC}}\left(\frac{T_1}{T_2}\right)$$

$$= \left(\frac{T_2}{T_1}\right)^{\frac{g}{LC}}\underbrace{\left(\frac{T_2}{T_1}\right)^{-1}}_{\text{by (1.4)}}$$

$$= \underbrace{\left(\frac{T_2}{T_1}\right)^{\frac{g}{LC}-1}}_{\text{by (1.5)}} = \left(\frac{T_2}{T_1}\right)^{\frac{g-LC}{LC}}$$

(1.4) $$\left(\frac{a}{b}\right)^{-1} = \frac{b}{a}$$

(1.5) $$a^m a^n = a^{m+n}$$