Complete solutions to Exercise 10(c)

1. Use a calculator to put v and Z into polar form

$$v = 2.9 + j6.2 = 6.84\angle 64.93°$$

$$Z = 8 + j1.9 = 8.22\angle 13.36°$$

$$i = \frac{6.84\angle 64.93°}{8.22\angle 13.36°} \underbrace{=}_{\text{by (10.18)}} \frac{6.84}{8.22}\angle\left(64.93° - 13.36°\right)$$

$$= 0.83\angle 51.57°$$

$$i = 0.83\angle 51.57°\text{ A}$$

2. We have

$$z = \frac{2\angle 45° \times 2\angle\left(-60°\right)}{4\angle\left(-22°\right) \times 5\angle 33°}$$

$$\underbrace{=}_{\text{by (10.17)}} \frac{2 \times 2\angle\left(45° + \left(-60°\right)\right)}{4 \times 5\angle\left(-22° + 33°\right)}$$

$$= \frac{4\angle\left(-15°\right)}{20\angle 11°} \underbrace{=}_{\text{by (10.18)}} \frac{4}{20}\angle\left(-15° - 11°\right) = 0.2\angle\left(-26°\right)$$

Hence modulus $|z| = 0.2$ and $\arg(z) = -26°$

3. (a) We use TABLE 1 to find the values of $\cos(30°)$ and $\sin(30°)$:

$$2\angle 30° = 2\left[\cos(30°) + j\sin(30°)\right]$$

$$= 2\left(\frac{\sqrt{3}}{2} + j\frac{1}{2}\right) = \sqrt{3} + j \qquad \left[\text{Cancelling } 2's\right]$$

(b) $2\angle\left(\frac{\pi}{6}\right) = \sqrt{3} + j$ [As (a) because $\frac{\pi}{6} = 30°$]

(c) $240\angle 0° = 240$

4. We have

$$z = r\angle\theta = r\left[\cos(\theta) + j\sin(\theta)\right]$$

$$z = r\cos(\theta) + jr\sin(\theta)$$

The complex conjugate, $\bar{z}$, is given by

$$\bar{z} = r\cos(\theta) - jr\sin(\theta)$$

$$= r\left[\cos(\theta) - j\sin(\theta)\right]$$

$$= r\left[\underbrace{\cos(-\theta)}_{\text{by (4.51)}} + j\underbrace{\sin(-\theta)}_{\text{by (4.50)}}\right] = r\angle(-\theta)$$

(4.50)	$\sin(-x) = -\sin(x)$
(4.51)	$\cos(-x) = \cos(x)$
(10.17)	$r\angle A \times q\angle B = rq\angle(A + B)$
(10.18)	$\frac{r\angle A}{q\angle B} = \frac{r}{q}\angle(A - B)$

5. Using the result of question 4 gives

(a) $6\angle(-45°)$ (b) $2\angle(-131°)$ (c) $15\angle 26°$

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6. (i) Let $z = a + jb$ then

$$|a+jb| = \sqrt{a^2+b^2} = 1 \text{ which gives } a^2+b^2 = 1$$

$$\frac{1}{z} = \frac{1}{a+jb} \underbrace{=}_{\text{by (10.13)}} \frac{a-jb}{a^2+b^2} = \frac{a-jb}{1} = a - jb = \bar{z}$$

(ii) We have $z = \frac{\sqrt{8}}{3} + \frac{j}{3}$ therefore

$$|z| = \sqrt{\left(\frac{\sqrt{8}}{3}\right)^2 + \left(\frac{1}{3}\right)^2} = \sqrt{\frac{8}{9}+\frac{1}{9}} = 1$$

Hence by part(i)

$$\frac{1}{z} = \bar{z} = \frac{1}{3}\left(\sqrt{8} - j\right)$$

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7. We have $z = r\left[\cos(\alpha) + j\sin(\alpha)\right]$ $w = q\left[\cos(\beta) + j\sin(\beta)\right]$

$$\frac{z}{w} = \frac{r}{q}\left[\frac{\cos(\alpha)+j\sin(\alpha)}{\cos(\beta)+j\sin(\beta)}\right]$$

$$\underbrace{=}_{\text{by (10.13)}} \frac{r}{q}\left[\frac{\left[\cos(\alpha)+j\sin(\alpha)\right]\left[\cos(\beta)-j\sin(\beta)\right]}{\cos^2(\beta)+\sin^2(\beta)}\right]$$

$$= \frac{r}{q}\left[\cos(\alpha)+j\sin(\alpha)\right]\left[\cos(\beta)-j\sin(\beta)\right] \quad \left(\text{because } \cos^2(\beta)+\sin^2(\beta)=1\right)$$

$$= \frac{r}{q}\left[\cos(\alpha)\cos(\beta) - j\cos(\alpha)\sin(\beta) + j\sin(\alpha)\cos(\beta) - j^2\sin(\alpha)\sin(\beta)\right]$$

$$= \frac{r}{q}\left[\left(\cos(\alpha)\cos(\beta) + \underbrace{\sin(\alpha)\sin(\beta)}_{\text{because } j^2=-1}\right) + j\left(\sin(\alpha)\cos(\beta) - \cos(\alpha)\sin(\beta)\right)\right]$$

$$\frac{z}{w} = \frac{r}{q}\left[\underbrace{\cos(\alpha-\beta)}_{\text{by (4.40)}} + j\underbrace{\sin(\alpha-\beta)}_{\text{by (4.38)}}\right]$$

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8. Substituting $\omega = 2\times 10^3$ into T gives

$$T = \frac{1000}{1 + j\left(2\times10^3\right)\left(0.5\times10^{-3}\right)}$$

$$= \frac{1000}{1+j} = \frac{1000\angle 0°}{\sqrt{2}\angle 45°} \underbrace{=}_{\text{by (10.18)}} \frac{1000}{\sqrt{2}}\angle(-45°)$$

(4.38) $\sin(A-B) = \sin(A)\cos(B) - \cos(A)\sin(B)$

(4.40) $\cos(A-B) = \cos(A)\cos(B) + \sin(A)\sin(B)$

(10.13) $\dfrac{a+jb}{c+jd} = \dfrac{(a+jb)(c-jd)}{c^2+d^2}$

$\text{Gain} = \frac{1000}{\sqrt{2}}, \ \text{Phase} = -45°$

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9. Substituting $\omega = 45$ into T gives

$$T = \frac{50}{j45(1+j0.45)(1+j9)}$$

$$\underbrace{=}_{\text{putting each number in polar form}} \frac{50\angle 0°}{45\angle 90° \times 1.10\angle 24.23° \times 9.06\angle 83.66°}$$

$$\text{Gain of } T = |T| = \frac{50}{45 \times 1.10 \times 9.06} = 0.11$$

$$\text{Phase of } T = 0° - 90° - 24.23° - 83.66° = -197.89°$$

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10. Let

$$(6+j2)I_1 + (2+j2)I_2 = 10 \qquad (*)$$

$$(2+j2)I_1 + 7I_2 = 10 \qquad (**)$$

Then $(*) - (**)$ gives

$$4I_1 + (j2-5)I_2 = 0$$

Making I_1 the subject

$$I_1 = \frac{5-j2}{4} I_2 \qquad (***)$$

Substituting this into (**) gives

$$(2+j2)\frac{5-j2}{4} I_2 + 7I_2 = 10$$

$$\left[\frac{7+j3}{2} + 7\right] I_2 = 10$$

$$\frac{21+j3}{2} I_2 = 10$$

$$I_2 = \frac{20}{21+j3}$$

$$= \frac{20\angle 0°}{21.21\angle 8.13°} \underbrace{=}_{\text{by (10.18)}} \frac{20}{21.21}\angle(0° - 8.13°)$$

$I_2 = 0.94\angle(-8.13°)A$. Substituting for I_2 into (***)

$$I_1 = \frac{5-j2}{4} \times 0.94\angle(-8.13°)$$

$$= \frac{5.39\angle(-21.8°) \times 0.94\angle(-8.13°)}{4\angle 0°}$$

$$\underbrace{=}_{\text{by (10.17) and (10.18)}} \frac{5.39 \times 0.94}{4}\angle(-21.8° - 8.13° - 0°)$$

$$I_1 = 1.27\angle(-29.93°)\text{A}$$

(10.18) $$\frac{r\angle A}{q\angle B} = \frac{r}{q}\angle(A-B)$$