Complete solutions to Exercise 10(d)

1.(a) $(1+j)^2 = j2$

(b)

$$(1-j)^3 = \left(\sqrt{2}\angle(-45°)\right)^3 \underbrace{=}_{\text{by (10.19)}} \left(\sqrt{2}\right)^3 \angle\left(3\times(-45°)\right)$$

$$= 2^{3/2}\angle(-135°) = -2 - j2$$

(c)

$$(1+j)^4 = \left(\sqrt{2}\angle 45°\right)^4 \underbrace{=}_{\text{by (10.19)}} \left(\sqrt{2}\right)^4 \angle(4\times 45°)$$

$$= 4\angle 180° = -4$$

(d) Using a calculator to put the number into polar form:

$$(6-j8)^7 = \left(10\angle(-53.13°)\right)^7 \underbrace{=}_{\text{by (10.19)}} 10^7\angle\left(7\times(-53.13°)\right)$$

$$= 10^7\angle(-371.91°) = 9784704 - j2063872$$

(e) $(-3+j4)^7 = (5\angle 126.87°)^7 = 5^7\angle(7\times 126.87°) = -76443 + j16123$

(f) $(-3-j)^5 = (3.16\angle(-161.57°))^5 = 12 - j316$

2. The numbers 1 and -1 are the obvious roots of $z^4 - 1 = 0$. From $1 = 1\angle 0°$, the other roots are found by adding $\left(\frac{360}{4}\right)^\circ = 90°$ to the first root $1\angle 0°$.

Hence the four roots are

$$1\angle 0°,\ 1\angle 90°,\ 1\angle 180° \text{ and } 1\angle 270°$$

In rectangular form the roots are 1, j, -1 and $-j$.

3. We have

$$1+j = \sqrt{2}\angle 45° = 2^{1/2}\angle\left(\frac{\pi}{4}\right)$$

$$(1+j)^n = \left(2^{1/2}\angle\left(\frac{\pi}{4}\right)\right)^n \underbrace{=}_{\text{by (10.19)}} 2^{n/2}\angle\left(\frac{n\pi}{4}\right)$$

Substituting $n = 12$ gives

$$(1+j)^{12} = 2^{12/2}\angle\left(\frac{12\pi}{4}\right) = 2^6\angle 3\pi = -2^6 = -64$$

4. Substituting $R = 10$, $\omega = 10\,000$, $L = 0.1\times 10^{-3}$, $C = 1\times 10^{-9}$ and $G = 1\times 10^{-6}$ gives

$$Z_0 = \left(\frac{R+j\omega L}{G+j\omega C}\right)^{1/2} = \left(\frac{10+j}{10^{-6}+j10^{-5}}\right)^{1/2} = \frac{1}{10^{-3}}\left(\frac{10+j}{1+j10}\right)^{1/2}$$

$$= 1000\left(\frac{10.05\angle 5.71°}{10.05\angle 84.29°}\right)^{1/2}$$

(10.18) $$\frac{r\angle A}{q\angle B} = \frac{r}{q}\angle(A-B)$$

(10.19) $$(r\angle\theta)^n = r^n\angle n\theta$$

$$\underbrace{=}_{\text{by (10.18)}} 1000\left(\angle\left(5.71° - 84.29°\right)\right)^{1/2}$$

$$\underbrace{=}_{\text{by (10.19)}} 1000\angle\left(\frac{1}{2}\times\left(-78.58°\right)\right)$$

$$Z_0 = 1000\angle\left(-39.3°\right)\ \Omega$$

(ii) Using the polar form of $R + j\omega L$ and $G + j\omega C$ from part (i) gives

$$\gamma = \left(10.05\angle 5.71°\times\left(10.05\times 10^{-6}\right)\angle 84.29°\right)^{1/2} \underbrace{=}_{\text{by (10.17)}} \left(10.05^2\times 10^{-6}\angle\left(5.71° + 84.29°\right)\right)^{1/2}$$

$$\underbrace{=}_{\text{by (10.19)}} \left(10.05\times 10^{-3}\right)\angle\left(\frac{1}{2}\times 90°\right)$$

$$= 0.01005\angle 45°/\text{m}$$

(iii) We have $\gamma = 0.01005\angle 45° = 0.007106 + j0.007106$, equating real and imaginary parts gives

$$\beta = 0.007106/\text{m}$$

Substituting $\beta = 0.007106$ into λ gives

$$\lambda = \frac{2\pi}{0.007106} = 884.2\ \text{m}$$

(iv) $\nu = \dfrac{10000}{0.007106} = 1.407\times 10^6\ \text{m/s}$

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5. First we evaluate $(\omega + j\omega)^4$

$$(\omega + j\omega)^4 = \left(\sqrt{2}\omega\angle\left(\frac{\pi}{4}\right)\right)^4 \underbrace{=}_{\text{by (10.19)}} 4\omega^4\angle\pi$$

Substituting $(\omega + j\omega)^4 = 4\omega^4\angle\pi$ into G gives

$$G = \frac{40\angle 0}{4\omega^4\angle\pi} \underbrace{=}_{\text{by (10.18)}} \frac{40}{4\omega^4}\angle\left(0 - \pi\right) = \frac{10}{\omega^4}\angle\left(-\pi\right) = -\frac{10}{\omega^4}$$

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6. (i) Poles occur at $z^3 = -64$ which gives

$$z = (-64)^{1/3} = -4 = 4\angle 180° = p_1$$

How do we find the other roots?

Add $\left(\dfrac{360}{3}\right)^{\circ} = 120°$ to find the other two roots:

$$p_2 = 4\angle 300° \text{ and } p_3 = 4\angle 420° = 4\angle 60°$$

In rectangular form the poles are given by

$$p_1 = -4,\ p_2 = 2 + j3.46 \text{ and } p_3 = 2 - j3.46$$

(ii) Zeros occur where $z^4 = -16 = 16\angle 180°$ which gives

$$z = \left(16\angle 180°\right)^{1/4} \underbrace{=}_{\text{by (10.19)}} 2\angle 45° = z_1$$

(10.17) $$r\angle A\times q\angle B = rq\angle\left(A + B\right)$$

(10.18) $$\frac{r\angle A}{q\angle B} = \frac{r}{q}\angle\left(A - B\right)$$

(10.19) $$\left(r\angle\theta\right)^n = r^n\angle n\theta$$

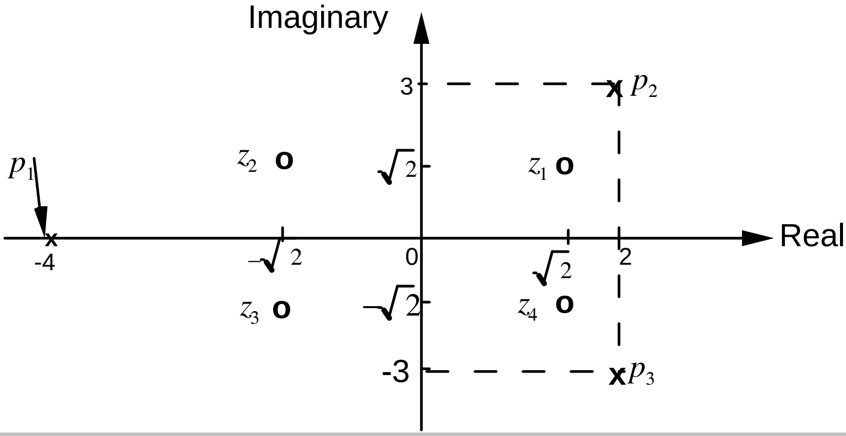
Add $\left(\frac{360}{4}\right)^{\circ} = 90°$ to find the other roots

$$z_2 = 2\angle 135°,\ z_3 = 2\angle 225° \text{ and } z_4 = 2\angle 315°$$

In rectangular form

$$z_1 = \sqrt{2} + j\sqrt{2},\ z_2 = -\sqrt{2} + j\sqrt{2},\ z_3 = -\sqrt{2} - j\sqrt{2} \text{ and } z_4 = \sqrt{2} - j\sqrt{2}$$

On an Argand diagram



7.(i) We have

$$z^6 = \left(\cos\left(\frac{\pi}{3}\right) + j\sin\left(\frac{\pi}{3}\right)\right)^6 \underbrace{=}_{\text{by (10.19)}} \cos(2\pi) + j\sin(2\pi) = 1$$

Hence z is a root of $z^6 - 1 = 0$.

(ii)

$$z^4 + z = \left(\cos\left(\frac{\pi}{3}\right) + j\sin\left(\frac{\pi}{3}\right)\right)^4 + \left(\cos\left(\frac{\pi}{3}\right) + j\sin\left(\frac{\pi}{3}\right)\right)$$

$$= \underbrace{\left(\cos\left(\frac{4\pi}{3}\right) + j\sin\left(\frac{4\pi}{3}\right)\right)}_{\text{by (10.19)}} + \left(\cos\left(\frac{\pi}{3}\right) + j\sin\left(\frac{\pi}{3}\right)\right) = 0$$

(iii) By parts (i) and (ii) we have $z^6 + z^4 + z = 1 + 0 = 1$

8. We have $s^2 = a + j\sqrt{3}a = 2a\angle\left(\frac{\pi}{3}\right)$. Thus taking the square root

$$s = \left(2a\angle\left(\frac{\pi}{3}\right)\right)^{1/2}$$

$$\underbrace{=}_{\text{by (10.19)}} (2a)^{1/2}\angle\left(\frac{\pi}{6}\right)$$

$$= (2a)^{1/2}\left(\cos\left(\frac{\pi}{6}\right) + j\sin\left(\frac{\pi}{6}\right)\right)$$

$$\underbrace{=}_{\text{using TABLE 1}} (2a)^{1/2}\left(\frac{\sqrt{3}}{2} + j\frac{1}{2}\right) = \frac{2^{1/2}a^{1/2}}{2}\left(\sqrt{3} + j\right) \underbrace{=}_{\text{by (10.4)}} \frac{a^{1/2}}{2^{1/2}}\left(\sqrt{3} + j\right) = \sqrt{\frac{a}{2}}\left(\sqrt{3} + j\right)$$

(10.4)	$a^m / a^n = a^{m-n}$
(10.19)	$(r\angle\theta)^n = r^n\angle n\theta$

9. We first evaluate each component

$$\left(3+j3\right)^3=\left(\sqrt{18}\angle 45°\right)^3 \underbrace{=}_{\text{by (10.19)}} 18^{3/2}\angle 135°$$

$$\left(1-j\right)^4=\left(\sqrt{2}\angle\left(-45°\right)\right)^4 \underbrace{=}_{\text{using (10.19)}} 4\angle\left(-180°\right)=-4$$

$$\left(1+j\sqrt{3}\right)^9=\left(2\angle 60°\right)^9 \underbrace{=}_{\text{by (10.19)}} 2^9\angle 540°=2^9\angle 180°=-2^9$$

Substituting each of these we have

$$\frac{\left(3+j3\right)^3\left(1-j\right)^4}{\left(1+j\sqrt{3}\right)^9}=\frac{-4\times 18^{3/2}\angle 135°}{-2^9}$$

$$=0.6\angle 135°=-0.42+j0.42$$

10.(i) Solving the equation by the quadratic formula gives

$$t=\frac{2\pm\sqrt{4-16}}{2}$$

$$=\frac{2\pm\sqrt{-12}}{2}=\frac{2\pm j2\sqrt{3}}{2}=1\pm j\sqrt{3}$$

Thus

$$t=1\pm j\sqrt{3}$$

(ii) Let $z^3=t$ then

$$z^6-2z^3+4=0$$

becomes the quadratic equation of part(i)

$$t^2-2t+4=0$$

which has the roots $1\pm j\sqrt{3}$. Hence $z^3=1+j\sqrt{3}$ or $z^3=1-j\sqrt{3}$
Putting $1+j\sqrt{3}$ into polar form gives

$$z^3=1+j\sqrt{3}=2\angle 60°$$

This gives

$$z=\left(2\angle 60°\right)^{1/3} \underbrace{=}_{\text{by (10.19)}} 2^{1/3}\angle 20°$$

For the other roots add $\left(\dfrac{360}{3}\right)^{°}=120°$:

$$2^{1/3}\angle 140° \quad\text{and}\quad 2^{1/3}\angle 260°$$

Similarly the equation $z^3=1-j\sqrt{3}$ gives the three roots

$$2^{1/3}\angle\left(-20°\right),\ 2^{1/3}\angle 100° \ \text{ and }\ 2^{1/3}\angle 220°$$

All six roots are

$$2^{1/3}\angle 20°,\ 2^{1/3}\angle 100°,\ 2^{1/3}\angle 140°,\ 2^{1/3}\angle 220°,\ 2^{1/3}\angle 260° \text{ and } 2^{1/3}\angle 340°$$

The last root is $2^{1/3}\angle(-20°)$ because

$$2^{1/3}\angle\left(340°\right)=2^{1/3}\angle\left(-20°\right)$$

(10.19) $$\left(r\angle\theta\right)^n=r^n\angle n\theta$$