Complete solutions to Exercise 10(e)

1. (a) $10\angle\pi \underbrace{=}_{\text{by (10.24)}} 10e^{j\pi}$

(b) Need to write 180° in radians

$$180^\circ = \pi \text{ radians}$$

$$10\left(\cos(180^\circ)+j\sin(180^\circ)\right)=10\angle 180^\circ=10\angle\pi=10e^{j\pi}$$

(c) $j2=2\angle 90^\circ=2\angle(\pi/2)=2e^{j\frac{\pi}{2}}$

(d) $1+j=\sqrt{2}\angle 45^\circ=\sqrt{2}\,\angle(\pi/4)=\sqrt{2}e^{j\frac{\pi}{4}}$

(e) $\sqrt{3}-j=2\angle(-30^\circ)=2\angle(-\pi/6)=2e^{-j\frac{\pi}{6}}$

2.(a) By (10.3) we have

$$e^{1+j\frac{\pi}{2}}=ee^{j\frac{\pi}{2}}$$

$$\underbrace{=}_{\text{by (10.25)}} e\left(\cos\left(\frac{\pi}{2}\right)+j\sin\left(\frac{\pi}{2}\right)\right)=e(0+j1)=je$$

(b) We have

$$e^{\pi(1+j)}=e^{\pi+j\pi}$$

$$\underbrace{=}_{\text{by (10.3)}} e^{\pi}e^{j\pi}$$

$$\underbrace{=}_{\text{by (10.25)}} e^{\pi}\left(\cos(\pi)+j\sin(\pi)\right)=e^{\pi}(-1+j0)=-e^{\pi}$$

(c) By using the rules of indices (10.3) we have

$$2e^{2+j\frac{\pi}{6}}=2e^{2}e^{j\frac{\pi}{6}}$$

$$\underbrace{=}_{\text{by (10.25)}} 2e^{2}\left(\cos\left(\frac{\pi}{6}\right)+j\sin\left(\frac{\pi}{6}\right)\right)$$

$$\underbrace{=}_{\text{by TABLE 1}} 2e^{2}\left(\frac{\sqrt{3}}{2}+j\frac{1}{2}\right)=e^{2}\left(\sqrt{3}+j\right) \qquad [\text{Cancelling } 2\text{'}s]$$

$$=\sqrt{3}e^{2}+je^{2} \qquad [\text{Expanding}]$$

(d)

$$e^{1000-j\frac{\pi}{4}}=e^{1000}e^{-j\frac{\pi}{4}} \underbrace{=}_{\text{by (10.26)}} e^{1000}\left(\cos\left(\frac{\pi}{4}\right)-j\sin\left(\frac{\pi}{4}\right)\right)$$

$$=e^{1000}\left(\frac{1}{\sqrt{2}}-j\frac{1}{\sqrt{2}}\right) \qquad [\text{By TABLE 1}]$$

$$=\frac{e^{1000}}{\sqrt{2}}(1-j) \qquad [\text{Factorizing}]$$

(10.3) $a^m a^n=a^{m+n}$

(10.24) $r\angle\theta=re^{j\theta}$

(10.25) $e^{j\theta}=\cos(\theta)+j\sin(\theta)$

(10.26) $e^{-j\theta}=\cos\theta-j\sin\theta$

3. All parts have the same value of 1. For example part(b) is

$$e^{j4\pi} \underbrace{=}_{\text{by (10.25)}} \cos(4\pi) + j\sin(4\pi) = 1 + j0 = 1$$

There is a general formula for evaluating complex exponentials of the above form:
For any whole number n

$$e^{j2\pi n} = 1$$

4. All parts have the same solution, -1. For example part(c) is

$$e^{j5\pi} \underbrace{=}_{\text{by (10.25)}} \cos(5\pi) + j\sin(5\pi) = -1 + j0 = -1$$

The general formula:
For any whole number n

$$e^{j(2n+1)\pi} = -1$$

$(2n+1)\pi$ means any odd number multiple of π.

5. Substituting for v and i we have

$$z = \frac{Ve^{j(wt+\alpha)}}{Ie^{j(wt+\beta)}} = \frac{Ve^{jwt}e^{j\alpha}}{Ie^{jwt}e^{j\beta}}$$

$$= \frac{Ve^{j\alpha}}{Ie^{j\beta}} \underbrace{=}_{(10.4)} \frac{V}{I}e^{j(\alpha-\beta)}$$

$$\underbrace{=}_{(10.25)} \frac{V}{I}\left[\cos(\alpha-\beta) + j\sin(\alpha-\beta)\right]$$

6. We use the rules of indices for complex exponential form:

$$G = \frac{10}{5e^{j\frac{\pi}{2}}e^{j\frac{\pi}{3}}} \underbrace{=}_{\text{by (10.3)}} \frac{10}{5e^{j\frac{\pi}{2}+j\frac{\pi}{3}}}$$

$$\underbrace{=}_{\text{by (10.4)}} \frac{10}{5}e^{-j\left(\frac{\pi}{2}+\frac{\pi}{3}\right)} = 2e^{-j\frac{5\pi}{6}}$$

7. We have

$$G = \frac{100}{10e^{j\frac{\pi}{2}}1.02e^{j0.2}1.8e^{j0.99}}$$

$$= \frac{100}{(10\times1.02\times1.8)e^{j\left(\frac{\pi}{2}+0.2+0.99\right)}}$$

$$= \frac{100}{10\times1.02\times1.8}e^{-j\left(\frac{\pi}{2}+0.2+0.99\right)} = 5.45e^{-j2.76}$$

(10.3) $a^m a^n = a^{m+n}$

(10.4) $\dfrac{a^m}{a^n} = a^{m-n}$

(10.25) $e^{j\theta} = \cos(\theta) + j\sin(\theta)$

8. (a)

$$(1+j)e^{j\frac{\pi}{4}}+(1-j)e^{-j\frac{\pi}{4}}=(1+j)\underbrace{\left(\cos\left(\frac{\pi}{4}\right)+j\sin\left(\frac{\pi}{4}\right)\right)}_{\text{by (10.25)}}+(1-j)\underbrace{\left(\cos\left(\frac{\pi}{4}\right)-j\sin\left(\frac{\pi}{4}\right)\right)}_{\text{by (10.26)}}$$

$$\underbrace{=}_{\text{by TABLE 1}}(1+j)\left(\frac{1}{\sqrt{2}}+j\frac{1}{\sqrt{2}}\right)+(1-j)\left(\frac{1}{\sqrt{2}}-j\frac{1}{\sqrt{2}}\right)$$

$$=\frac{1}{\sqrt{2}}\left[(1+j)^2+(1-j)^2\right]=0$$

$$(3+j4)e^{(1+j)t}+(3-j4)e^{(1-j)t}=(3+j4)\underbrace{e^te^{jt}}_{by\ (10.3)}+(3-j4)\underbrace{e^te^{-jt}}_{by\ (10.3)}$$

$$=e^t\left\{(3+j4)e^{jt}+(3-j4)e^{-jt}\right\}\qquad\left[\text{Factorizing } e^t\right]$$

$$=e^t\left\{(3+j4)\underbrace{\left(\cos(t)+j\sin(t)\right)}_{\text{by (10.25)}}+(3-j4)\underbrace{\left(\cos(t)-j\sin(t)\right)}_{\text{by (10.26)}}\right\}$$

$$=e^t\left(6\cos(t)-8\sin(t)\right)$$

9. (a)

$$V=\mathrm{Im}\left[\omega LIe^{j(\omega t+\pi/2)}\right]$$

$$=\mathrm{Im}\left[\omega LI\underbrace{\left\{\cos\left(\omega t+\frac{\pi}{2}\right)+j\sin\left(\omega t+\frac{\pi}{2}\right)\right\}}_{\text{by (10.25)}}\right]$$

$$=\underbrace{\omega LI\sin\left(\omega t+\frac{\pi}{2}\right)}_{\text{only need the imaginary part}}$$

$$=\omega LI\underbrace{\left(\sin(\omega t)\cos\left(\frac{\pi}{2}\right)+\cos(\omega t)\sin\left(\frac{\pi}{2}\right)\right)}_{\text{by (4.42)}}$$

$$=\omega LI\left[\sin(\omega t)(0)+\cos(\omega t)(1)\right]=\omega LI\cos(\omega t)$$

(4.42) $\sin(A+B)=\sin(A)\cos(B)+\cos(A)\sin(B)$

(10.3) $a^{m+n}=a^ma^n$

(10.25) $e^{j\theta}=\cos(\theta)+j\sin(\theta)$

(10.26) $e^{-j\theta}=\cos(\theta)-j\sin(\theta)$

(b)

$$\operatorname{Im}\left[\frac{I}{\omega C}e^{j(\omega t-\pi/2)}\right]\underbrace{=}_{\text{by (10.25)}}\operatorname{Im}\left[\frac{I}{\omega C}\left[\cos\left(\omega t-\frac{\pi}{2}\right)+j\sin\left(\omega t-\frac{\pi}{2}\right)\right]\right]$$

$$=\frac{I}{\omega C}\sin\left(\omega t-\frac{\pi}{2}\right)\qquad\begin{bmatrix}\text{Selecting Only}\\ \text{The Imaginary Part}\end{bmatrix}$$

$$=\frac{I}{\omega C}\underbrace{\left(\sin(\omega t)\cos\left(\frac{\pi}{2}\right)-\cos(\omega t)\sin\left(\frac{\pi}{2}\right)\right)}_{\text{by (4.43)}}=-\frac{I}{\omega C}\cos(\omega t)$$

10. Putting $z=\ln(3)+j\dfrac{\pi}{2}$ into e^z gives

$$e^{\ln(3)+j\frac{\pi}{2}}\underbrace{=}_{\text{by (10.3)}}e^{\ln(3)}e^{j\frac{\pi}{2}}$$

$$=\underbrace{3}_{\text{by (5.16)}}e^{j\frac{\pi}{2}}=\underbrace{3\left(\cos\left(\frac{\pi}{2}\right)+j\sin\left(\frac{\pi}{2}\right)\right)}_{\text{by (10.25)}}=j3$$

Hence equating the real and imaginary parts of

$$a+jb=j3 \text{ gives } a=0,\quad b=3$$

11. Using the Maclaurin series (7.15) for e^x with $x=j\theta$ we have

$$e^{j\theta}=1+j\theta+\frac{(j\theta)^2}{2!}+\frac{(j\theta)^3}{3!}+\frac{(j\theta)^4}{4!}+\frac{(j\theta)^5}{5!}+\ldots$$

$$=1+j\theta+\frac{j^2\theta^2}{2!}+\frac{j^3\theta^3}{3!}+\frac{j^4\theta^4}{4!}+\frac{j^5\theta^5}{5!}+\ldots$$

$$=1+j\theta-\frac{\theta^2}{2!}-\frac{j\theta^3}{3!}+\frac{\theta^4}{4!}+\frac{j\theta^5}{5!}+\ldots\qquad[\text{Simplifying Powers of } j]$$

$$=\underbrace{\left(1-\frac{\theta^2}{2!}+\frac{\theta^4}{4!}+..\right)}_{=\cos(\theta)\text{ by (7.17)}}+j\underbrace{\left(\theta-\frac{\theta^3}{3!}+\frac{\theta^5}{5!}+\ldots\right)}_{=\sin(\theta)\text{ by (7.16)}}\qquad\begin{bmatrix}\text{Collecting Real and}\\ \text{Imaginary Parts}\end{bmatrix}$$

$$=\cos(\theta)+j\sin(\theta)$$

Hence $re^{j\theta}=r\left[\cos(\theta)+j\sin(\theta)\right]$.

(4.43) $\quad\sin(A-B)=\sin(A)\cos(B)-\cos(A)\sin(B)$

(5.16) $\quad e^{\ln(a)}=a$

(7.15) $\quad e^x=1+x+\dfrac{x^2}{2!}+\dfrac{x^3}{3!}+\dfrac{x^4}{4!}+\dfrac{x^5}{5!}+\ldots$

(7.16) $\quad\sin(x)=x-\dfrac{x^3}{3!}+\dfrac{x^5}{5!}+\ldots$

(7.17) $\quad\cos(x)=1-\dfrac{x^2}{2!}+\dfrac{x^4}{4!}+\ldots$

(10.3) $\quad a^{m+n}=a^m a^n$

(10.25) $\quad e^{j\theta}=\cos(\theta)+j\sin(\theta)$