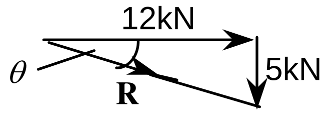
Complete solutions to Exercise 12(a)

1. We have



By Pythagoras

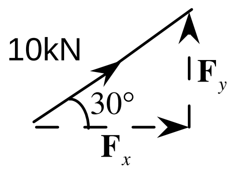
$$|R| = \sqrt{12^2 + 5^2} = 13 \text{ kN}$$

The angle θ is given by

$$\theta = \tan^{-1}\left(\frac{5}{12}\right) = 22.62°$$

R is the force of magnitude $13kN$ and at an angle of $22.62°$ below the horizontal.

2. Let $\mathbf{F}_x$ and $\mathbf{F}_y$ be the horizontal and vertical components of $\mathbf{F}$.



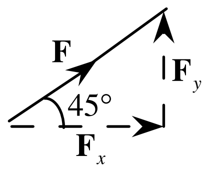
By (4.5)

$$\mathbf{F}_x = 10\cos(30°) = 8.66kN \text{ horizontal}$$

By (4.4)

$$\mathbf{F}_y = 10\sin(30°) = 5kN \text{ vertical}$$

3. Let $\mathbf{F}_x$ and $\mathbf{F}_y$ be the horizontal and vertical components of $\mathbf{F}$.



Remember $12000 = 12kN$. Applying (4.5) and (4.4) gives

$$|\mathbf{F}_x| = 12\cos(45°) = 8.49kN$$

$$|\mathbf{F}_y| = 12\sin(45°) = 8.49kN$$

The force **F** has the components $8.49kN$ horizontally and $8.49kN$ vertically.

4. The angle θ is given by

$$\theta = \tan^{-1}(7/3) = 66.80°$$

Since $|\mathbf{F}| = 10kN$ we have

$$|\mathbf{F}_x| = 10\cos(66.80°) = 3.94kN$$

$$|\mathbf{F}_y| = 10\sin(66.80°) = 9.19kN$$

$3.94kN$ horizontally and $9.19kN$ vertically.

(4.4) $o = h\sin(\theta)$

(4.5) $a = h\cos(\theta)$

5. We have

$$\vec{AM} = \vec{AO} + \vec{OM} = -\vec{OA} + \vec{OM} = -\mathbf{a} + \frac{1}{2}\mathbf{b} = \frac{1}{2}\mathbf{b} - \mathbf{a}$$

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6. We have

$$\vec{MN} = \vec{MC} + \vec{CN} \qquad (\dagger)$$

Since M is the midpoint of AC

$$\vec{MC} = \frac{1}{2}\vec{AC}$$

Similarly $\vec{CN} = \frac{1}{2}\vec{CB}$. Also $\vec{CB} = -\vec{BC}$. So

$$\vec{CN} = -\frac{1}{2}\vec{BC}$$

By observing Fig 26

$$\vec{AC} = \vec{AB} + \vec{BC}$$

which gives

$$\vec{MC} = \frac{1}{2}\left(\vec{AB} + \vec{BC}\right)$$

Substituting $\vec{CN} = -\frac{1}{2}\vec{BC}$ and $\vec{MC} = \frac{1}{2}\left(\vec{AB} + \vec{BC}\right)$ into $(\dagger)$ gives

$$\vec{MN} = \frac{1}{2}\left(\vec{AB} + \vec{BC}\right) - \frac{1}{2}\vec{BC}$$

$$= \frac{1}{2}\vec{AB} + \underbrace{\frac{1}{2}\vec{BC} - \frac{1}{2}\vec{BC}}_{=0} = \frac{1}{2}\vec{AB}$$

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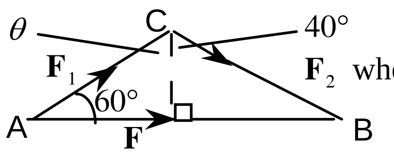
7. Drawing $\mathbf{F}_1$, $\mathbf{F}_2$ and $\mathbf{F}$ in a triangle we have



where $|\mathbf{F}_2| = 10$

$$\theta = 180° - (60° + 90°) = 30°$$

$$\text{angle } C = 30° + 40° = 70°$$

We label a and c to the sides opposite the angles A and C respectively.
To find $|\mathbf{F}|$ we can use the sine rule $\frac{a}{\sin(A)} = \frac{c}{\sin(C)}$

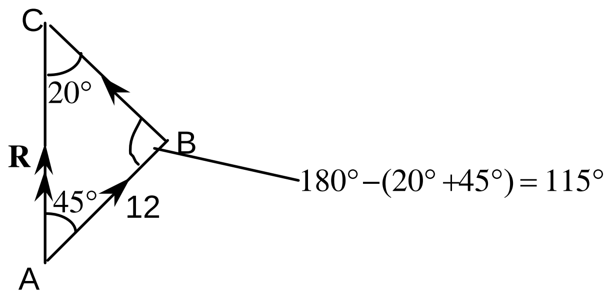
Substituting $a = 10$, $A = 60°$, $C = 70°$ into the sine rule gives

$$\frac{10}{\sin(60°)} = \frac{c}{\sin(70°)}$$

$$c = \frac{10}{\sin(60°)} \times \sin(70°) = 10.851$$

$\mathbf{F}$ is $10.85N$ horizontally.

8. We have



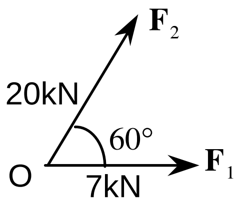
To find the magnitude of **R** we use the sine rule (4.16). Substituting $c = 12$, angle C $= 20°$ and angle B $= 115°$ into (4.16) gives

$$\frac{b}{\sin(115°)} = \frac{12}{\sin(20°)}$$

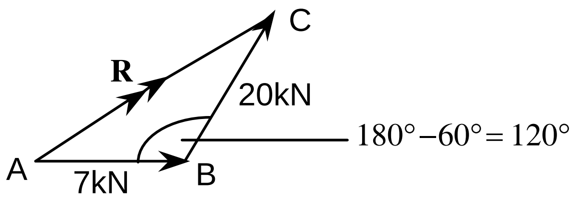
$$b = \frac{12}{\sin(20°)} \times \sin(115°) = 31.80$$

The magnitude of **R** is $31.80\,kN$

9. We can rotate the forces as follows:



By shifting $\mathbf{F}_2$ to the end of $\mathbf{F}_1$ we have the resultant force, **R**, given by:



We can find the magnitude of **R** by using the cosine rule (4.18). We have $a = 20,\ c = 7$ and angle B $= 120°$. Substituting into (4.18) gives

$$b^2 = 20^2 + 7^2 - \left(2 \times 20 \times 7 \times \cos(120°)\right) = 589$$

$$b = \sqrt{589} = 24.27$$

$|\mathbf{R}| = 24.27\,kN$. Need to find angle A. We can use the sine rule(4.16). Substituting $a = 20,\ B = 120°$ and $b = 24.27$ into (4.16) gives

$$\frac{20}{\sin(A)} = \frac{24.27}{\sin(120°)}$$

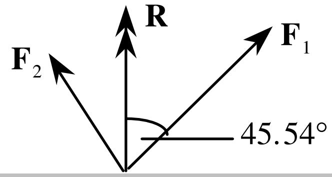
$$\sin(A) = \frac{20 \times \sin(120°)}{24.27} = 0.7137$$

$$A = \sin^{-1}(0.7137) = 45.54°$$

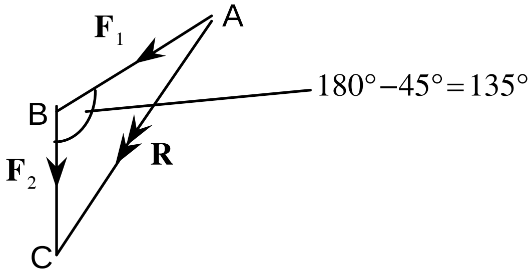
(4.16) $$\frac{a}{\sin(A)} = \frac{b}{\sin(B)} = \frac{c}{\sin(C)}$$

(4.18) $$b^2 = a^2 + c^2 - 2ac\cos(B)$$

The resultant force has a magnitude of $24.27kN$ and is at an angle of $45.54°$ from force $\mathbf{F}_1$. Plotting the resultant on the original diagram gives:



10. The resultant force is determined by $\mathbf{F}_1 + \mathbf{F}_2$.



Consider the triangle ABC. We can find $\mathbf{R}$ by using the cosine rule on ABC.
The magnitude of $\mathbf{R}$ is represented by b. Substitute $a = 10$, $c = 15$ and $B = 135°$ into

$$b^2 = a^2 + c^2 - 2ac\cos(B)$$

gives

$$b^2 = 10^2 + 15^2 - (2\times10\times15)\cos(135°) = 537.132$$

$$b = \sqrt{537.132} = 23.18$$

So $|\mathbf{R}| = 23.18kN$. How can we find angle A?
Use sine rule (4.16). Substituting $a = 10$, $b = 23.18$ and $B = 135°$ gives

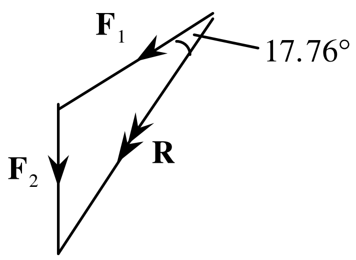
$$\frac{10}{\sin(A)} = \frac{23.18}{\sin(135°)}$$

$$\sin(A) = \frac{10}{23.18}\times\sin(135°) = 0.305$$

Hence taking the inverse sin gives

$$A = 17.76°$$

We have



(4.16) $$\frac{a}{\sin(A)} = \frac{b}{\sin(B)}$$