Complete solutions to Exercise 12(b)

1. (a) Putting $\mathbf{F}_1$ and $\mathbf{F}_2$ into **i** and **j** components gives

$$\mathbf{F}_1 = 20\mathbf{i}$$

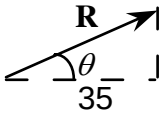
$$\mathbf{F}_2 = \left[30\cos(60°)\right]\mathbf{i} + \left[30\sin(60°)\right]\mathbf{j}$$

$$\mathbf{R} = \mathbf{F}_1 + \mathbf{F}_2 = \left[20 + 30\cos(60°)\right]\mathbf{i} + \left[30\sin(60°)\right]\mathbf{j} = 35\mathbf{i} + 25.981\mathbf{j}$$

The magnitude of **R** is determined by Pythagoras;

$$|\mathbf{R}| = \sqrt{35^2 + 25.981^2} = 43.59 \text{ kN}$$

By using trigonometry



$$\theta = \tan^{-1}\left(\frac{25.981}{35}\right) = 36.59°$$

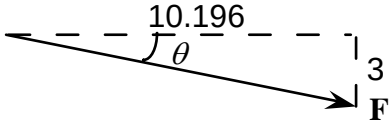
So **R** has a magnitude of $43.59kN$ at an angle of $36.59°$ to the horizontal.

(b) Writing $\mathbf{F}_1$ and $\mathbf{F}_2$ into **i** and **j** components,

$$\mathbf{F}_1 = 5\mathbf{i}$$

$$\mathbf{F}_2 = \left[6\cos(30°)\right]\mathbf{i} \underbrace{-}_{\substack{\text{minus because} \\ \mathbf{F}_2 \text{ is down}}} \left[6\sin(30°)\right]\mathbf{j}$$

$$\mathbf{F} = \mathbf{F}_1 + \mathbf{F}_2 = \left[5 + 6\cos(30°)\right]\mathbf{i} - \left[6\sin(30°)\right]\mathbf{j} = 10.196\mathbf{i} - 3\mathbf{j}$$



The magnitude, $|\mathbf{F}|$, and angle θ are

$$|\mathbf{F}| = \sqrt{10.196^2 + (-3)^2} = 10.63 \text{ kN}$$

$$\theta = \tan^{-1}(-3/10.196) = -16.40°$$

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2. We have

$$\mathbf{F} = 31(4\mathbf{i} - 6\mathbf{j}) - 3(7\mathbf{i} - 12\mathbf{j})$$

$$= \left[(31\times 4)\mathbf{i} - (31\times 6)\mathbf{j}\right] - (3\times 7)\mathbf{i} - (3\times(-12))\mathbf{j}$$

$$= \left[124\mathbf{i} - 186\,\mathbf{j}\right] - 21\mathbf{i} + 36\,\mathbf{j} = 103\mathbf{i} - 150\,\mathbf{j}$$

By using Pythagoras, $|\mathbf{F}| = \sqrt{103^2 + (-150)^2} = 181.96kN$

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3. We first put $\mathbf{F}_1$, $\mathbf{F}_2$ and $\mathbf{F}_3$ into **i** and **j** components by using our calculator.

$$\mathbf{F}_1 = 5\angle 30° = 4.33\mathbf{i} + 2.5\mathbf{j}$$

$$\mathbf{F}_2 = 7\angle 60° = 3.5\mathbf{i} + 6.062\,\mathbf{j}$$

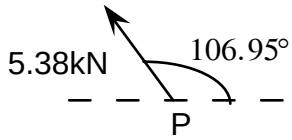
Note that $\mathbf{F}_3$ is $-160°$ from the positive horizontal axis. The minus sign signifies the angle is measured clockwise from the positive horizontal axis.

$$\mathbf{F}_3 = 10\angle(-160°) = -9.397\mathbf{i} - 3.42\,\mathbf{j}$$

$$\mathbf{R} = \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3$$

$$= (4.33 + 3.5 - 9.397)\mathbf{i} + (2.5 + 6.062 - 3.42)\mathbf{j}$$

$$= -1.567\mathbf{i} + 5.142\,\mathbf{j} \underbrace{=}_{\text{by calculator}} 5.38\angle 106.95°$$



4. Writing each force into polar form:
$\mathbf{F}_1 = 12\angle 10°$, $\mathbf{F}_2 = 15\angle 80°$, $\mathbf{F}_3 = 18\angle 160°$, $\mathbf{F}_4 = 19\angle(-150°)$, $\mathbf{F}_5 = 16\angle(-110°)$ and $\mathbf{F}_6 = 5\angle(-40°)$.
Using a calculator to add these forces gives a resultant force of magnitude $21.13kN$ and angle of $-167.04°$. Remember the minus sign indicates the angle is measured clockwise from the positive horizontal axis as shown: