Complete solutions to Exercise 12(c)

1. Since in each case the particle is in equilibrium then

$$\mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3 + \mathbf{F}_4 = \mathbf{0}$$

$$\mathbf{F}_4 = -(\mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3) \qquad (*)$$

(a) Using (*) we have

$$\begin{aligned}\mathbf{F}_4 &= -[(2\mathbf{i} - \mathbf{j} - 3\mathbf{k}) + (7\mathbf{i} - 7\mathbf{j} - 7\mathbf{k}) + (9\mathbf{i} - 6\mathbf{j} - \mathbf{k})] \\ &= -[18\mathbf{i} - 14\mathbf{j} - 11\mathbf{k}] \\ \mathbf{F}_4 &= -18\mathbf{i} + 14\mathbf{j} + 11\mathbf{k}\end{aligned}$$

(b) Using (*) again we have

$$\begin{aligned}\mathbf{F}_4 &= -[(61\mathbf{i} - 64\mathbf{j} - 89\mathbf{k}) + (93\mathbf{i} - 98\mathbf{j}) + (22\mathbf{i} - 41\mathbf{j} + 43\mathbf{k})] \\ &= -(176\mathbf{i} - 203\mathbf{j} - 46\mathbf{k}) = -176\mathbf{i} + 203\mathbf{j} + 46\mathbf{k}\end{aligned}$$

---

2. Since the body is in equilibrium

$$\mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3 + \mathbf{F}_4 = \mathbf{0}$$

Hence

$$\begin{aligned}\mathbf{F}_1 &= -(\mathbf{F}_2 + \mathbf{F}_3 + \mathbf{F}_4) \\ &= -[(3\mathbf{i} + \mathbf{j} - \mathbf{k}) + (-2\mathbf{i} - 3\mathbf{j} - 10\mathbf{k}) + (-\mathbf{i} - 5\mathbf{j} + 3\mathbf{k})] \\ &= -[0\mathbf{i} - 7\mathbf{j} - 8\mathbf{k}] = 7\mathbf{j} + 8\mathbf{k}\end{aligned}$$

---

3. We have

$$\begin{aligned}\mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3 + \mathbf{F}_4 &= (2 + 12 + 7 - 9)\mathbf{i} + (-1 - 3 + 1 - 4)\mathbf{j} + (10 + 22 - 7 + 5)\mathbf{k} \\ &= 12\mathbf{i} - 7\mathbf{j} + 30\mathbf{k}\end{aligned}$$

Hence

$$\mathbf{F} = 12\mathbf{i} - 7\mathbf{j} + 30\mathbf{k}$$

---

4. We first find the unit vectors of $12\mathbf{i} + 5\mathbf{k}$, $2\mathbf{i} - 2\mathbf{j} + \mathbf{k}$ and $4\mathbf{i} - 4\mathbf{j} - 2\mathbf{k}$ and then multiply each unit vector by the corresponding magnitude. The unit vector in the direction of $12\mathbf{i} + 5\mathbf{k}$ is

$$\frac{1}{\sqrt{12^2 + 5^2}}(12\mathbf{i} + 5\mathbf{k}) = \frac{1}{13}(12\mathbf{i} + 5\mathbf{k})$$

Hence

$$\mathbf{F}_2 = 26 \times \frac{1}{13}(12\mathbf{i} + 5\mathbf{k}) = 2(12\mathbf{i} + 5\mathbf{k}) = 24\mathbf{i} + 10\mathbf{k}$$

The unit vector in the direction of $2\mathbf{i} - 2\mathbf{j} + \mathbf{k}$ is

$$\frac{1}{\sqrt{2^2 + (-2)^2 + 1^2}}(2\mathbf{i} - 2\mathbf{j} + \mathbf{k}) = \frac{1}{3}(2\mathbf{i} - 2\mathbf{j} + \mathbf{k})$$

Thus

$$\mathbf{F}_3 = 18 \times \frac{1}{3}(2\mathbf{i} - 2\mathbf{j} + \mathbf{k}) = 6(2\mathbf{i} - 2\mathbf{j} + \mathbf{k}) = 12\mathbf{i} - 12\mathbf{j} + 6\mathbf{k}$$

Similarly the unit vector in the direction of $4\mathbf{i} - 4\mathbf{j} - 2\mathbf{k}$ is

$$\frac{1}{\sqrt{4^2 + (-4)^2 + (-2)^2}}(4\mathbf{i} - 4\mathbf{j} - 2\mathbf{k}) = \frac{1}{6}(4\mathbf{i} - 4\mathbf{j} - 2\mathbf{k})$$

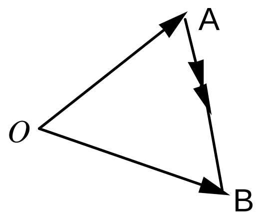
Therefore

$$\mathbf{F}_4 = 12 \times \frac{1}{6}(4\mathbf{i} - 4\mathbf{j} - 2\mathbf{k}) = 2(4\mathbf{i} - 4\mathbf{j} - 2\mathbf{k}) = 8\mathbf{i} - 8\mathbf{j} - 4\mathbf{k}$$

Collecting these forces together gives (in N)

$$\mathbf{F}_1+\mathbf{F}_2+\mathbf{F}_3+\mathbf{F}_4=(\mathbf{i}-\mathbf{j}+3\mathbf{k})+(24\mathbf{i}+10\mathbf{k})+(12\mathbf{i}-12\mathbf{j}+6\mathbf{k})+(8\mathbf{i}-8\mathbf{j}-4\mathbf{k})$$
$$=(1+24+12+8)\mathbf{i}+(-1+0-12-8)\mathbf{j}+(3+10+6-4)\mathbf{k}$$
$$=(45\mathbf{i}-21\mathbf{j}+15\mathbf{k})\text{ N}$$

5. We have



$$\vec{AB}=\vec{OB}-\vec{OA} \qquad (\dagger)$$

The vector $\vec{OA}$ with $A=(7,5,14)$ is written as $\vec{OA}=7\mathbf{i}+5\mathbf{j}+14\mathbf{k}$

and $\vec{OB}$ with $B=(-1,1,15)$

$$\vec{OB}=-\mathbf{i}+\mathbf{j}+15\mathbf{k}$$

Substituting these into (†) gives

$$\vec{AB}=(-\mathbf{i}+\mathbf{j}+15\mathbf{k})-(7\mathbf{i}+5\mathbf{j}+14\mathbf{k})$$
$$=-8\mathbf{i}-4\mathbf{j}+\mathbf{k}$$

The unit vector, $\mathbf{u}$, in the direction of $\vec{AB}=-8\mathbf{i}-4\mathbf{j}+\mathbf{k}$ is found by using (12.5)

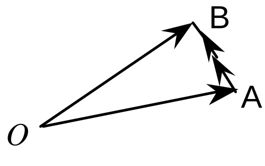
$$\mathbf{u}=\frac{1}{\sqrt{(-8)^2+(-4)^2+1^2}}(-8\mathbf{i}-4\mathbf{j}+\mathbf{k})=\frac{1}{9}(-8\mathbf{i}-4\mathbf{j}+\mathbf{k})$$

Hence the force $\mathbf{F}$ in kN is

$$\mathbf{F}=36\times\frac{1}{9}(-8\mathbf{i}-4\mathbf{j}+\mathbf{k})$$
$$=4(-8\mathbf{i}-4\mathbf{j}+\mathbf{k})=(-32\mathbf{i}-16\mathbf{j}+4\mathbf{k})kN$$

---

6. Similar to solution 5.



$$\vec{AB}=\vec{OB}-\vec{OA}$$
$$=(2.2\mathbf{i}+5.6\mathbf{j}+7.1\mathbf{k})-(1.1\mathbf{i}-2.6\mathbf{j}-4.3\mathbf{k})$$
$$=1.1\mathbf{i}+8.2\mathbf{j}+11.4\mathbf{k}$$

The unit vector, $\mathbf{u}$, is

$$\mathbf{u}=\frac{1}{\sqrt{1.1^2+8.2^2+11.4^2}}(1.1\mathbf{i}+8.2\mathbf{j}+11.4\mathbf{k})=\frac{1}{14.086}(1.1\mathbf{i}+8.2\mathbf{j}+11.4\mathbf{k})$$

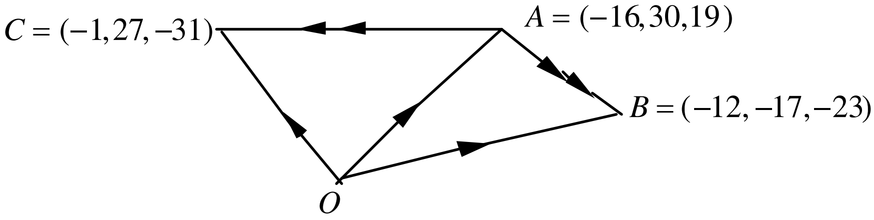
(12.5) $$\mathbf{u}=\frac{1}{\sqrt{a^2+b^2+c^2}}(a\mathbf{i}+b\mathbf{j}+c\mathbf{k})$$

The force $\mathbf{F}$ in kN is

$$\mathbf{F} = 11.3 \times \frac{1}{14.086}(1.1\mathbf{i} + 8.2\mathbf{j} + 11.4\mathbf{k})$$

$$= 0.802(1.1\mathbf{i} + 8.2\mathbf{j} + 11.4\mathbf{k}) = 0.88\mathbf{i} + 6.58\mathbf{j} + 9.14\mathbf{k}$$

7. Plotting the vectors:



$$\vec{AB} = \vec{OB} - \vec{OA}$$
$$= (-12, -17, -23) - (-16, 30, 19)$$
$$= (4, -47, -42)$$

Similarly we have

$$\vec{AC} = \vec{OC} - \vec{OA}$$
$$= (-1, 27, -31) - (-16, 30, 19)$$
$$= (15, -3, -50)$$

Next we find the unit vector in the directions of $\vec{AB}$ and $\vec{AC}$. Rather than work through the laborious arithmetic, it is easier to use a calculator. To find the unit vector , $\mathbf{v}$, in the direction of $\vec{AB} = (4, -47, -42)$ we use a calculator and obtain:

$$\mathbf{v} = 0.063\mathbf{i} - 0.744\mathbf{j} - 0.665\mathbf{k}$$

To find the force $\mathbf{F}_{AB}$ we multiply this, $\mathbf{v}$, by 850:

$$\mathbf{F}_{AB} = 53.833\mathbf{i} - 632.535\mathbf{j} - 565.244\mathbf{k}$$

Similarly we find $\mathbf{F}_{AC}$. The unit vector, $\mathbf{u}$, in the direction of $\vec{AC} = (15, -3, -50)$ is

$$\mathbf{u} = 0.287\mathbf{i} - 0.057\mathbf{j} - 0.956\mathbf{k}$$

Hence

$$\mathbf{F}_{AC} = 700 \times \mathbf{u} = 200.812\mathbf{i} - 40.162\mathbf{j} - 669.374\mathbf{k}$$

$$\mathbf{F} = \mathbf{F}_{AB} + \mathbf{F}_{AC}$$
$$= (53.833 + 200.812)\mathbf{i} + (-632.535 - 40.162)\mathbf{j} + (-565.244 - 669.374)\mathbf{k}$$
$$\mathbf{F} = (254.645\mathbf{i} - 672.697\mathbf{j} - 1234.618\mathbf{k}) \text{ kN}$$

The magnitude $|\mathbf{F}| = 1428.86$ kN.