Complete solutions to Miscellaneous Exercise 13

Throughout these solutions I have assumed the argument, x, in $\ln(x)$ to be positive.

1. Rearranging gives

$$C\frac{d(pV)}{pV}=\frac{dm}{m}$$

Integrating by using (8.2) and taking the constant of integration to be 0:

$$C\ln(pV)=\ln(m) \text{ gives } \ln\left[(pV)^C\right]=\ln(m)$$

Taking exponentials of both sides $(pV)^C=m$

2. Substituting for acceleration, a, we have

$$\frac{dv}{dt}=v^2$$

Separating variables

$$\frac{dv}{v^2}=dt \text{ which gives } \int v^{-2}dv=\int dt$$

$$-v^{-1}=t+C, \text{ thus } -\frac{1}{v}=t+C$$

Putting $t=0,\ v=10$ gives $-0.1=C$. Hence

$$-\frac{1}{v}=t-0.1 \text{ multiplying by } -1 \text{ gives } \frac{1}{v}=0.1-t$$

$$v=\frac{1}{0.1-t}$$

3. We have $\frac{dv}{dt}=-kv^2$. Separating variables gives $\frac{dv}{v^2}=-(k)dt$

We can rewrite this and integrate

$$v^{-2}dv=-(k)dt,\quad \int v^{-2}dv=-\int(k)dt$$

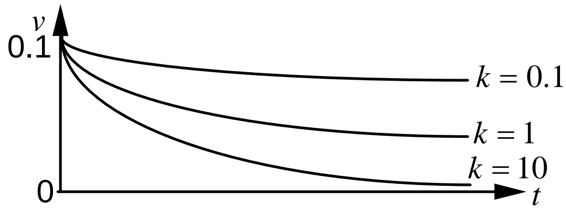
Thus

$$-\frac{1}{v}=-kt+C$$

Substituting the given initial condition $t=0,\ v=0.1$ gives $C=-10$. Therefore

$$-\frac{1}{v}=-kt-10, \text{ multiplying through by } -1 \text{ gives } \frac{1}{v}=kt+10$$

We have $v=\frac{1}{kt+10}$. The graphs for different values of k are



The larger the value of k the larger the drop in velocity, v, with time, t.

(8.2) $$\int\frac{du}{u}=\ln|u|$$

4. Separating variables

$$\frac{dv}{v} = -(k)\,dt$$

Integrating, $\int \frac{dv}{v} = \int -(k)dt$, gives

$$\ln(v) = -kt + C \qquad (*)$$

Substituting $t = 0,\ v = V_0$ yields

$$\ln(V_0) = C$$

Putting this, $\ln(V_0) = C$, into (*)

$$\ln(v) - \ln(V_0) = -kt$$

$$\ln\left(\frac{v}{V_0}\right) = -kt$$

Taking exponentials gives $\frac{v}{V_0} = e^{-kt}$. Multiplying both sides by V_0 gives $v = V_0 e^{-kt}$

5. We have $\frac{dP}{P} = -k\frac{dV}{V}$. Integrating by (8.2) yields

$$\ln(P) = -k\ln(V) + C \text{ rearranging } \ln(P) + k\ln(V) = C$$

By using the properties of logs we have

$$\ln(P) + \ln(V^k) = C$$

$$\ln(PV^k) = C$$

Taking exponentials gives the required result $PV^k = e^C = \text{constant}$

6. We can rearrange the given differential equation to the same differential equation of question 5 and solve as above.

7. Substituting $v = 240\sin(100t)$ and $L = 1\times 10^{-3}$ gives

$$\left(1\times 10^{-3}\right)\frac{di}{dt} = 240\sin(100t) \text{ rearranging } \frac{di}{dt} = \frac{240}{1\times 10^{-3}}\sin(100t)$$

$$di = \left[\left(240\times 10^{3}\right)\sin(100t)\right]dt$$

Integrating both sides

$$i = \int\left[\left(240\times 10^{3}\right)\sin(100t)\right]dt$$

$$\underbrace{=}_{\text{by (8.39)}} \left(240\times 10^{3}\right)\left(-\frac{\cos(100t)}{100}\right) + C = -2400\cos(100t) + C = i$$

Using the initial condition $t = 0,\ i = 0$ gives

$$-2400 + C = 0, \quad C = 2400$$

Substituting $C = 2400$ into $i = -2400\cos(100t) + C$ yields the required result

$$i = -2400\cos(100t) + 2400 = 2400\left[1 - \cos(100t)\right]$$

(8.2) $\qquad \int \frac{du}{u} = \ln|u|$

(8.39) $\qquad \int \sin(kt)dt = -\cos(kt)/k$

8. Subtracting V from both sides we have $RC\frac{dV}{dt} = E - V$. Separating variables

$$\frac{RC}{E-V}dV = dt$$

Integrating $RC\int\frac{dV}{E-V} = \int dt$ gives

$$-RC\ln\left(E-V\right) = t + C \qquad (*)$$

Substituting $t = 0,\ V = 0$

$$-RC\ln\left(E\right) = C$$

Putting this into (*)

$$-RC\ln\left(E-V\right) = t - RC\ln\left(E\right)$$

$$-t = RC\ln\left(E-V\right) - RC\ln\left(E\right) = RC\left[\ln\left(E-V\right) - \ln\left(E\right)\right]$$

$$-\frac{t}{RC} = \ln\left(\frac{E-V}{E}\right)$$

Taking exponentials

$$e^{-\frac{t}{RC}} = \frac{E-V}{E} \quad \text{gives} \quad Ee^{-\frac{t}{RC}} = E - V$$

Hence $V = E - Ee^{-t/RC} = E\left(1 - e^{-t/RC}\right)$.

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9. (i) Dividing both sides by 100 gives $\frac{dv}{dt} = 2 - 0.1v$. Separating variables

$$\frac{dv}{2-0.1v} = dt$$

Integrating, $\int\frac{dv}{2-0.1v} = \int dt$, by using (8.42) on the left hand side we have

$$-10\ln\left(2-0.1v\right) = t + C$$

Substituting $t = 0,\ v = 0$ gives $-10\ln(2) = C$. We have

$$-10\ln\left(2-0.1v\right) = t - 10\ln\left(2\right)$$

$$-t = 10\ln\left(2-0.1v\right) - 10\ln\left(2\right) = 10\left[\ln\left(2-0.1v\right) - \ln\left(2\right)\right]$$

Dividing both sides by 10 and using the properties of logs gives

$$-\frac{t}{10} = \ln\left(\frac{2-0.1v}{2}\right) \quad \text{we have} \quad -0.1t = \ln\left(1-0.05v\right)$$

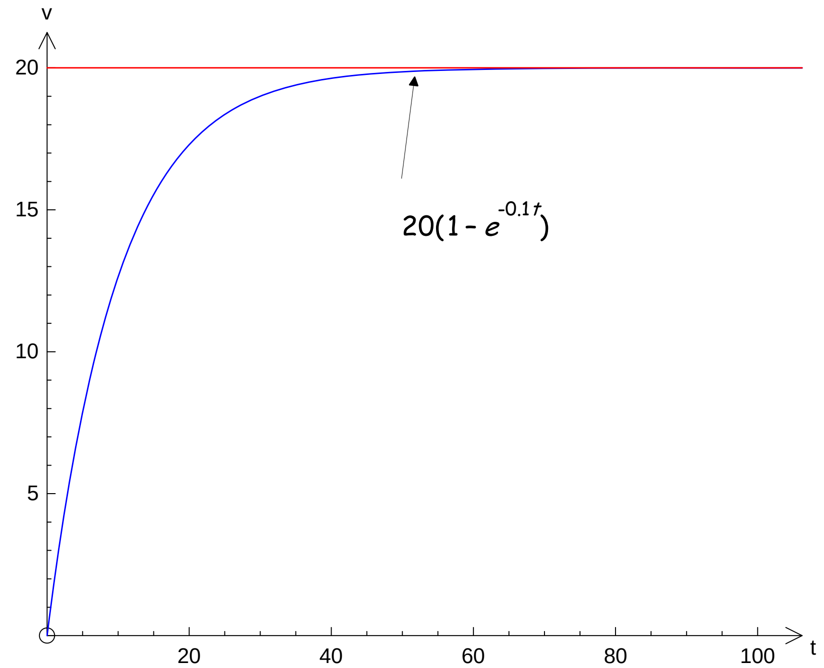
Taking exponentials of both sides

$$e^{-0.1t} = e^{\ln(1-0.05v)} = 1 - 0.05v \quad \text{rearranging} \quad v = \frac{1-e^{-0.1t}}{0.05} = 20\left(1-e^{-0.1t}\right)$$

(ii) As $t \to \infty,\ e^{-0.1t} \to 0$ so $v \to 20$.

(iii) Plotting $v = 20\left(1 - e^{-0.1t}\right)$ gives

(8.42) $$\int\frac{f'(x)}{f(x)}dx = \ln\left|f(x)\right|$$



10. Dividing through by m gives $\frac{dv}{dt}=g-\frac{kv}{m}$. Separating variables

$$\frac{dv}{g-\frac{k}{m}v}=dt$$

Integrating $\int \frac{dv}{g-\frac{k}{m}v}=\int dt$ gives $\underbrace{-\frac{m}{k}\ln\left(g-\frac{k}{m}v\right)}_{\text{by (8.42)}}=t+C$

Using the initial condition $t=0,\ v=0$ gives $-\frac{m}{k}\ln(g)=C$. So we have

$$-\frac{m}{k}\ln\left(g-\frac{k}{m}v\right)=t-\frac{m}{k}\ln(g)$$

$$-t=\frac{m}{k}\ln\left(g-\frac{k}{m}v\right)-\frac{m}{k}\ln(g)=\frac{m}{k}\left[\ln\left(g-\frac{k}{m}v\right)-\ln(g)\right]$$

$$-\frac{kt}{m}=\ln\left(\frac{g-\frac{k}{m}v}{g}\right)$$

$$=\ln\left(\frac{g}{g}-\frac{kv}{mg}\right)=\ln\left(1-\frac{kv}{mg}\right)$$

Taking exponentials of both sides

$$e^{-kt/m}=1-\frac{kv}{mg},\quad \frac{kv}{mg}=1-e^{-kt/m}\quad \text{rearranging}\quad v=\frac{mg}{k}\left(1-e^{-kt/m}\right)$$

As $t\to\infty$, $e^{-kt/m}\to 0$, hence $v\to mg/k$.

(8.42) $$\int\frac{f'(x)}{f(x)}dx=\ln\left|f(x)\right|$$

11. Dividing the given differential equation by m and separating variables

$$\frac{dv}{dt}=g-\frac{k}{m}v^2,\quad dv=\left(g-\frac{k}{m}v^2\right)dt$$

We can factorize the bracket term as $\left(g-\frac{k}{m}v^2\right)=\frac{k}{m}\left(\frac{mg}{k}-v^2\right)$. Thus

$$dv=\frac{k}{m}\left(\frac{mg}{k}-v^2\right)dt \text{ rearranging } \frac{dv}{\frac{mg}{k}-v^2}=\frac{k}{m}dt \qquad (*)$$

Need to integrate both sides. How do we integrate the left hand side?

By using the hint in question $\int\frac{du}{u^2-a^2}=\frac{1}{2a}\ln\left|\frac{u-a}{u+a}\right|$. Denominator in (*) is

$$\frac{mg}{k}-v^2=\left(\sqrt{\frac{mg}{k}}\right)^2-v^2,\text{ let } \alpha=\sqrt{\frac{mg}{k}} \text{ we have } \frac{mg}{k}-v^2=\alpha^2-v^2=-\left(v^2-\alpha^2\right)$$

Integrating (*) becomes $-\int\frac{dv}{v^2-\alpha^2}=\int\frac{k}{m}dt$. Using hint

$$-\frac{1}{2\alpha}\ln\left(\frac{v-\alpha}{v+\alpha}\right)=\frac{k}{m}t+C,\qquad \ln\left(\frac{v-\alpha}{v+\alpha}\right)=-2\alpha\frac{k}{m}t-2\alpha C$$

Taking exponentials of both sides gives

$$\frac{v-\alpha}{v+\alpha}=e^{-2\alpha\frac{k}{m}t-2\alpha C}=e^{-2\alpha\frac{k}{m}t}e^{-2\alpha C}=Ae^{-2\alpha\frac{k}{m}t} \text{ where } A=e^{-2\alpha C} \text{ (constant)}$$

The index can be simplified as

$$2\alpha\frac{k}{m}=2\sqrt{\frac{mg}{k}}\frac{k}{m}\underbrace{=}_{\text{using rules of indices}}2\sqrt{\frac{gk}{m}}=2\beta \text{ where } \beta=\sqrt{\frac{gk}{m}}$$

Putting this into the above we have

$$\frac{v-\alpha}{v+\alpha}=Ae^{-2\beta t}$$

Rearranging to make v the subject gives

$$v-\alpha=Ae^{-2\beta t}\left(v+\alpha\right)=Ae^{-2\beta t}v+\alpha Ae^{-2\beta t}$$

$$v-Ae^{-2\beta t}v=\alpha+\alpha Ae^{-2\beta t} \text{ factorizing } v\left(1-Ae^{-2\beta t}\right)=\alpha\left(1+Ae^{-2\beta t}\right)$$

Hence we have the required result $v=\alpha\left(\frac{1+Ae^{-2\beta t}}{1-Ae^{-2\beta t}}\right)=\sqrt{\frac{mg}{k}}\left(\frac{1+Ae^{-2\beta t}}{1-Ae^{-2\beta t}}\right)$.

As $t\to\infty$, $e^{-2\beta t}\to 0$, so $v\to\sqrt{mg/k}$.

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12. Dividing by L gives $\frac{di}{dt}+\frac{R}{L}i=\frac{t}{L}$. Using the integrating factor method, $i\left(\text{I.F.}\right)=\int\left[\left(\text{I.F.}\right)f\left(t\right)\right]dt$, yields

$$\text{I.F.}=e^{\int\frac{R}{L}dt}=e^{\frac{R}{L}t} \text{ we have } ie^{Rt/L}=\int\left(\frac{t}{L}e^{Rt/L}\right)dt \qquad (*)$$

How do we find $\int\left(\frac{t}{L}e^{Rt/L}\right)dt$ *?*

Use integration by parts formula, (8.45), with

$$u = t/L \qquad v' = e^{Rt/L}$$

$$u' = 1/L \qquad v = \int e^{Rt/L}dt \underbrace{=}_{\text{by (8.41)}} \frac{e^{Rt/L}}{R/L} = \frac{Le^{Rt/L}}{R}$$

Applying (8.45) we have

$$\int\left(\frac{t}{L}e^{Rt/L}\right)dt = \frac{t}{L}\frac{Le^{Rt/L}}{R} - \int\left(\frac{Le^{Rt/L}}{R}\frac{1}{L}\right)dt$$

$$= \frac{te^{Rt/L}}{R} - \underbrace{\frac{e^{Rt/L}}{R(R/L)}}_{\text{by (8.41)}} + C = e^{Rt/L}\left(\frac{t}{R} - \frac{L}{R^2}\right) + C$$

Substituting into (*) gives

$$ie^{Rt/L} = e^{Rt/L}\left(\frac{t}{R} - \frac{L}{R^2}\right) + C$$

Dividing through by $e^{Rt/L}$ yields

$$i = \frac{e^{Rt/L}}{e^{Rt/L}}\left(\frac{t}{R} - \frac{L}{R^2}\right) + \frac{C}{e^{Rt/L}} = \frac{t}{R} - \frac{L}{R^2} + Ce^{-Rt/L}$$

Using the initial condition $t = 0,\ i = 0$ gives $C = L/R^2$. Hence we have

$$i = \frac{t}{R} - \frac{L}{R^2} + \frac{L}{R^2}e^{-Rt/L} = \frac{t}{R} + \frac{L}{R^2}\left(e^{-Rt/L} - 1\right)$$

13. Let v be the voltage across the capacitor, then

$$\left(13 \times 10^3\right)i + v = 5e^{-t}\cos(100\pi t) \qquad (*)$$

where i is the current flowing through the circuit. We have

$$i = C\frac{dv}{dt} = \left(0.9 \times 10^{-6}\right)\frac{dv}{dt}$$

Substituting this into (*) gives

$$\left(13\times10^3\right)\left(0.9\times10^{-6}\right)\frac{dv}{dt} + v = 5e^{-t}\cos(100\pi t)$$

$$\left(11.70\times10^{-3}\right)\frac{dv}{dt} = 5e^{-t}\cos(100\pi t) - v$$

$$\frac{dv}{dt} = 427.35e^{-t}\cos(100\pi t) - 85.47v$$

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14. Dividing through by L we have $\frac{di}{dt} + \frac{R}{L}i = \frac{E}{L}\cos(\omega t)$. Using the integrating factor method, with $\text{I.F.} = e^{Rt/L}$ we have

$$ie^{Rt/L} = \frac{E}{L}\int\left[e^{Rt/L}\cos(\omega t)\right]dt \qquad (\dagger)$$

How do we find $\int\left[e^{Rt/L}\cos(\omega t)\right]dt$*?*

(8.41) $$\int e^{kt}dt = \frac{e^{kt}}{k}$$

(8.45) $$\int uv\,dt = uv - \int(u'v)dt$$

Use (8.33)

$$\int\left[e^{Rt/L}\cos(\omega t)\right]dt=\frac{e^{Rt/L}}{(R/L)^2+\omega^2}\left(\frac{R}{L}\cos(\omega t)+\omega\sin(\omega t)\right)+C$$

$$=\frac{L^2e^{Rt/L}}{R^2+\omega^2L^2}\left(\frac{R}{L}\cos(\omega t)+\omega\sin(\omega t)\right)+C$$

Substituting this into (†) gives

$$ie^{Rt/L}=\frac{E}{L}\frac{L^2e^{Rt/L}}{R^2+\omega^2L^2}\left(\frac{R}{L}\cos(\omega t)+\omega\sin(\omega t)\right)+\frac{CE}{L}$$

$$=\frac{Ee^{Rt/L}}{R^2+\omega^2L^2}\underbrace{\left[R\cos(\omega t)+\omega L\sin(\omega t)\right]}_{\text{multiplying by } L}+\frac{CE}{L}$$

Dividing through by $e^{Rt/L}$ yields

$$i=\frac{E}{R^2+\omega^2L^2}\left[R\cos(\omega t)+\omega L\sin(\omega t)\right]+\frac{CE}{L}e^{-Rt/L} \qquad (*)$$

Substituting $t=0,\ i=0$

$$0=\frac{ER}{R^2+\omega^2L^2}+\frac{CE}{L}, \qquad -\frac{LR}{R^2+\omega^2L^2}=C$$

The last term on the right hand side of (*) becomes

$$\frac{CE}{L}e^{-Rt/L}=-\frac{LR}{R^2+\omega^2L^2}\frac{E}{L}e^{-Rt/L}=\frac{E}{R^2+\omega^2L^2}\left(-\mathrm{Re}^{-Rt/L}\right)$$

Thus putting this into (*) and factorizing

$$i=\frac{E}{R^2+\omega^2L^2}\left[R\cos(\omega t)+\omega L\sin(\omega t)-\mathrm{Re}^{-Rt/L}\right]$$

For Maple Solutions see next page.

(8.33) $$\int\left[e^{au}\cos(bu)\right]du=\frac{e^{au}}{a^2+b^2}\left[a\cos(bu)+b\sin(bu)\right]$$

15. We have

```
> de_15:=diff(y(x),x)=(x^2)+(y(x)^2);
```

$$de\_15 := \frac{d}{dx}\,y(x) = x^2 + y(x)^2$$

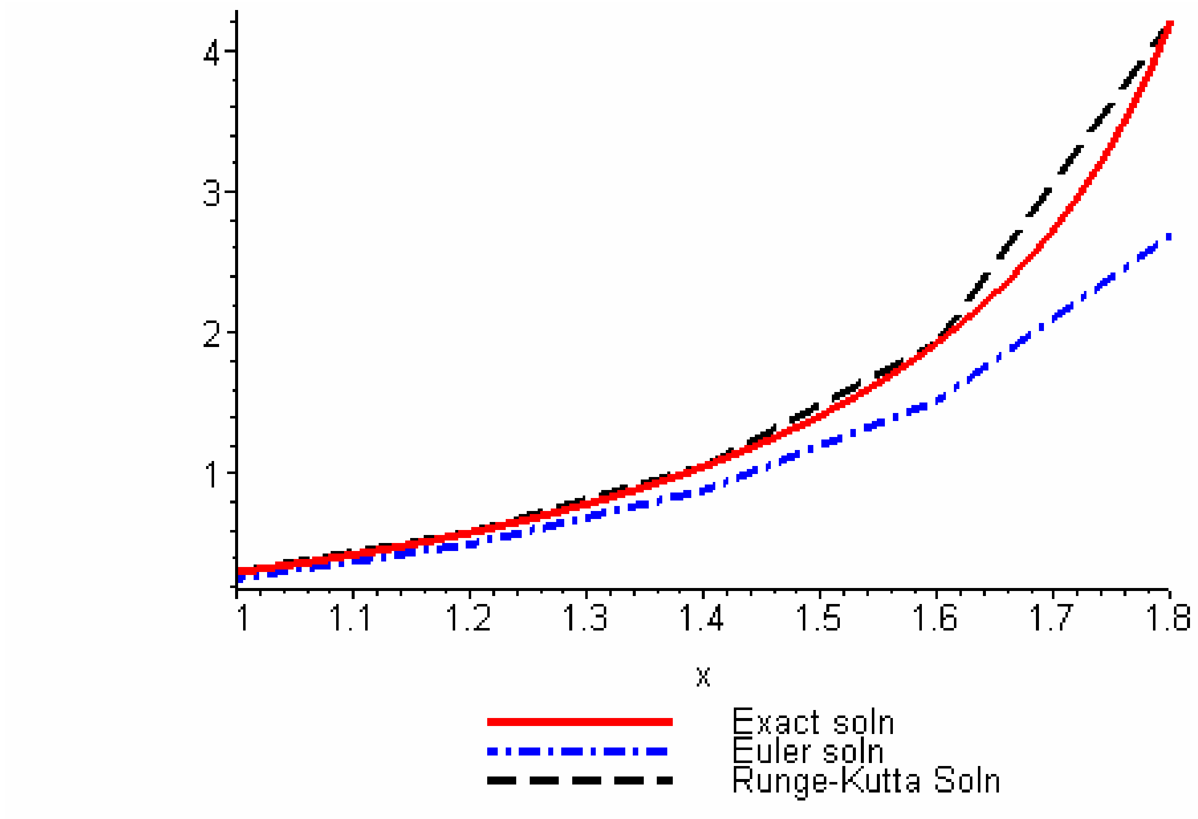
```
> soln:=dsolve({de_15,y(.5)=0},y(x)):
>
e_soln:=dsolve({de_15,y(0.5)=0},y(x),type=numeric,method=
classical,output=array([1,1.2,1.4,1.6,1.8]),stepsize=0.1)
;
```

$$e\_soln := \begin{bmatrix} [x, y(x)] \\ \begin{bmatrix} 1 & 0.259740426319531758 \\ 1.2 & 0.500918202595131201 \\ 1.4 & 0.883901461534468913 \\ 1.6 & 1.51713290582614335 \\ 1.8 & 2.69362407411532654 \end{bmatrix} \end{bmatrix}$$

```
>
rk_soln:=dsolve({de_15,y(0.5)=0},y(x),type=numeric,method
=classical[rk4],output=array([1,1.2,1.4,1.6,1.8]),stepsiz
e=0.1);
```

$$rk\_soln := \begin{bmatrix} [x, y(x)] \\ \begin{bmatrix} 1 & 0.302233703026492328 \\ 1.2 & 0.583441888688835730 \\ 1.4 & 1.05277303452914506 \\ 1.6 & 1.93047072561334687 \\ 1.8 & 4.20779606250247706 \end{bmatrix} \end{bmatrix}$$

```
> with(plots):
>
p_1:=plots[odeplot](e_soln,[x,y(x)],linestyle=4,color=blu
e):
>
p_2:=plots[odeplot](rk_soln,[x,y(x)],linestyle=6,color=bl
ack):
> p_3:=plot(rhs(soln),x=1..1.8):
> plots[display]({p_1,p_2,p_3});
```



Clearly the graphs show that the Runge-Kutta solution is closer to the actual solution then the Euler solution.

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16. The Maple solution is

```
> de_16:=diff(y(x),x)=(x^2)+(y(x)*exp(x));
```

$$de\_16 := \frac{d}{dx}\,\mathrm{y}(x) = x^2 + \mathrm{y}(x)\,\mathbf{e}^x$$

```
> soln:=dsolve({de_16,y(0)=-1},y(x)):
> e_soln:=dsolve({de_16,y(0)=-1},y(x),type=numeric,method=classical[heunform],value=array([0.2,0.4,0.6,1.2]),stepsize=0.2);
```

Warning, the 'value' option is deprecated, use 'output' instead

$$e\_soln := \left[\begin{array}{c} [x, \mathrm{y}(x)] \\ \left[\begin{array}{cc} 0.2 & -1.24256833097922037 \\ 0.4 & -1.60379406468626851 \\ 0.6 & -2.16464286106246372 \\ 1.2 & -8.27127617937256332 \end{array}\right] \end{array}\right]$$

```
> rk_soln:=dsolve({de_16,y(0)=-1},y(x),type=numeric,method=classical[rk4],value=array([0.2,0.4,0.6,1.2]),stepsize=0.2);
```

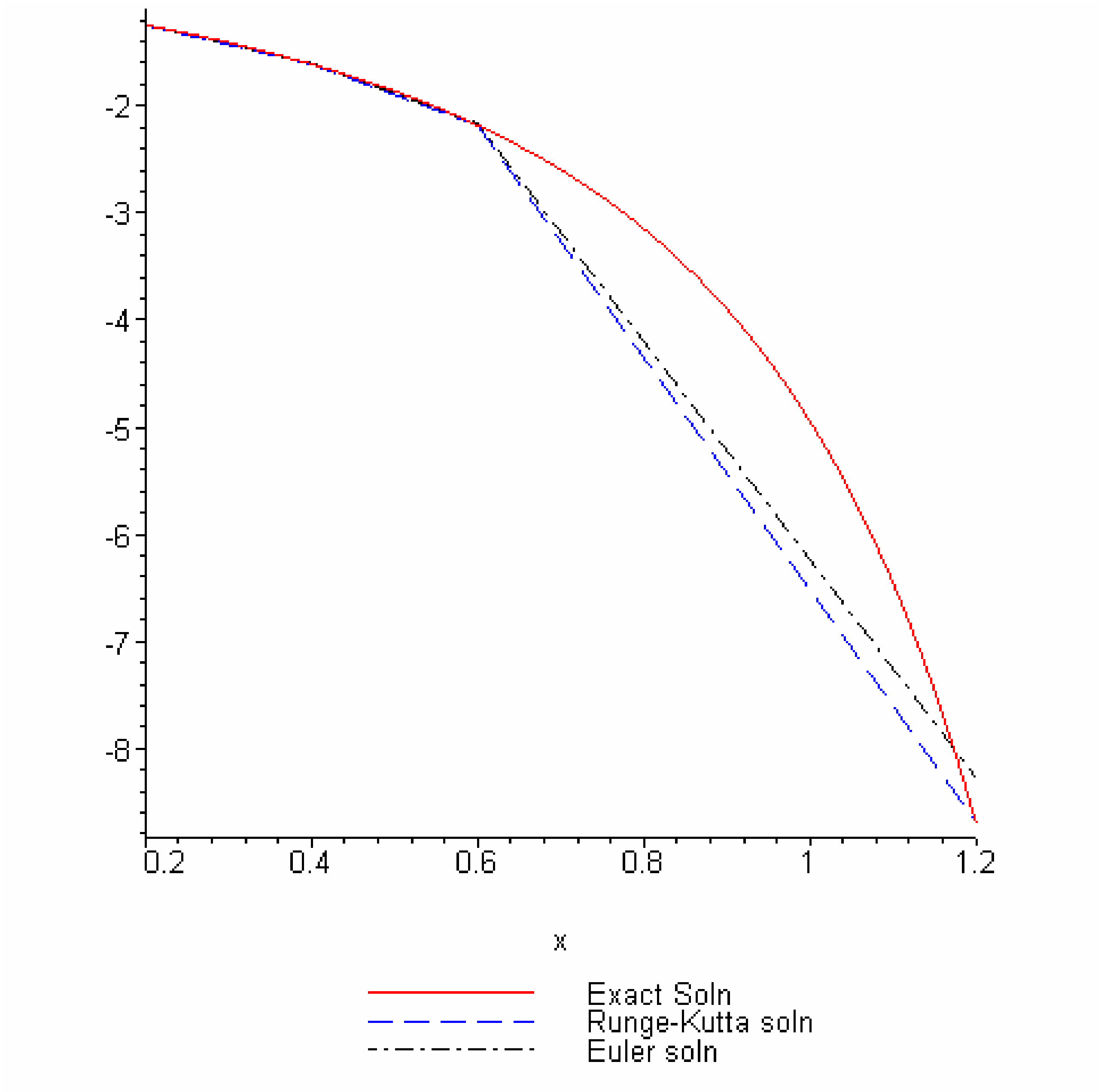
Warning, the 'value' option is deprecated, use 'output' instead

$$rk\_soln := \begin{bmatrix} [x, \mathrm{y}(x)] \\ \begin{bmatrix} 0.2 & -1.244989933554081 72 \\ 0.4 & -1.61065506915090206 \\ 0.6 & -2.18191431959185688 \\ 1.2 & -8.68499181916465446 \end{bmatrix} \end{bmatrix}$$

```
> with(plots):
```

Warning, the name changecoords has been redefined

```
> 
p_1:=plots[odeplot](e_soln,[x,y(x)],linestyle=4,color=bla
ck):
> 
p_2:=plots[odeplot](rk_soln,[x,y(x)],linestyle=6,color=bl
ue):
> p_3:=plot(rhs(soln),x=0.2..1.2):
> plots[display]({p_1,p_2,p_3});
```



17. We have

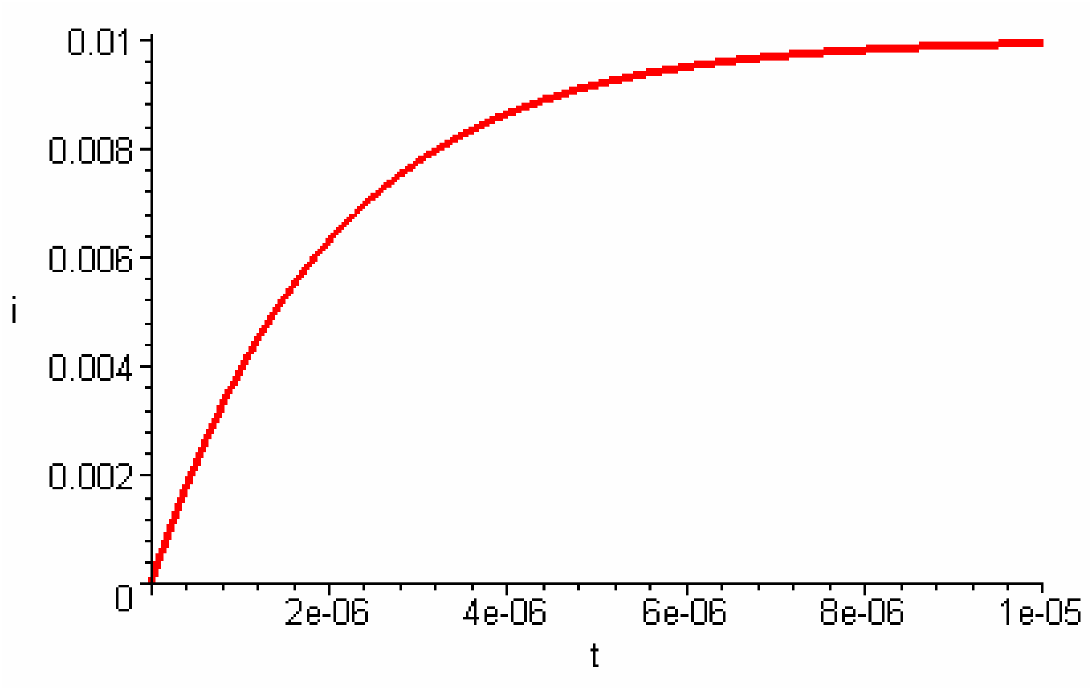
```
> de_17:=(2*10^(-3))*diff(i(t),t)+(1*10^3)*i(t)=10;
```

$$de\_17 := \frac{1}{500}\left(\frac{d}{dt}\,\mathrm{i}(t)\right) + 1000\,\mathrm{i}(t) = 10$$

```
> soln:=dsolve({de_17,i(0)=0},i(t));
```

$$soln := \mathrm{i}(t) = \frac{1}{100} - \frac{1}{100}\,\mathbf{e}^{(-500000\,t)}$$

```
> plot(rhs(soln),t=0..10*10^(-6),labels=['t','i']);
```



18. We have

```
> de_18:=(10*10^3)*C*diff(v(t),t)+v(t)=9;
```

$$de\_18 := 10000\, C \left(\frac{d}{dt} v(t) \right) + v(t) = 9$$

```
> sol:=dsolve({de_18,v(0)=0},v(t));
```

$$sol := v(t) = 9 - 9\,\mathbf{e}^{\left(-1/10000\,\frac{t}{C}\right)}$$

```
> C_vls:=seq(10*i*10^(-6),i=1..5);
```

$$C\_vls := \frac{1}{100000}, \frac{1}{50000}, \frac{3}{100000}, \frac{1}{25000}, \frac{1}{20000}$$

```
> solns:={seq(subs({C=C_vls},rhs(sol)),C=C_vls)};
```

$$solns := \{9 - 9\,\mathbf{e}^{(-10\,t)}, 9 - 9\,\mathbf{e}^{(-5\,t)}, 9 - 9\,\mathbf{e}^{(-5/2\,t)}, 9 - 9\,\mathbf{e}^{(-2\,t)}, 9 - 9\,\mathbf{e}^{(-10/3\,t)}\}$$

```
> f_1:=op(1,solns);
```

$$f\_1 := 9 - 9\,\mathbf{e}^{(-10\,t)}$$

```
> f_2:=op(2,solns);
```

$$f\_2 := 9 - 9\,\mathbf{e}^{(-5\,t)}$$

```
> f_3:=op(3,solns);
```

$$f\_3 := 9 - 9\,\mathbf{e}^{(-5/2\,t)}$$

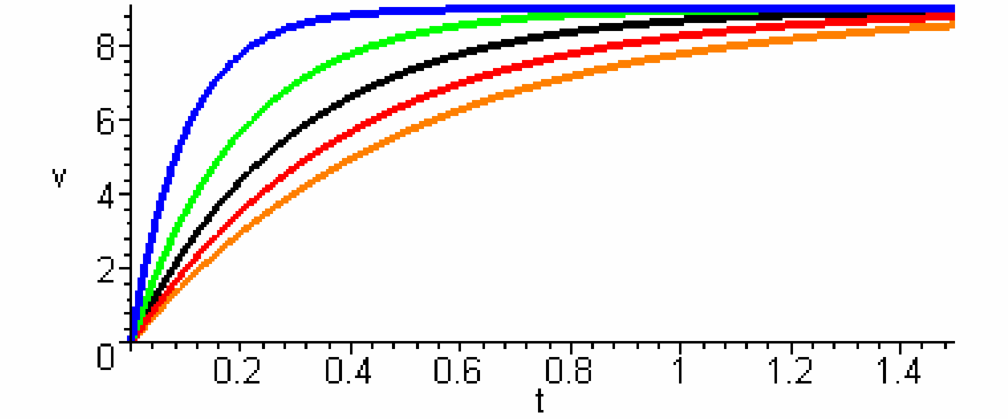
```
> f_4:=op(4,solns);
```

$$f\_4 := 9 - 9\,\mathbf{e}^{(-2\,t)}$$

```
> f_5:=op(5,solns);
```

$$f\_5 := 9 - 9\,\mathbf{e}^{(-10/3\,t)}$$

```
> plot({f_1,f_2,f_3,f_4,f_5},t=0..1.5,color=[blue,green,red,coral,black],labels=['t','v']);
```



(iv) As t gets large the voltage v tends to 9 volt, (the 'final value').
(v)The graphs show that the smaller the value of C the quicker the voltage (v) reaches its final value of 9 volt.

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19. The Maple solution is

```
> de:=(10*10^3)*i(t)+L*diff(i(t),t)=9;
```

$$de := 10000\ \mathrm{i}(t) + L\left(\frac{d}{dt}\mathrm{i}(t)\right) = 9$$

```
> sol:=dsolve({de,i(0)=0},i(t));
```

$$sol := \mathrm{i}(t) = \frac{9}{10000} - \frac{9}{10000}\,\mathrm{e}^{\left(-\frac{10000\,t}{L}\right)}$$

```
> L_vls:=seq(1/2^k,k=1..5);
```

$$L\_vls := \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \frac{1}{32}$$

```
> solns:={seq(subs({L=L_vls},rhs(sol)),L=L_vls)};
```

$$solns := \{\frac{9}{10000} - \frac{9}{10000}\,\mathrm{e}^{(-80000\,t)}, \frac{9}{10000} - \frac{9}{10000}\,\mathrm{e}^{(-20000\,t)}, \frac{9}{10000} - \frac{9}{10000}\,\mathrm{e}^{(-320000\,t)},$$
$$\frac{9}{10000} - \frac{9}{10000}\,\mathrm{e}^{(-160000\,t)}, \frac{9}{10000} - \frac{9}{10000}\,\mathrm{e}^{(-40000\,t)}\}$$

```
> p_1:=op(1,solns);
```

$$p\_1 := \frac{9}{10000} - \frac{9}{10000}\,\mathrm{e}^{(-80000\,t)}$$

```
> p_2:=op(2,solns);
```

$$p\_2 := \frac{9}{10000} - \frac{9}{10000} e^{(-20000\,t)}$$

```
> p_3:=op(3,solns);
```

$$p\_3 := \frac{9}{10000} - \frac{9}{10000} e^{(-320000\,t)}$$

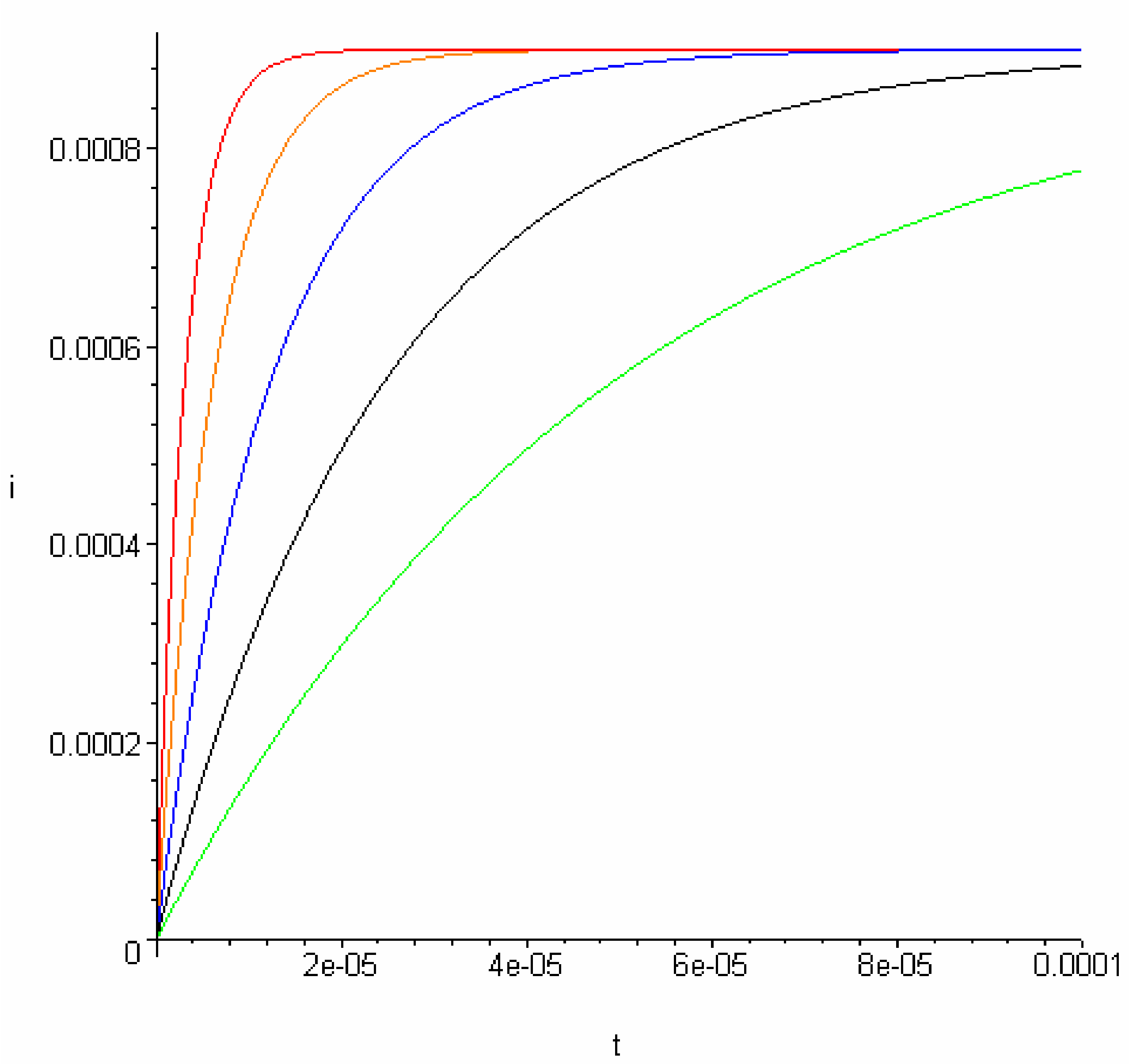
```
> p_4:=op(4,solns);
```

$$p\_4 := \frac{9}{10000} - \frac{9}{10000} e^{(-160000\,t)}$$

```
> p_5:=op(5,solns);
```

$$p\_5 := \frac{9}{10000} - \frac{9}{10000} e^{(-40000\,t)}$$

```
>
plot({p_1,p_2,p_3,p_4,p_5},t=0..0.0001,color=[blue,green,
red,coral,black],labels=['t','i']);
```



(iv) AS t gets larger i tends to 0.9 mA
(v) The graph shows that the larger the inductance value (L), the longer it takes the current (i) to reach its final value.