Complete solutions to Exercise 16(b)

1. To find the median we need to place the data into some order.

5, 8, 12, 15, 19, 27, 29, 34, 36, 39, 50

Hence median =27. By calculator mean $= 24.91$ (2 d.p.) and s.d. $= 13.61$ (2 d.p.)

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2. (a) By calculator mean=15.2, s.d.=11.46 (2 d.p.)

(b) Notice the data has been shifted by 100 from part (a) so the standard deviation is the same 11.46 but mean 115.2

(c) Mean 1.52, standard deviation 1.146 (3 d.p.)

(d) Mean 155, standard deviation 114.6 (1 d.p.)

The data of (b), (c) and (d) has been shifted or multiplied by some factor of (a).

The data of (d) is obtained by: Multiplying (a) by 10 and then adding 3.

The mean of (d) is obtained by multiplying by 10 and adding 3 to the mean of (a).

If every term of the data has been shifted by 3, the S.D. will not change, but the multiplication changes the S.D.

S.D. of (d) = S.D. of (a) $\times 10$

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3. We have $y_j = kx_j$. By (16.1)

$$\bar{y} = \frac{y_1 + y_2 + \ldots + y_n}{n}$$

$$= \frac{kx_1 + kx_2 + \ldots + kx_n}{n} = k\left(\frac{x_1 + x_2 + \ldots + x_n}{n}\right) = k\underbrace{\bar{x}}_{\text{by (16.1)}}$$

By (16.2) we have

$$s_y^2 = \frac{\sum_{j=1}^{n}\left(y_j - \bar{y}\right)^2}{n}$$

$$= \frac{\sum_{j=1}^{n}\left(kx_j - k\bar{x}\right)^2}{n} = k^2\frac{\sum_{j=1}^{n}\left(x_j - \bar{x}\right)^2}{n} = k^2\underbrace{s_x^2}_{\text{by (16.2)}}$$

Hence $s_y = \sqrt{k^2}\, s_x = |k| s_x$.

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4. By (16.1) we have

$$\bar{y} = \frac{y_1 + y_2 + \ldots + y_n}{n}$$

$$= \frac{\left(kx_1 + a\right) + \left(kx_2 + a\right) + \ldots + \left(kx_n + a\right)}{n}$$

$$= \frac{k\left(x_1 + x_2 + \ldots + x_n\right) + \left(a + a + \ldots + a\right)}{n} = k\left(\frac{x_1 + x_2 + \ldots + x_n}{n}\right) + \frac{na}{n}$$

$$\bar{y} = k\underbrace{\bar{x}}_{\text{by (16.1)}} + a$$

By (16.2)

(16.1) $$\bar{x} = \frac{x_1 + x_2 + \ldots + x_n}{n}$$

$$s_y^2 = \frac{\sum_{j=1}^{n}\left(y_j - \bar{y}\right)^2}{n}$$

$$= \frac{\sum_{j=1}^{n}\left[\left(kx_j + a\right) - \underbrace{\left(k\bar{x} + a\right)}_{\text{by part (i)}}\right]^2}{n}$$

$$= \frac{\sum_{j=1}^{n}\left[kx_j - k\bar{x} + a - a\right]^2}{n}$$

$$= \frac{\sum_{j=1}^{n}\left[kx_j - k\bar{x}\right]^2}{n} = \frac{k^2\sum_{j=1}^{n}\left(x_j - \bar{x}\right)^2}{n}$$

$$s_y^2 = k^2 s_x^2$$

$$s_y = \sqrt{k^2}s_x = |k|s_x$$

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5. All working are to 4 s.f. (i) Let $\bar{x}$ and s be the mean and standard deviation respectively for data of part (i). Then by using a calculator we obtain $\bar{x} = 69.29$ and $s = 23.68$.

(ii) Data of part (ii) =10 × part (i). By question 3,

$$\text{Mean} = 10 \times 69.29 = 692.9$$
$$\text{S.D.} = 10 \times 23.68 = 236.8$$

(iii) Data of part (iii) $= \dfrac{\text{data of part (i)}}{100}$. Similarly by question 3 or

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$$\text{Mean} = \frac{69.29}{100} = 0.6929$$
$$\text{S.D.} = \frac{23.68}{100} = 0.2368$$

(iv) Data of part (iv) = part (i) + 5. By question 4 with $k = 1$ and $a = 5$

$$\text{Mean} = \bar{x} + 5 = 69.29 + 5 = 74.29$$
$$\text{S.D.} = 23.68$$

(v) Data of part (v)= $\left[100 \times \text{part (i)}\right] + 100$. By question 4 with $k = 100$, $a = 100$

$$\text{Mean} = \left(100 \times 69.29\right) + 100 = 7029$$
$$\text{S.D.} = \left(100 \times 23.68\right) = 2368$$

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6. By calculator mean is $159.932kN$ (3 d.p.) and S.D. is $4.656kN$ (3 d.p.).

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7. (i) and (ii). The mean is approximately $10k\Omega$. We need to use the mid-point value for resistance. We expect a small S.D. because the data are quite close to $10k\Omega$.

(iii)

Resistance R ($k\Omega$)	Mid-point	Frequency
$9.6 \le R < 9.7$	9.65	1
$9.7 \le R < 9.8$	9.75	2
$9.8 \le R < 9.9$	9.85	5
$9.9 \le R < 10.0$	9.95	17
$10.0 \le R < 10.1$	10.05	18
$10.1 \le R < 10.2$	10.15	5
$10.2 \le R < 10.3$	10.25	1
$10.3 \le R < 10.4$	10.35	1

(iii) We use a calculator which gives

Mean $= 9.996k\Omega$ S.D.$= 0.122k\Omega$

8. (a) Similarly to solution 7.

Tensile strength $\left(MN/m^2\right)$	Mid-point	Frequency
$320 \le T < 350$	335	4
$350 \le T < 380$	365	12
$380 \le T < 410$	395	18
$410 \le T < 440$	425	16
$440 \le T < 470$	455	15
$470 \le T < 500$	485	10
$500 \le T < 530$	515	3
$530 \le T < 560$	545	7

Calculator gives mean$= 431.35\,MN/m^2$ (2 d.p.), S.D. $= 55.62\,MN/m^2$ (2 d.p.)

(b) Use calculator

mean$= 196.29\,GN/m^2$ (2 d.p.)

S.D. $= 1.86\,GN/m^2$ (2 d.p.)