Complete solutions to Exercise 16(c)

1. (a)

$$P(\text{Not faulty resistor}) \underbrace{=}_{\text{by (16.8)}} 1-P(\text{faulty resistor})=1-0.1=0.9$$

(b) $P(\text{All three components are faulty})$

$$=P([\text{resistor is faulty}] \text{ and } [\text{inductor is faulty}] \text{ and } [\text{capacitor is faulty}])$$

$$=P(\text{resistor is faulty})\times P(\text{inductor is faulty})\times P(\text{capacitor is faulty})$$

$$=\frac{10}{100}\times\frac{5}{100}\times\frac{3}{100}=\frac{3}{20\,000}$$

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2. Let D denote defective

$$P(\text{All three } D)=P(D \text{ and } D \text{ and } D)$$
$$=P(D)\times P(D)\times P(D)$$
$$=\frac{2}{100}\times\frac{2}{100}\times\frac{2}{100}=8\times10^{-6}$$

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3. We have

$$P(\text{Connection between } X \text{ and } \text{Y})=P(A \text{ or } B)$$
$$=P(A)+P(B)-P(A)\times P(B)$$
$$=0.8+0.65-(0.8\times0.65)=0.93$$

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4. (i) $P(L\le 200)=\dfrac{3000}{10\,000}=0.3$

(ii) Note that all these events are mutually exclusive.

$$P(L\le 500)=P(0<L\le 200 \text{ or } 200<L\le 500)$$
$$=P(0<L\le 200)+P(200<L\le 500)$$
$$=0.3+0.35=0.65$$

(iii) $P(L>500) \underbrace{=}_{\text{by (16.8)}} 1-P(L\le 500)=1-0.65=0.35$

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5. Let M and L denote a calculator not having a memory button and log button respectively.

(a) We need to find $P(M \text{ or } L)$:

$$P(M \text{ or } L)=P(M)+P(L)-P(M \text{ and } L)$$
$$=\frac{5}{100}+\frac{3}{100}-\frac{1}{100}=\frac{7}{100}$$

(b) We use De Morgan's law

$$P([\text{not } M] \text{ and } [\text{not } L]) \underbrace{=}_{\text{by (16.15)}} P[\text{not } (M \text{ or } L)]$$

$$=1-P(M \text{ or } L)=1-\frac{7}{100}=\frac{93}{100}$$

(16.8) $$P(\text{not } A)=1-P(A)$$

6. Let A and B denote that system A and system B work respectively. Since the communication network fails if both systems fail we have

$$P([\text{not } A] \text{ and } [\text{not } B]) = P([\text{not } A]) \times P([\text{not } B])$$
$$= (1-0.88)\times(1-0.93) = 8.4\times10^{-3}$$

7. Let A denote the first component is good and B denote the second component is defective.

$$P(A) = \frac{46}{50}$$
$$P(B) = \frac{4}{49}$$

8. (i) Let A, B and C represent parts A, B and C being defective respectively.

$$P(\text{no defective part}) = P([\text{not } A] \text{ and } [\text{not } B] \text{ and } [\text{not } C])$$
$$= P(\text{not } A)\times P(\text{not } B)\times P(\text{not } C)$$
$$= (1-0.15)(1-0.25)(1-0.02) = 0.62475$$

(ii) We have

$$P(\text{At least one defective part}) \underbrace{=}_{\text{by (16.17)}} 1 - P(\text{no defective part})$$
$$= 1 - 0.62475 = 0.37525$$

9. Let A, B and C denote A, B and C work respectively. Remember A, B and C are independent events

$$P(\text{connection}) = P([A \text{ or } B] \text{ and } C) = P(A \text{ or } B)\times P(C)$$
$$P(A \text{ or } B) = P(A) + P(B) - P(A)\times P(B)$$
$$= 0.7 + 0.75 - (0.7\times 0.75) = 0.925$$
$$P(\text{connection}) = 0.925\times 0.85 = 0.786 \quad \text{(3 d.p.)}$$

10. $$P(\text{At least one working}) \underbrace{=}_{\text{by (16.17)}} 1 - P(\text{none working}) \qquad (*)$$

$$P(\text{none working}) = P([\text{not } A] \text{ and } [\text{not } B] \text{ and } [\text{not } C])$$
$$= P(\text{not } A)\times P(\text{not } B)\times P(\text{not } C)$$
$$= (1-0.6)(1-0.75)(1-0.55) = 0.045$$

Substituting $P(\text{none working}) = 0.045$ into (*) gives

$$P(\text{At least one working}) = 1 - 0.045 = 0.955$$

11. $$P(\text{second component defective}) = \frac{3}{59}$$

(16.17) $$P(\text{at least } n \text{ successes}) = 1 - P(\text{less than } n \text{ successes})$$

12. Let D denote defective

(a) We have

$$P(\text{Both defective}) = P(D \text{ and } D) = P(D)\times P(D) = \frac{5}{100}\times\frac{4}{99} = \frac{1}{495}$$

(b)

$$P(\text{None defective}) = P([\text{not } D] \text{ and } [\text{not } D]) = P(\text{not } D)\times P(\text{not } D) = \frac{95}{100}\times\frac{94}{99} = \frac{893}{990}$$

(c)

$$P(\text{Only one is defective}) = P([\text{first } D] \text{ and } [\text{second not } D]) \text{ or } P([\text{first not } D] \text{ and } [\text{second } D]) = \left(\frac{5}{100}\times\frac{95}{99}\right)+\left(\frac{95}{100}\times\frac{5}{99}\right) = \frac{19}{198}$$

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13. Very similar to **EXAMPLE 16**.

$$P(B) = \frac{P(A \text{ and } B)}{P(A)} = \frac{0.88}{0.92} = 0.957 \text{ (3 d.p.)}$$

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14. Since choosing a component is an independent event, we have
5% of $120 = 6$. So 6 components are faulty.

$$P\text{(first faulty)} = \frac{6}{120}, \qquad P\text{(second faulty)} = \frac{5}{119}$$

Hence

$$P(\text{second faulty}) = \frac{5}{119}$$

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15. Let A, B and C denote defects of type I, II and III respectively.

(a) *What do we need to find?*

$$P(A) = \frac{P(A \text{ and } B)}{P(B)} = \frac{3/150}{4/150} = \frac{3}{4}$$

(b) We have

$$P(C) = \frac{P(C \text{ and } B)}{P(B)} = \frac{2/150}{4/150} = \frac{2}{4} = \frac{1}{2}$$

(c) We have

$$P(A \text{ and } C) = \frac{P(A \text{ and } B \text{ and } C)}{P(B)} = \frac{1/150}{4/150} = \frac{1}{4}$$

16.

$$P\,(\text{both defective}) = \frac{7}{96} \times \frac{6}{95} = \frac{7}{1520}$$

17. Let F and S denote the first and second engine fail respectively.

$$P\,(\text{at least one engine working}) = 1 - P\,(\text{no engine working})$$

Probability of both engines fail is given by

$$P(F \text{ and } S) = P(S) \times P(F) \qquad (*)$$

$$P(F) = 1 - 0.85 = 0.15$$

Substituting $P(F) = 0.15$ and $P(S) = 0.15$ into (*) gives

$$P(F \text{ and } S) = 0.15 \times 0.15 = 0.0225$$

Hence

$$P\,(\text{at least one engine working}) = 1 - 0.0225 = 0.9775$$

18. We have

$$P([A \text{ and } B] \text{ or } [C \text{ and } D]) = [P(A) \times P(B)] + [P(C) \times P(D)] - [P(A) \times P(B) \times P(C) \times P(D)]$$

$$= (0.7 \times 0.8) + (0.77 \times 0.95) - (0.7 \times 0.8 \times 0.77 \times 0.95) = 0.88186$$