Complete solutions to Exercise 16(d)

1. To find the probability that 3 are faulty means the rest (4) are good. So we choose 3 out of 10 and 4 out of 50. We have ${}^{10}C_3 \times {}^{50}C_4$

The total number of selections ${}^{60}C_7$. Hence

$$P(3\text{ faulty}) = \frac{{}^{10}C_3 \times {}^{50}C_4}{{}^{60}C_7} = 0.0716 \ \ (3\text{ s.f.})$$

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2. The number of selections ${}^{60}C_8$.

(i) For all 8 to be good, we need to choose 8 from $51(60-9)$, given by ${}^{51}C_8$. So

$$P(\text{all 8 are good}) = \frac{{}^{51}C_8}{{}^{60}C_8} = 0.249 \quad (3\text{ s.f.})$$

(ii) For all 8 to be defective, we need to choose 8 from 9, ${}^{9}C_8$.

$$P(\text{all 8 are defective}) = \frac{{}^{9}C_8}{{}^{60}C_8} = 3.52\times 10^{-9} \ \ (3\text{ s.f.})$$

(iii)

$$\begin{aligned} P(\text{at least one defective}) &\underbrace{=}_{\text{by (16.17)}} 1 - P(\text{none defective}) \\ &= 1 - P(\text{all good}) \\ &= 1 - \underbrace{0.249}_{\text{by part (i)}} = 0.751\,(3\text{ s.f.}) \end{aligned}$$

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3. There are ${}^{40}C_6$ possible selections.

(a) $P(\text{winning with 150 tickets}) = \dfrac{150}{{}^{40}C_6} = 3.908\times 10^{-5}$ (4 s.f.)

(b) Let £x be the amount you need to spend.

$$\frac{x}{{}^{40}C_6} = 0.5$$

$$x = 0.5 \times {}^{40}C_6 = 1\,919\,190$$

You need to spend more than £1 919 190 to obtain a probability of more than 0.5 of winning the jackpot.

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4. (a) Order does matter. Choosing the first letter to be A or B gives two different number plates. So choosing 1 letter out of 26 can be done in ${}^{26}P_1$ ways (or just 26 ways). Choosing 3 digits out of 10 can be done in ${}^{10}P_3$ ways and choosing 3 letters out of 25 can be done in ${}^{25}P_3$ ways.

The number of plates $= {}^{26}P_1 \times {}^{10}P_3 \times {}^{25}P_3 = 258\,336\,000$

(b) The first letter can be chosen in 26 ways. For the first number we have 10 choices, how many choices do we have for the second number?

Also 10 choices because we can repeat. For all 3 numbers we have 10^3 choices. Similarly $26\times 26\times 26 = 26^3$ choices for the last 3 letters.

$$\begin{aligned} \text{Number of plates} &= 26 \times (10\times 10\times 10)\times 26^3 \\ &= 26^4 \times 10^3 = 456\,976\,000 \end{aligned}$$

(16.17) $\quad P(\text{at least } n \text{ successes}) = 1 - P(\text{less than } n \text{ successes})$