Complete solutions to Exercise 16(e)

1. Let X be the number of resistor out of tolerance.
(a) Need to find $P(X=3)$ in the binomial distribution, (16.24), with $p=0.1$, $q=0.9$ and $n=5$.

$$P(X=3)={}^5C_3(0.1)^3(0.9)^2=8.1\times10^{-3}$$

(b) Need to find

$$\begin{aligned}P(X\le3)&=1-P(X>3)\\&=1-\left[P(X=4)+P(X=5)\right]\qquad(\dagger)\end{aligned}$$

By (16.24) we have

$$P(X=4)={}^5C_4(0.1)^4(0.9)=4.5\times10^{-4}$$
$$P(X=5)=0.1^5=1\times10^{-5}$$

Substituting $P(X=4)=4.5\times10^{-4}$ and $P(X=5)=1\times10^{-5}$ into (†) gives

$$P(X\le3)=1-\left[\left(4.5\times10^{-4}\right)+\left(1\times10^{-5}\right)\right]=0.99954$$

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2. Let X be the number of components that fail.
(a) Using (16.24) with $p=\dfrac{1}{25}=0.04$, $q=1-0.04=0.96$, $n=5$ and $x=1$ gives

$$P(X=1)={}^5C_1\left(0.04\times0.96^4\right)=0.170\ \text{(3 s.f.)}$$

(b) In this case we have $X>3$.

$$\begin{aligned}P(X>3)&=P(X=4)+P(X=5)\\&={}^5C_4(0.04)^4(0.96)+(0.04)^5\\&=1.239\times10^{-5}\ \text{(4 s.f.)}\end{aligned}$$

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3. Let X be the number of integrated circuits that fail. Using (16.24) with $p=1-0.86=0.14$, $q=0.86$ and $n=10$ ($p\neq0.86$ because X is the number of failures).
(i) We have

$$\begin{aligned}P(2\text{ will fail})&=P(X=2)\\&={}^{10}C_2(0.14)^2(0.86)^8=0.264\ \text{(3 d.p.)}\end{aligned}$$

(ii) The probability of 8 will pass is the same as the probability of 2 failing, which is 0.264
(iii)

$$\begin{aligned}P(\text{at least 2 will fail})&=1-P(X\le1)\\&=1-\left[P(X=0)+P(X=1)\right]\\&=1-\left[0.86^{10}+{}^{10}C_1(0.14)(0.86)^9\right]\\&=1-0.5816=0.418\ \text{(3 d.p.)}\end{aligned}$$

(16.24) $$P(X=x)={}^nC_x\,p^xq^{n-x}$$

(iv) We have

$$P(\text{less than 2 will fail}) = P(X=0)+P(X=1)$$
$$= 0.582 \quad (3 \text{ d.p.})$$

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4. Let X be the number of boys.

$$P(\text{having more boys}) = P(X=4)+P(X=5)+P(X=6)+P(X=7)$$

Using (16.24) with $n=7$, $p=0.48$ and $q=1-0.48=0.52$ we have

$$P(\text{having more boys}) = {}^7C_4(0.48)^4(0.52)^3 + {}^7C_5(0.48)^5(0.52)^2 + {}^7C_6(0.48)^6(0.52) + (0.48)^7$$
$$= 0.456 \ (3 \text{ d.p.})$$

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5. We have

$$P(\text{passes more than 6 subjects}) = P(\text{passes 7 subjects}) + P(\text{passes 8 subjects})$$

Binomial distribution with $p=0.85$, $q=0.15$ and $n=8$

$$P(\text{passes 7 subjects}) = {}^8C_7(0.85)^7(0.15) = 0.3847$$
$$P(\text{passes 8 subjects}) = 0.85^8 = 0.2725$$

Hence

$$P(\text{passes more than 6 subjects}) = 0.3847 + 0.2725 = 0.6572 \ (4 \text{ d.p.})$$

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6. (a) Using (16.24) with $n=600$, $p=1\times 10^{-3}$ and $q=1-\left(1\times 10^{-3}\right)=0.999$.

(All solutions correct to 4 d.p.)

(i) $P(X=1) = {}^{600}C_1\left(1\times 10^{-3}\right)(0.999)^{599} = 0.3295$

(ii) $P(X=2) = {}^{600}C_2\left(1\times 10^{-3}\right)^2(0.999)^{598} = 0.0988$

(iii) $P(X=3) = {}^{600}C_3\left(1\times 10^{-3}\right)^3(0.999)^{597} = 0.0197$

(b) We have $nP = 600\times 1\times 10^{-3} = 0.6$. Using $P(X=x)=\dfrac{e^{-0.6}(0.6)^x}{x!}$ gives

(i) $P(X=1)=\dfrac{e^{-0.6}(0.6)^1}{1!}=0.3293$

(ii) $P(X=2)=\dfrac{e^{-0.6}(0.6)^2}{2!}=0.0988$

(iii) $P(X=3)=\dfrac{e^{-0.6}(0.6)^3}{3!}=0.0198$

The answers are more or less the same.

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7. Let X represent England player scores a penalty. Then $p=0.85$, $q=0.15$ and $n=5$. We need to find $P(X=4)+P(X=5)$

$$P(X=4)+P(X=5) = {}^5C_4(0.85)^4(0.15)+(0.85)^5$$
$$=0.83521$$

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8. Substituting $x=0, 1$ and 2 into $P(X=x)=kx^4$ gives

(16.24) $$P(X=x) = {}^nC_x p^x q^{n-x}$$

$$P(X=0)=0$$
$$P(X=1)=k$$
$$P(X=2)=2^4 k=16k$$

By (16.23)

$$16k+k=1 \text{ which gives } k=\frac{1}{17}$$

9. Similarly

$$P(X=0)=k.0^3=0$$
$$P(X=1)=k$$
$$P(X=2)=2^3 k=8k$$
$$P(X=3)=3^3 k=27k$$

By (16.23)

$$27k+8k+k=1$$
$$36k=1 \text{ which gives } k=\frac{1}{36}$$

10. We have

$$\sum_{x=1}^{n} x\,P(X=x)=\left[0\times P(X=0)\right]+\left[1\times P(X=1)\right]+\left[2\times P(X=2)\right]+\left[3\times P(X=3)\right]+\ldots+\left[n\times P(X=n)\right]$$

$$\underbrace{=}_{\text{by (16.24)}} 0+npq^{n-1}+\frac{2n(n-1)}{2!}p^2q^{n-2}+\frac{3n(n-1)(n-2)}{3!}p^3q^{n-3}+\ldots+np^n$$

$$=npq^{n-1}+n(n-1)p^2q^{n-2}+\frac{n(n-1)(n-2)}{2}p^3q^{n-3}+\ldots+np^n$$

$$=np\left[q^{n-1}+(n-1)pq^{n-2}+\frac{(n-1)(n-2)}{2!}p^2q^{n-3}+\ldots+p^{n-1}\right]$$

$$=np\underbrace{\left[(q+p)^{n-1}\right]}_{\text{by (7.23)}}$$

$$=np \text{ because } q+p=1 \text{ and } 1^{n-1}=1$$

(7.23) $$(a+b)^n=a^n+na^{n-1}b+\frac{n(n-1)}{2!}a^{n-2}b^2+\ldots+b^n$$

(16.24) $$P(X=x)=\frac{n!}{x!(n-x)!}p^xq^{n-x}$$