Complete solutions to Exercise 7(i)

1. (a) $r_1 = -2$; Let $f(x) = x^3 - 2x + 7$, $f'(x) = 3x^2 - 2$. So by applying (7.29) we have:

$$r_2 = -2 - \frac{f(-2)}{f'(-2)} = -2 - \frac{3}{10} = -2.3$$

$$r_3 = -2.3 - \frac{f(-2.3)}{f'(-2.3)} = -2.260$$

$$r_4 = -2.26 - \frac{f(-2.26)}{f'(-2.26)} = -2.258 = -2.26 \qquad (2 \text{ d.p.})$$

Since $r_3 = r_4$, the root of $x^3 - 2x + 7 = 0$ is -2.26 (2 d.p.).

(b) $r_1 = -2$; Let $f(x) = x^4 - 3x^2 - 2$, $f'(x) = 4x^3 - 6x$. Applying (7.29):

$$r_2 = -2 - \frac{f(-2)}{f'(-2)} = -1.90$$

$$r_3 = -1.9 - \frac{f(-1.9)}{f'(-1.9)} = -1.887$$

$$r_4 = -1.887 - \frac{f(-1.887)}{f'(-1.887)} = -1.887$$

Since $r_3 = r_4$ a root of $x^4 - 3x^2 - 2 = 0$ is -1.89 correct to two d.p. (Although -1.887 is correct to three d.p.).

(c) $r_1 = -2$; Let $f(x) = e^x - 2x - 5$, $f'(x) = e^x - 2$:

$$r_2 \underbrace{=}_{\text{by (7.25)}} -2 - \frac{f(-2)}{f'(-2)} = -2 - \left(\frac{-0.865}{-1.865}\right) = -2.464$$

$$r_3 \underbrace{=}_{\text{by (7.25)}} -2.464 - \frac{f(-2.464)}{f'(-2.464)} = -2.464 - \left(\frac{0.013}{-1.915}\right) = -2.457$$

$r_2 = -2.46$ correct to two d.p. and $r_3 = -2.46$ correct to two d.p. To the required accuracy $r_2 = r_3$ so a root of $e^x - 2x - 5 = 0$ is -2.46 (2 d.p.).

2. Let

$$f(\lambda) = \lambda^3 + 28\lambda^2 + 231\lambda + 541$$

$$f'(\lambda) = 3\lambda^2 + 56\lambda + 231$$

For the first root (close to -4), take $r_1 = -4$:

$$r_2 \underbrace{=}_{\text{by (7.25)}} -4 - \frac{f(-4)}{f'(-4)} = -4.0182$$

$$r_3 \underbrace{=}_{\text{by (7.25)}} -4.0182 - \frac{f(-4.0182)}{f'(-4.0182)} = -4.0183$$

$-4.018 = r_2 = r_3$ correct to three d.p., so the root close to -4 is -4.018.

For the 2nd root (close to -9) take $r_1 = -9$:

(7.29) $$r_{n+1} = r_n - \frac{f(r_n)}{f'(r_n)}$$

$$r_2 = -9 - \frac{f(-9)}{f'(-9)} = -8.9667$$

$$r_3 = -8.9667 - \frac{f(-8.9667)}{f'(-8.9667)} = -8.9666 = -8.967 \ (\text{to 3 d.p.})$$

$r_2 = r_3 = -8.967$ (to three d.p.), so the root close to -9 is -8.967.
The third root (close to -15), take $r_1 = -15$:

$$r_2 = -15 - \frac{f(-15)}{f'(-15)} = -15.0151$$

$$r_3 = -15.0151 - \frac{f(-15.0151)}{f'(-15.0151)} = -15.0151$$

$r_2 = r_3$, so the root close to -15 is -15.015 correct to three d.p.
All three roots correct to three d.p. are

$$-4.018, \ -8.967 \text{ and } \ -15.015.$$

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3. Take $r_1 = 10$ and let

$$f(v) = v^3 - 6v^2 - 348v + 3112 \qquad f'(v) = 3v^2 - 12v - 348$$

By (7.29)

$$r_2 = 10 - \frac{f(10)}{f'(10)} = 10 - \frac{32}{-168} = 10.1905$$

$$r_3 = 10.19 - \frac{f(10.19)}{f'(10.19)} = 10.1960$$

$$r_4 = 10.196 - \frac{f(10.196)}{f'(10.196)} = 10.1960$$

Since $r_3 = r_4$, so $v = 10.196 \text{ m / s}$, correct to three d.p.

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4. Let $f(x) = x^3 - 3x^2 + 2x - 1$, so we need to solve the equation $f(x) = 0$ because $\frac{WL}{EI} \neq 0$.

Try some obvious values of x: $x = 1, \ 2$ and 3

$x = 1; \quad f(1) = 1 - 3 + 2 - 1 = -1$

$x = 2; \quad f(2) = 2^3 - 3.(2)^2 + (2 \times 2) - 1 = -1$

$x = 3; \quad f(3) = 3^3 - 3.(3)^2 + (2 \times 3) - 1 = 5$

We know there is a root between $x = 2$ and $x = 3$ because $f(x)$ goes from negative to positive. Since -1 is closer to zero, try $r_1 = 2$.

$$f(x) = x^3 - 3x^2 + 2x - 1$$

$$f'(x) = 3x^2 - 6x + 2$$

By repeated use of (7.29) we have

(7.29) $\qquad r_{n+1} = r_n - \frac{f(r_n)}{f'(r_n)}$

$$r_2 = 2 - \frac{f(2)}{f'(2)} = 2.5$$

$$r_3 = 2.5 - \frac{f(2.5)}{f'(2.5)} = 2.348$$

$$r_4 = 2.348 - \frac{f(2.348)}{f'(2.348)} = 2.325$$

$$r_5 = 2.325 - \frac{f(2.325)}{f'(2.325)} = 2.325$$

Since $r_4 = r_5$, the distance along the beam, where there is zero deflection is $2.325m$ (3 d.p.).